ENGINEERING DESIGN HANDBOOK

FIRE CONTROL SERIES

SECTION 1

FIRE CONTROL SYSTEMS—GENERAL

HEADQUARTERS, U.S. ARMY MATERIEL COMMAND JANUARY 1968

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ENGINEERING DESIGN HANDBOOK

FIRE CONTROL SERIES
SECTION 1, FIRE CONTROL SYSTEMS--GENERAL

This pamphlet is published for the information and guidance of all concerned.

(AMCRD-R)

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FOREWORD

INTRODUCTION

The Fire Control Series forms part of the Engineering Design Handbook Series which presents engineering information and quantitative data for the design and construction of Army equipment. In particular, the handbooks of the Fire Control Series have been prepared to aid the designers of Army fire control equipment and systems, and to serve as a reference guide for all military and civilian personnel who may be interested in the design aspects of such material.

The handbooks of the Fire Control Series are based on the fundamental parameters of the fire control problem and its solution. In all problems of control over the accuracy of weapon fire, some method or system of fire control is employed that derives its intelligence from the acquisition and tracking of a target; evaluates this system-input intelligence by computation; and, finally, applies the output information to the positioning of a weapon along the line of fire. Primary emphasis is laid on the systematic approach required in the design of present-day fire control equipment and systems. This approach involves (1) thorough analysis of the particular fire control problem at hand, (2) establishment of the most suitable mathematical model, and (3) mechanization of this mathematical model.

ORGANIZATIONAL BREAKDOWN

To accomplish the aforenoted objectives, the Fire Control Series will consist primarily of the following four main sections, each published as a separate handbook:

a. Section 1, Fire Control Systems – General (AMCP 706-327)
b. Section 2, Acquisition and Tracking Systems (AMCP 706-328)
c. Section 3, Fire Control Computing Systems (AMCP 706-329)
d. Section 4, Weapon-Pointing Systems (AMCP 706-330)

An additional handbook of the Fire Control Series is AMC Pamphlet AMCP 706-331, Compensating Elements. The following paragraphs summarize the content of each of these five handbooks.

Section 1 introduces the subject of fire control systems, discloses the basic fire control problem and its solution (in functional terms), delineates system-design philosophy, and discusses the application of maintenance and human engineering principles and standard design practices to fire control system design.

Section 2 is devoted to the first aspect of fire control, i.e., gathering intelligence on target position and motion.

Section 3, because of the complexity of the subject of computing systems, is divided into three parts that are preceded by an introductory discussion of the roles of computing systems in Army fire control and by a description of specific roles played in particular fire-control applications. Part I discusses the first step in system design, i.e., the establishment of a mathematical model for the solution of a fire control problem. Emphasis is given to the basis, derivation, and manipulation of mathematical models. Part II discusses the various computing devices that perform useful functions in fire control computing systems. The discussion ranges from simple mechanical linkages to complex digital computers. Types of devices in each classification are briefly described; external sources are referenced for detailed information where practical. Part III discusses the various ways in which the computing devices described in Part II can be applied to the mechanization of the mathematical models described in Part I., It stresses that a fire control computing system designer needs to apply his talents in three special ways: (1) to improvise and innovate as needed to meet particular problems that may arise, (2) to use ingenuity in obtaining the simplest and most economical devices for the particular requirement at hand, and (3) to master the many problems that result from intrasystem

interactions when individually satisfactory components are combined in complex computing systems. Examples culled from actual fire-control-system design work illustrate the concepts given.

Section 4 of the Fire Control Series discusses weapon-pointing systems with respect to (1) input intelligence and its derivation, (2) the means of implementing weapon-pointing for the two basic types of weapon-pointing systems from the standpoint of system stability, (3) general design considerations, and (4) the integration of components that form a complete fire control system.

AMCP 706-331 presents information on: (1) the effects of out-of-level conditions and a displacement between a weapon and its aiming device, and (2) the instrumentation necessary to correct the resulting errors. It also presents general reference information on compensating elements that pertains to accuracy considerations, standard design practices; and considerations of general design, manufacture, field use, maintenance, and storage.

PREPARATION

The handbooks of the Fire Control Series have been prepared under the direction of the Engineering Handbook Office, Duke University, under contract to the Army Research Office-Durham. With the exception of the handbook titled Compensating Elements, the material for the Fire Control Series was prepared by the Jackson & Moreland Division of United Engineers and Constructors Inc., Boston, Massachusetts under subcontract to the Engineering Handbook Office. The Jackson & Moreland Division was assisted in its work by consultants who are recognized authorities in various aspects of fire control. Specific authorship is indicated where appropriate. Overall technical guidance and assistance were rendered by Frankford Arsenal; coordination and direction of this effort were provided by Mr. Leon G. Pancoast of the Fire Control Development & Engineering Laboratories at Frankford Arsenal.

PREFACE

The Engineering Design Handbook Series of the Army Materiel Command is a coordinated series of handbooks containing basic information and fundamental data useful in the design and development of Army materiel and systems. The handbooks are authoritative reference books of practical information and quantitative facts helpful in the design and development of Army materiel so that it will meet the tactical and the technical needs of the Armed Forces.

Comments and suggestions on this handbook are welcome and should be addressed to Army Research Office-Durham, Box CM, Duke Station, Durham, North Carolina 27706,

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CHAPTER 1
INTRODUCTION TO FIRE CONTROL SYSTEMS

1-1 DEFINITION AND NATURE OF FIRE CONTROL [1-6]

1-1.1 GENERAL

This chapter defines and describes the nature of fire control; covers the chronological development of Army fire control equipments; discusses recent developments in Army fire control; and summarizes the applications of fire control to modern warfare. It is hoped that this information will both interest the fire control designer and prove of direct value to him, by giving him a better perspective on his work through a knowledge of past developments and present trends. For more detailed information, consult the General Fire Control Bibliography at the end of the chapter.

1-1.2 DEFINITION AND GOALS OF FIRE CONTROL

Fundamentally, all fire control problems are variations of the same basic situation: launching a missile* from a weapon station so as to hit a selected target. The target, the weapon station, or both may be moving. Fire control is the science of offsetting the direction of weapon fire from the line of site to the target so as to hit the target (see Fig. 1-1). The angle of offset is called the prediction angle; it is the angle between the line of site (the line from the weapon station to the target at the instant of firing) to the weaponline (the extension of the weapon axis). It represents the fire control system's prediction of the best solution to the fire control problem at hand when the available input information is taken into account to the extent made possible by the particular fire control system involved. † As discussed subsequently, the prediction angle is achieved as the result of offset components in elevation and deflection.

With guns and rocket launchers, solution data are applied up to the instant of firing. With guided missiles, solution data are also applied at intervals or continuously after firing. The Fire Control Series is concerned with weapons-laying before and during firing and does not cover in-flight control of guided missiles, though many of the same principles apply.

Fire control is accomplished by (1) accumulating appropriate input data, (2) calculating the elevation and deflection components required for the projectile to intersect the target, and (3) applying these components to correctly position the weapon. These three functions are primarily associated, respectively, with (1) acquistion and tracking sys-

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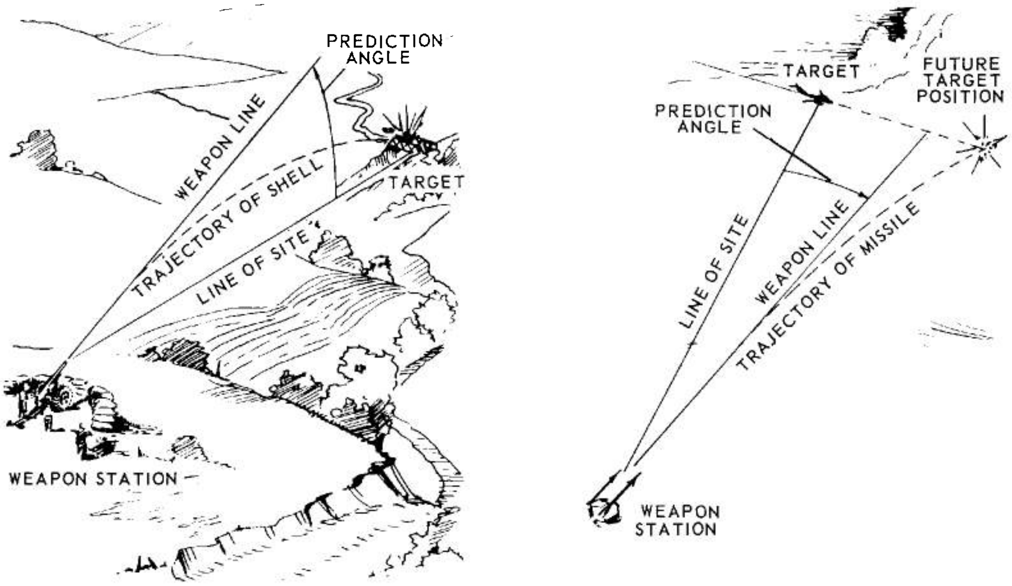
* A missile, in the general sense used here, identifies an object that is thrown, dropped, projected, or propelled toward a target (though sometimes the term is used more narrowly, to define a self-propelled guided missile or a ballistic missile). A projectile, in the general sense employed throughout the Fire Control Series, is a missile projected by an exterior force and continuing in motion by its own inertia. The term primarily applies to projectiles, bullets, etc., shot from guns and recoilless weapons but is also extended to rockets (which, by definition, cannot be controlled in flight), even though a rocket does not strictly fit the stated definition during the period between the time of launching and the time of burnout.

† The use of the word "prediction" in its present context relates to its definition as "an inference regarding a future event based on probability theory" (see Reference 42). Thus, the prediction angle computed at a given moment in a given fire control situation is the inferred best solution to the problem of obtaining a hit on the target. The term prediction is particularly appropriate to fire control since (see Chapter 4) the very nature of weapon-fire accuracy is probabilistic as a result of such factors as dispersion and other random errors associated with weapon-system operation. Further, as emphasized in Reference 43 by the late Dr. John G. Tappert of Frankford Arsenal (see the acknowledgement and tribute at the end of Chapter 4), the general case of fire control for a moving target involves extrapolation, or prediction, in its broadest sense.



(A) Surface-to-surface with guns

(B) Surface-to-air with missiles

NOTE

THE TERM "LINE OF SITE" USED HERE IN CONNECTION WITH THE GEOMETRY OF A FIRE CONTROL PROBLEM IS DEFINED AS THE STRAIGHT LINE BETWEEN THE ORIGIN OF THE TRAJECTORY (THAT IS, THE WEAPON STATION) AND THE TARGET. IT CORRESPONDS TO THE TERM "LINE OF SIGHT" USED IN CONNECTION WITH THE MEANS EMPLOYED TO SOLVE A FIRE CONTROL PROBLEM AND DEFINED AS THE STRAIGHT LINE BETWEEN THE EYE OF AN OBSERVER SIGHTING FROM THE WEAPON STATION (OR A TRANSMITTING RADAR ANTENNA) AND THE TARGET.

Fig. 1-1. Factors common to typical Army fire control problems. (Adapted from FIRE CONTROL PRINCIPLES by W. Wrigley and J. Hovorka. Copyright © 1959 by McGraw-Hill, Inc. Used by permission of McGraw-Hill Book Company.)

tems, (2) computing systems, and (3) weapon-pointing systems. As discussed in the Foreword, these three systems are treated in Sections 2, 3 and 4, respectively, of the Fire Control Series.

In some situations, fire control also includes the solution of two other types of problems: (1) maintaining cognizance of the weapon-target situation and controlling the time and volume of fire so as to achieve maximum effectiveness of fire and minimize waste of ammunition, and (2) causing projectiles to explode when they reach the vicinity of the target by means of time fuzes preset with the aid of a fuze-time computer. The latter problem does not arise, of course, with impact and proximity fuzes.

Thus fire control may be broadly defined as quantitative control over one or more of the following, to deliver effective weapon fire on a selected target:

1. The direction of launch.
2. The time and volume of fire.
3. The detonation of the missile.

However, fire control is primarily concerned with item (1), the direction of launch, and the Fire Control Series will be mainly concerned with this aspect.

In Army parlance, the terms "weapon fire control" and "weapon control" are used interchangeably with the expression "fire control". To indicate specific applications to certain types of weapons, terms such as "gun fire control" and "missile fire control" are frequently employed.

1-1.3 SUMMARY OF FIRE CONTROL METHODS

The fundamental problem of fire control is to orient a weapon in such a way that the missile it fires will achieve hits on the selected target. For weapons of the present era, fire control varies in complexity from the simple aiming of a pistol to the intricate prob-

lem of destroying an incoming ICBM.

Two general methods of fire control are employed in connection with Army weapons: (1) direct fire control and (2) indirect fire control.

1-1.3.1 Direct Fire Control

Direct fire control is used for the control of weapon fire delivered at a target that can be observed (by optical instruments, radar, etc.) either from the weapon itself or from nearby elements in a director-controlled type of weapon system. When the target is thus visible from the weapon, a line of site is established between the gun and the target. The weapon can then be aimed in elevation and deflection with reference to this line of site either by means of sighting instruments mounted on the weapon or by means of the director fire control system.

The following types of direct fire are used in Army combat situations:

1. Antiaircraft fire
2. Personal weapon fire, e.g., rifles and bazookas
3. Tank weapon fire
4. Airborne weapon fire
5. Field-artillery weapon fire.

It should be noted that types 1 through 4, inclusive, are typical direct-fire situations. On the other hand, fire from field-artillery weapons (type 5) is direct fire only under exceptional conditions of short range.

1-1.3.2 Indirect Fire Control

Indirect fire control is used for the control of weapon fire delivered at a target that cannot be seen from the weapon position. When the target is invisible from the weapon (e.g., when it lies behind a hill), an observation post is usually established from which the target can be seen.* Fire-control intelligence is then obtained and computed for the gun at a fire-direction center. The transmission of firing data to the weapon may be by telephone communication or by remote control. In the latter case, the gun is pointed in azimuth and in elevation automatically in accordance with the established firing data.

The following types of indirect fire are used in Army combat situations:

1. Mortar fire
2. Field-artillery weapon fire
3. Tank weapon fire.

It should be noted that type 1 is a typical indirect-fire situation and that type 2 consists of indirect fire mainly; by contrast, type 3 only occasionally involves the use of indirect fire.

1-1.3.3 Geometry of a Typical Fire Control Problem

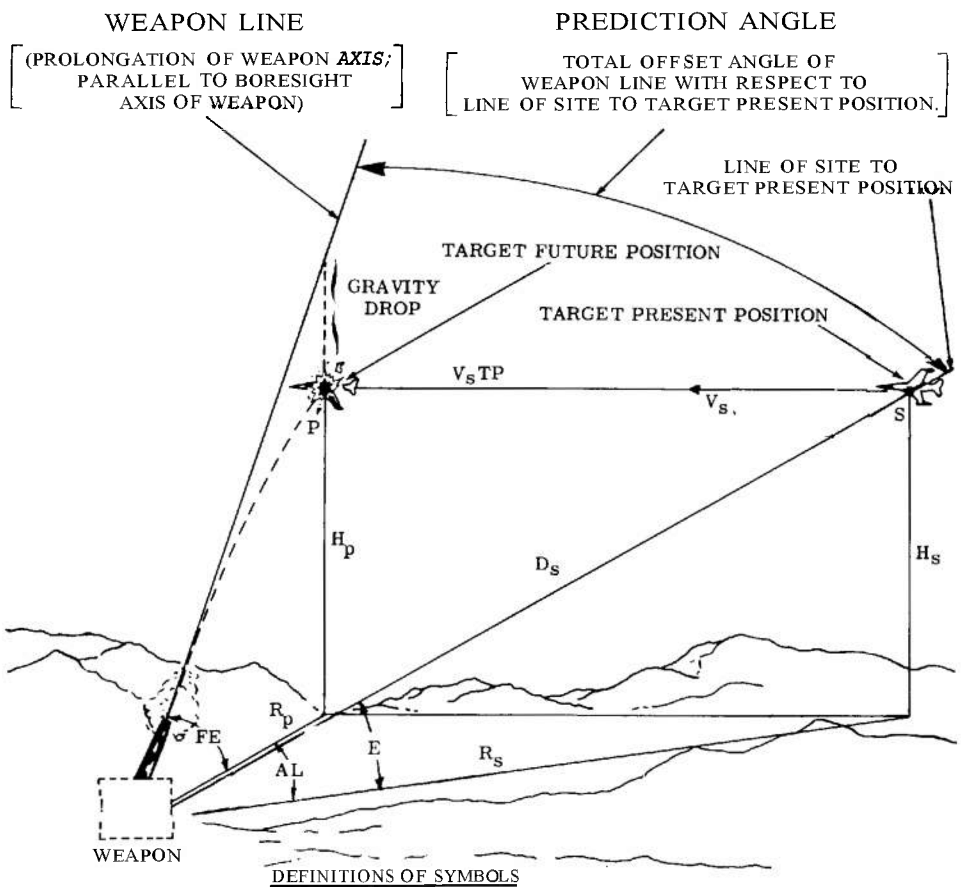
Figure 1-2 shows the fundamental geometry of a typical direct fire control problem in terms of the prediction angle – the total offset angle measured from the present line of site to the weapon line. As indicated, it has an elevation component and a deflection component. The prediction angle is uniquely determined at each instant of a fire control situation by two factors:

1. The motion of the target relative to the weapon.
2. The exterior ballistics affecting the path of the projectile after the weapon is fired.

The fire control problem pictured is that associated with the Vigilante Antiaircraft Weapon System which is described in Chapter 4 in connection with a discussion of design procedures for fire control systems.

The theoretical fire control problem consists of finding the magnitude of the required prediction angle between the weapon line and the line of site, together with the direction of the axis about which it represents a rotation, as functions of measurable physical quantities. It should be noted that, because of practical considerations involved in achieving the required offset of the weapon line from the line of site, the prediction angle is defined in terms of, and implemented by means of, two separate components – an elevation component and a deflection component – whose combination yields the required offset angle. For example, in the illustrative fire control problem depicted in Fig. 1-2 it can be seen that the elevation component of the prediction angle is equal to the fire-elevation angle FE, minus the target-elevation angle E, where

* It should be noted that radar observations of parts of the trajectories of enemy weapon fire and appropriate extrapolation techniques have also proved to be an effective means of locating weapon targets, e.g., enemy mortars.



DEFINITIONS OF SYMBOLS

S-OPERATOR'S SMOOTH TRACKING POINT
D_s-SLANT RANGE
E-ELEVATION OF TARGET
P-PREDICTED POINT
V_s-SMOOTH-TRACKING-POINT VELOCITY
FE-FIRE-ELEVATION ANGLE
AL-AZIMUTH LEAD ANGLE
TP-TIME OF FLIGHT OF PROJECTILE
V_sTP-DISTANCE THE TARGET TRAVELS FROM SMOOTH TRACKING POINT S TO THE PREDICTED TARGET POINT P
H_s-SMOOTH-TRACKING ALTITUDE (PARAMETER)
H_p-PREDICTED-TRACKING ALTITUDE (PARAMETER)
R_s-SMOOTH-HORIZONTAL RANGE
R_p-PREDICTED-HORIZONTAL RANGE

NOTE

THE SYMBOLS AND NOMENCLATURE USED HERE ARE REPRESENTATIVE ONLY. THE BROAD TECHNOLOGICAL BASE UPON WHICH FIRE-CONTROL SYSTEM DESIGN IS BASED HAS PRECLUDED THE ESTABLISHMENT OF A FORMALLY ACCEPTED SET OF SYMBOLS AND DEFINITIONS THAT APPLIES UNIVERSALLY TO ALL ARMY FIRE-CONTROL APPLICATIONS.

Fig. 1-2. Fundamental geometry of a typical fire control problem for surface-to-air fire.

FE is the vertical angle between the axis of the weapon bore (the weapon line) and its projection onto the horizontal plane and E is the vertical angle between the line of site to the present target position and its projection onto the horizontal plane. The deflection component of the prediction angle in Fig. 1-2 is equal to the azimuth lead angle AL, the angle between the horizontal projections of the weapon line and the line of site to the present target position. (As pointed out in Fig. 1-2, these symbols and nomenclature are representative only; no formally accepted set of definitions and symbols to cover all Army fire-control applications has ever been established.) The theoretical aspects of the fire control problem and its solution are discussed in Chapter 2.

The practical fire control design problem consists of analyzing the fire control problem, establishing a mathematical model, and mechanizing this model. The end product is equipment that is suitable for receiving the available inputs for particular fire control applications and generating the correct prediction angle as the output (in the form of the aforenoted elevation and deflection components). This practical design problem is discussed in Chapters 3 through 5.

1-1.4 CLASSIFICATIONS OF FIRE CONTROL EQUIPMENT

Fire control objectives are obtained by the use of specially designed aids, instruments, and systems. One or more basic types of operation may be involved—electrical, electronic, mechanical, optical, hydraulic, or combinations of these.

Aids to fire control are devices that help the aimer judge or correct the prediction angle. They include such items as rifle sights and, in a broader sense, the use tracer bullets. Fire control instruments are used for more exact, quantitative acquisition, calculation, and application of data than aids. They include range finders, compasses, telescopes, and radars; predictors, directors, and other computers; and servos and other devices for positioning the weapon in azimuth and elevation. A fire control system is defined as an assemblage of interacting or interdependent fire control equipment that receives data concerning the present position and motion (if any) of a selected target, calculates the future target position, correlates this information with information concerning projectile flight (exterior ballistics data), and controls the aiming of the weapon to bring effective fire upon the target. Human operators may be considered elements of a system and in any evaluation of overall system effectiveness they should be so considered.*

Fire control equipment is frequently classified by its location as either on-carriage or off-carriage equipment.

On-carriage describes instruments such as sighting telescopes and elevation quadrants that are mounted directly on the weapon or weapon carriage. The designation includes indicators having graduated dials and pointers that are connected to the elevating and traversing mechanism of a gun. In certain systems, the designation may also include range finders, computers, etc.

Off-carriage equipment includes all fire control instruments that are not mounted on weapons or weapon carriages. These instruments supply the data to be set on the sights, drums, quadrants, indicators and other aiming and laying devices mounted on the weapon itself. Typical off-carriage instruments are the aiming circle, the battery commander's telescope, plotting boards, various types of range finders, radars, computers, directors, and illuminating devices.

1-1.5 APPLICATIONS OF MODERN FIRE CONTROL SYSTEMS

The various types of weapon fire that are controlled by fire control equipment may be classified by establishing (1) the relationship of the physical area of weapon fire to the physical area of the target and (2) the type of weapon involved. Thus, the various types of weapon fire covered in this handbook may be classified as follows:

1. Surface-to-surface with guns
2. Surface-to-surface with rockets
3. Surface-to-air with guns
4. Surface-to-air with rockets
5. Air-to-surface with guns

* The word "system" does not necessarily imply complexity. Strictly speaking, a man and his rifle constitute a simple system (which could be further broken down into rifle, rifle sight, eye, visual cortex, nerves, muscles, etc.).

6. Air-to-surface with rockets
7. Air-to-air with guns
8. Air-to-air with rockets.

Each classification applies to all types of weaponfire implied by the name of the classification. That is, the classifications do not differentiate between the types of weapon or launcher (except for the broad differentiation between guns and rockets), the types of missile, warhead or propellant, the types of fire control system used, the types of target, or (in the case of ground targets) between stationary and moving targets. In this handbook, it is also assumed that air-to-air and air-to-surface classifications are limited to aircraft available to the U.S. Army.

1-1.5.1 Surface-to-Surface With Guns

This classification of surface-to-surface weapon fire includes all projectiles, except rockets and guided missiles, that are fired from the surface of the earth and whose purpose is to destroy a target also on the surface of the earth.

The destructive intent of surface-to-surface gunfire is directed in defense and attack against either of the two basic categories of targets: the stationary target and the moving target. With respect to the stationary target the firing data required for accurate weapon laying is based on providing the required projectile trajectory in accordance with known ballistic data, information on variables of the firing situation, such as wind, and information concerning the location of the target relative to the location of the weapon. Target position relative to weapon position is generally determined by sight but can also be obtained from maps under certain conditions. An increase in the range of fire between the gun and the stationary target generally results in an increase in the errors of gunfire.

With respect to moving surface targets, information on present position and motion is usually derived from direct observation of the target. Firing data is based on predicting the future target position and then providing a projectile trajectory that passes through that position. From the time a projectile is fired, its trajectory is irrevocably dependent on projectile ballistics, wind effects, and gravity. Also, a finite time is required for the projectile to reach the target. Accordingly, if the time of flight is short and the target motion is slow, the probability of a hit is reasonably good. If on the other hand, the target is moving rapidly, with a high degree of maneuverability, the probability of destruction is low. Inadequate target observation and inaccurate range measurement correspondingly reduce the hit probability (see Chapter 4 for a discussion of hit probability).

1-1.5.2 Surface-to-Surface With Rockets

Surface-to-surface weapon fire with rockets includes any rocket-propelled missile, whose trajectory cannot be controlled during flight, that is launched from the surface of the earth, and whose purpose is to destroy a target also on the surface of the earth. The comments on surface-to-surface gunfire (see par 1-1.5.1) apply to this category also. Essential differences between the two categories exist mainly in the important characteristics of the military rocket. Military rockets utilize a method of propulsion that, unlike the gunfired projectile, results in the propellant and its gases traveling with the rocket during the period of propellant burning. Military rockets also are characterized by low velocity, which results in greater time of flight to target and thus provides the target greater time to maneuver, with a concomitant reduction in the probability of obtaining hits. Dispersion of rockets as a result of in-flight turbulence of the burning propellant, inaccuracies in the symmetry or alignment of the rocket nozzle, and the use of stabilizing fins limit their ground use largely to area targets.

In the tactical employment of military weapons, guns are generally utilized where great accuracy and a large range adaptability are required; rockets, because of their relative inefficiency, are limited in both respects. Conversely, the volume, rate of fire, and the high mobility of rocket weapons justify their use against area targets or large troop concentrations – although rockets, logistically, are not suited for high rate of fire over extended periods. Used in infantry antitank weapons for direct fire by infantrymen at short ranges, the bazooka rocket is well suited for delivering a small, light HEAT warhead, at low velocity, that is capable of destroying a tank.

1-1.5.3 Surface-to-Air With Guns

This classification of surface-to-air weapon fire includes any projectile, exclusive of rockets and guided missiles, that is fired from the surface of the earth and whose purpose is to destroy a target in the air.

The basic mission of antiaircraft artillery is defense against aerial attack. Because of the high speed, the maneuvering capabilities in three dimensions, and the relatively small size of such airborne targets as propeller- or jet-powered aircraft, the unique problem of this type of fire control is the determination of the target's future position because the target may move a very considerable distance after the projectile has been fired. The fire control problem in surface-to-air gunfire, therefore, consists of: (1) predicting the future course of the target on the basis of its behavior in the time just preceding the firing of the gun, (2) determining the target's probable position at the end of the time of flight, and (3) preparing the firing data required to burst a projectile at that point.

Of major significance in this category of weapon fire is that computation of a single set of firing data will not suffice for application to the aiming point. Rather, firing data suitable for any instant must be available during the entire period of time that the target is flying within the range of the weapon being employed. As this period may be short, firing data must be produced continuously and instantaneously throughout the interval. Further, since the target capability for changing direction and speed quickly reduces its vulnerability, the maximum practicable volume of fire must be employed in a minimum amount of time to increase the probability of target destruction.

1-1.5.4 Surface-to-Air With Rockets

This classification of surface-to-air weapon fire includes any rocket-propelled missile launched from the earth's surface whose trajectory cannot be controlled during flight and whose purpose is to destroy an airborne target.

1-1.5.5 Air-to-Surface With Guns

This classification of air-to-surface weapon fire includes all projectiles, except rockets and guided missiles, that are fired from an aircraft and whose purpose is to destroy a target on the surface of the earth.

1-1.5.6 Air-to-Surface With Rockets

This classification of air-to-surface weapon fire includes any rocket-propelled missile that is launched from an aircraft and whose trajectory cannot be controlled during flight and whose purpose is to destroy a target on the earth's surface.

Rockets fired from fast-moving aircraft are more effective than those launched from the ground because of their greater accuracy which stems from the additional speed with respect to the air that the high-speed forward movement of the aircraft imparts to the rocket at the time of launching. The accompanying aerodynamic effects on the rocket fins result in increased stability of the rocket, thereby tending to minimize dispersion and permit relatively accurate fire at point targets. For effective fire control, however, sighting for accurate forward-firing of aircraft rockets must allow compensating considerations for altitude, indicated air speed, dive angle of the aircraft, and slant range from the target.

1-1.5.7 Air-to-Air With Guns

This classification of air-to-air weapon fire includes all projectiles, except rockets and guided missiles, that are fired from an aircraft and whose purpose is to destroy an aerial target.

1-1.5.8 Air-to-Air With Rockets

This classification of air-to-air weapon fire includes any rocket-propelled missile launched from an aircraft and whose trajectory cannot be controlled during flight, and whose purpose is to destroy an aerial target.

1-1.6 THE INPUT-OUTPUT CONCEPT

Fire control may be viewed in terms of certain inputs that are available for measurement and certain outputs that are required to position a weapon for firing at a stationary or moving target. The input-output concept is

implied at once when consideration is given to gathering data on the position of a target, calculating the target's future position, correlating exterior ballistics information, and controlling the aiming of a weapon to bring fire on the target.

Basic input data can be enumerated as range, elevation, azimuth, and motion of the target, measurable with respect to weapon position. Supplementary input data include information on the wind and such items affecting initial projectile velocity as gun-barrel erosion and propellant temperature. In any fire control system, all available input data are employed to the extent of the system's capability to produce as outputs such firing data as are applicable to the aiming of the weapon being controlled.

1-1.6.1 Primary Factors in Establishing Input-Output Relationships

There are two main categories:

1. Factors affecting the projectile path.
2. Target motion with respect to the weapon.

These factors are discussed in some detail in Chapter 2 but are summarized in the paragraphs which follow to clarify the input-output concept. Each of these factors must be considered as an integral part of each individual fire control problem. The emphasis allotted to a given factor in establishing the solution to a particular fire control problem is determined primarily by its relative effect on the outputs and by the accuracy requirements of the fire control system. Chapter 3 describes the various functional elements of fire control systems and cites functional arrangements that provide desired input-output relationships for specific fire control systems. Chapter 4 presents the conceptual approach for achieving the actual designs of these fire control systems most effectively.

1-1.6.1.1 Factors Affecting the Projectile Path

The factors that affect the projectile path and so determine projectile trajectory can be classified as those that contribute (1) to curvature of the trajectory and (2) to projectile jump. The first type are active during the entire course of the projectile trajectory and cause the projectile to change its path continuously. They include gravity, air resistance, wind, and drift. Factors of type (2) are active only at the instant the projectile is fired or launched; they cause the direction of the projectile velocity vector to differ from the direction in which the weapon is aimed. Thus jump effect is comprised of two components: vertical jump and horizontal jump. While the total jump effect can be quite large for air-to-air weapon fire from fast-moving aircraft, the jump associated with most types of Army weapon fire is relatively small.

1-1.6.1.2 Target Motion With Respect to the Weapon

Obviously, anything projected at a moving target, whether it be a projectile from a weapon or a football rifled at a fast-moving end, must incorporate some allowance, or lead, to account for target motion if a hit is to be achieved. With all other conditions of a fire control situation remaining unchanged, the amount of lead required to correct for target motion increases with the magnitude of the target velocity and varies with the target's relative direction; a target traveling at right angles to the line of site requires a larger lead angle than a target traveling at the same speed on some other path.

Target range and projectile time of flight also affect lead. The required lead increases as time of flight increases; or stated another way, the greater the projectile velocity, the less is the lead required. On the other hand, lead angle decreases as the range increases.

1-1.6.2 Secondary Factors in Establishing Input-Output Relationships

As indicated in Chapter 2, the firing tables that form the basis of correcting for the individual characteristics of projectile trajectories are of necessity computed by assuming certain standard conditions. In an actual fire control problem, various nonstandard conditions must be corrected for if their omission would seriously affect the fire control solution. The following nonstandard conditions are typical:

1. Corrections to elevation firing data to account for:

a. Differences in projectile weight

b. Increase or decrease in muzzle velocity, and
c. Ballistic head and tail winds.

2. Corrections to azimuth firing data to correct for cross wind.

Because of the constant improvement that is being made in fire control equipment, one-time insignificant sources of error may subsequently become significant. Examples of this are the corrections for cant and gun-tube distortion required for modern tank fire control systems.

1-2 CHRONOLOGICAL DEVELOPMENT OF ARMY FIRE-CONTROL EQUIPMENTS [4-30]

1-2.1 INTRODUCTION

Missile hurling was a skilled craft thousands of years before writing was developed, and ballistics--the study of the motion and behavior characteristics of missiles--was elevated from a technical art to a science following the introduction of firearms to Western Europe in the 15th century A. D. Fire control, on the other hand, reached scientific status quite recently; accurate fire control became practical only with the development of accurate weapons in the last century and a half, and the vastly increased ranges of weapons and mobility of targets during the same period made it a practical necessity.

1-2.2 PRE-19TH CENTURY FIRE CONTROL

1-2.2.1 A Word on Nomenclature

Originally, the term artillery was applied to all devices used to propel missiles through the air. With the initial development of firearms, however, all guns were called cannon, to distinguish them from mechanically operated missile-throwing weapons. As firearms developed further, those using projectiles of small diameter were termed small arms, while all other firearms retained the original terminology of cannon. Eventually, the term artillery came to mean cannon in this sense and to identify the arm of the Army that mans and operates cannon. (See Reference 2 for modern definitions of gun, cannon, small arms, howitzer, mortar, etc.).

1-2.2.2 Control of Weapons Prior to Firearms

A large variety of missile weapons were used from the Stone Age through the Middle Ages, ranging from the earliest hurled stones, spears, and javelins, to weapons using stored energy (the simple bow and later the longbow) and weapons that were elastically operated and mechanically retracted (the catapult, ballista, and later the crossbow). Control over the accuracy of all these weapons, however, was primarily a matter of skill and judgment.

1-2.2.3 Development and Control of Early Firearms

The medieval Chinese invention of gunpowder probably became known in the Near East early in 1200 A. D. to Moslems who had fought with Mongols during the reign of Ghenghis Khan. Gunpowder led eventually to the invention of firearms in the form of heavy, crude cannon, which were introduced into warfare in Western Europe about 1310 A. D.

The end of the 14th century witnessed the appearance of the earliest hand firearm, the hand cannon (see Fig. 1-3), which was devised from the early crude cannon. The hand cannon evolved as a simple wrought-iron or bronze tube of large caliber and smooth bore, mounted on a crude stock. It was muzzle-loaded, had no trigger or sight,, and was fired by lighting a touch hole of exposed powder. Weapon fire was, of course, quite inaccurate.

Weapon improvements in the early days of ordnance engineering were primarily refinements in the ignition of powder charges for small arms, exemplified by the introduction of triggering devices and the elimination of the objectionable match by the invention of the flintlock. These developments resulted in only small improvements in the accuracy of weapon fire, however. For example, as late as the Rattle of Bunker Hill, the Continental troops insured effectiveness of fire by the simple expedient of holding it until they could see the whites of the enemy's eyes.

Until the 19th century, control of gun fire from cannon, as from small arms, was rudimentary (see Fig. 1-4). It consisted chiefly of aligning the cannon with the target for azimuth control and elevating it by eye. Sometimes the curvature of the trajectory was al-

Fig. 1-3. Aiming an early type of hand cannon.

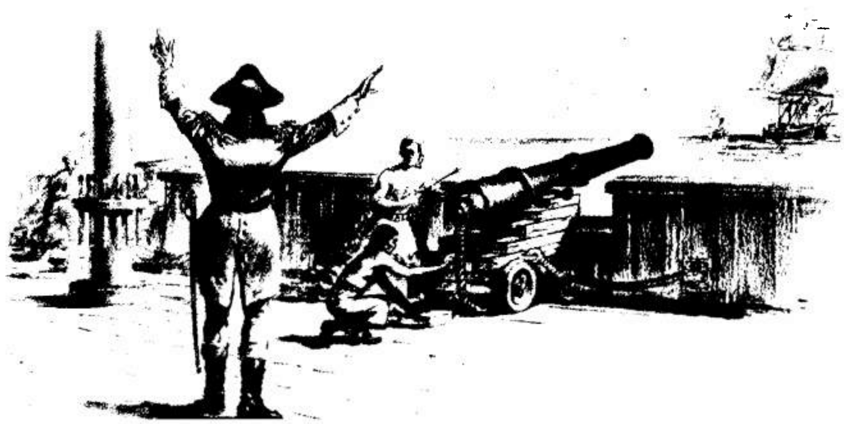


Fig. 1-4. Aiming an early type of cannon.

lowed for by "sighting along the line of metal." This was accomplished by aligning the top of the muzzle with the point of aim, thus causing the cannon to be elevated by the amount of taper from breech to muzzle.

As early as the 16th century, mathematicians had established approximate solutions for the trajectory of projectiles. Galileo, Tartaglia, Newton, Bernouilli, Euler, and others prepared the foundations for accurate weapon laying in their treatises by bringing theoretical ballistics "to a degree of perfection capable of directing fire in all circumstances" (to quote Tartaglia). Many of the mathematicians of this period were even able to prepare rudimentary forms of firing data from range observations.

However, through the 18th century, inconsistent gun performance and lack of interest on the part of military authorities combined to prevent any advance in fire control corresponding to the advances in the science of ballistics. Little attempt was made to regulate initial velocity; powder charges were estimated and the projectile load was variable. These inaccuracies, combined with poor workmanship on the guns and human fallibility in laying them, severely limited the accurate range. Battle ranges were spoken of as pistol shot (about 50 yards) and half pistol shot (about 20 yards). Targets were slow; the fastest was a charging troop of cavalry. Fire control is not mentioned in tactical treatises or directives of the period.

1-2.3 DEVELOPMENTS IN THE 19th CENTURY

1-2.3.1 Improvements in Weapons

In the 19th century, weapons improved so much in consistency of performance, rapidity of fire, and range that improved fire control became both practical and necessary. Techniques improved generally in manufacturing powder and in fabricating gun components with greater precision and durability. However, perhaps the most significant developments were rifling and breech loading.

Rifling imparts rotation to the projectile. It thus gives the projectile stability in flight, prevents tumbling, and reduces dispersion. Rifling had been used in small arms since the late 18th century in such more-or-less single-shot applications as hunting. However, as long as muzzle loading was used, rifling was impractical in military weapons because of the difficulty of devising a projectile that could both be loaded through a fouled bore (especially after rapid firing) and at the same time fit closely enough to expand into the rifling when fired.

Then came the development of percussion primers, which used a new powder that exploded when crushed, and metal cartridge cases, which would "obturate" (i.e., close up) the gun breech to prevent propellant gases from escaping. These made breech loading, and hence rifling practical for artillery. The first workable artillery rifle appeared about 1846. Five years later, an elongated bullet with an expandable base was developed, which led to the cylindrical-ogival artillery projectile with a rotating band of copper or soft metal alloy to engage the spiral grooves of the gun.

Breech loading not only made rifling practical; it made rapid fire safe for the first time.

1-2.3.2 Improvements in Fire Control

The first gunsights, introduced in the Napoleonic Wars, consisted of fixed front and rear sight points parallel to the bore of the gun. They were mainly suitable for leveling the gun at the point of aim.

Rifling introduced a drift to the right during flight, resulting from the combined effects of right-hand twist (all rifling in U. S. weapons is right-handed) and gravity. In big guns, it was first approximately compensated for by inclining the rear sight bracket. However, during and after the Civil War, increases in range and consistent performance of guns made graduated and adjustable gunsights a necessity. The simple gunsight gave rise to the tangent sight which consisted of a fixed foresight near the muzzle and a rear sight movable in the vertical plane. The reference point—a notch or aperture—of the rear sight was supported on a swinging leaf. Vertical movement of the rear sight was restricted to permit it to follow a drift curve cut out in the sight leaf, which thereby compensated for any lateral deviation of the projectile due to drift. Further refinements permitted lateral adjustment to correct for the effects of wind.

Toward the end of the 19th century, fire

control was further improved by adding the sight telescope, mounted on the gun so that its line of sight could be offset from the axis of the bore of the gun to correct for the effects of range, drift, and relative motion between gun and target. Elevation scales were graduated in accordance with ordnance proving-ground data, and the weight and composition of powder charges were carefully regulated. A final improvement in operation was obtained by installing two sights and dividing the responsibility for keeping the line of sight on target between (1) the pointer, who controlled gun elevation, and (2) the trainer, who controlled gun azimuth.

By the end of the 19th century, refinements in the manufacture of guns, detailed studies of trajectories, and simple fire control sighting equipment had made possible much more accurate long-range shooting than had been possible at the start of the century.

1-2.4 DEVELOPMENTS IN THE 20TH CENTURY THROUGH WORLD WAR II

1-2.4.1 Introduction

Until recently, fire control concepts could be mastered through a detailed study of actual fire control systems. The increasing number and complexity of weapons and weapon systems – and hence of the associated fire control systems – have now made it impractical to learn general concepts by this method.

On the other hand, as a background for the approach pursued in the Fire Control Series (which is based on explaining fundamental principles, treating the necessary reasoning processes required to arrive at superior fire control system designs, and including pertinent illustrative examples), a brief exposure to the "hardware" approach, in the form of a survey of fire control development during the 20th century, should prove helpful. Sufficient detail is provided to indicate operation of the equipment and the functioning of the various mechanisms involved. Those desiring detailed information on its use with particular weapons should consult appropriate operation manuals.

1-2.4.2 Weapon-Laying Devices

The development of fire control began to intensify about the start of the 20th century as weapons generally improved and better ammunition became available. As the range and accuracy of guns increased, the use and improvement of weapon-laying devices became mandatory. Of particular significance were three different types of quadrants developed by the Army for aiming weapons in elevation when engaged in indirect fire*: (1) the gunner's quadrant, (2) the elevation quadrant, and (3) the range quadrant.

1-2.4.2.1 The Gunner's Quadrant

The gunner's quadrant was developed for use in artillery fire control. It was used to adjust a gun to an elevation predetermined by the firing officer from firing tables and range data. The operation of this device is based on the principle of offsetting a spirit level with respect to the gun-bore axis. Figure 1-5 shows the Gunner's Quadrant M1, which is a typical design for this type of aiming device.

For coarse elevation, the swing arm with the spirit level was set at the desired angle with respect to the leveling feet by means of the ratchet; for finer increments, the micrometer was employed. The quadrant's leveling feet were then set on machined leveling pads on the gun, parallel to its bore, and the gun was laid by moving it in elevation until the bubble was centered. For elevations higher than 800 mils (45°), a second scale, on the back of that shown, and a second pair of leveling feet were used.

1-2.4.2.2 The Elevation and Range Quadrants

The elevation and range quadrants, improved devices for laying guns in elevation, also used the principle of offsetting a level vial with respect to the gun-bore axis. They

* A device used for aiming weapons in azimuth during indirect fire is the panoramic telescope (see par 1-2.4.3.3), which is covered under the general heading of Optical Sighting Equipment.

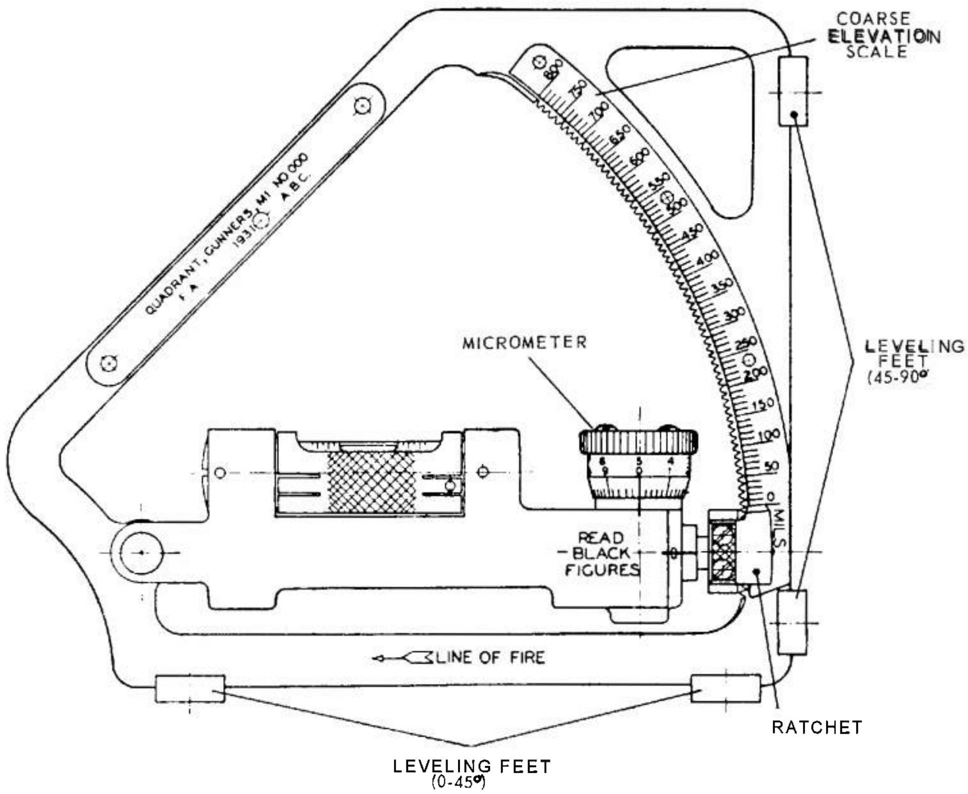


Fig. 1-5. Gunner's Quadrant M1.

differed from the gunner's quadrant in that (1) they were permanently attached to the gun carriage and (2) they incorporated two sets of elevation scales in order that the two components of the actual quadrant elevation could be set into the device separately.* These two devices increased materially the ease and quickness with which guns could be laid.

Elevation Quadrant M1917, designed for railway artillery, is shown in Fig. 1-6. With this device, the elevation corresponding to the range component was set in, being measured in degrees and minutes by means of the disc and drum, respectively. Then the angle of site was set in, and the gun moved in elevation until a bubble on the angle-of-site arm was centered. A knob and bubble were also provided for cross-leveling the quadrant, in a plane 90° from the elevation plane.

The range quadrant speeded up the process of laying the gun by using a matched pointer system and by permitting the direct setting of range, thereby eliminating the need to use firing tables. On the Range Quadrant M1 (see Fig. 1-7), for example, which was mounted on the 75 mm Gun Carriage M2, the angle of site was set on the scale and drum, the range was "cranked in" with the lower right-hand knob and the level-vial bubble was then centered by either of the two knobs provided. These actions offset the quadrant pointer from its normal zero-range position by an angle equal to the required quadrant elevation. The gun was then moved in elevation until the gun pointer, which moved with the gun, matched the quadrant pointer. Clearly, it was easier and quicker to match two pointers than to elevate a large gun until a bubble was centered, especially for guns that were lowered for each loading. The direct range scale was, of course, only good for

* As noted in Chapter 2, the two components are: (1) the component corresponding to the horizontal range, and (2) the angle of site - the component corresponding to the difference in altitude between gun and target. The angle of site can be determined by devices such as the battery commander's telescope (see par 1-2.4.3.4) and the range finder (see par 1-2.4.5).

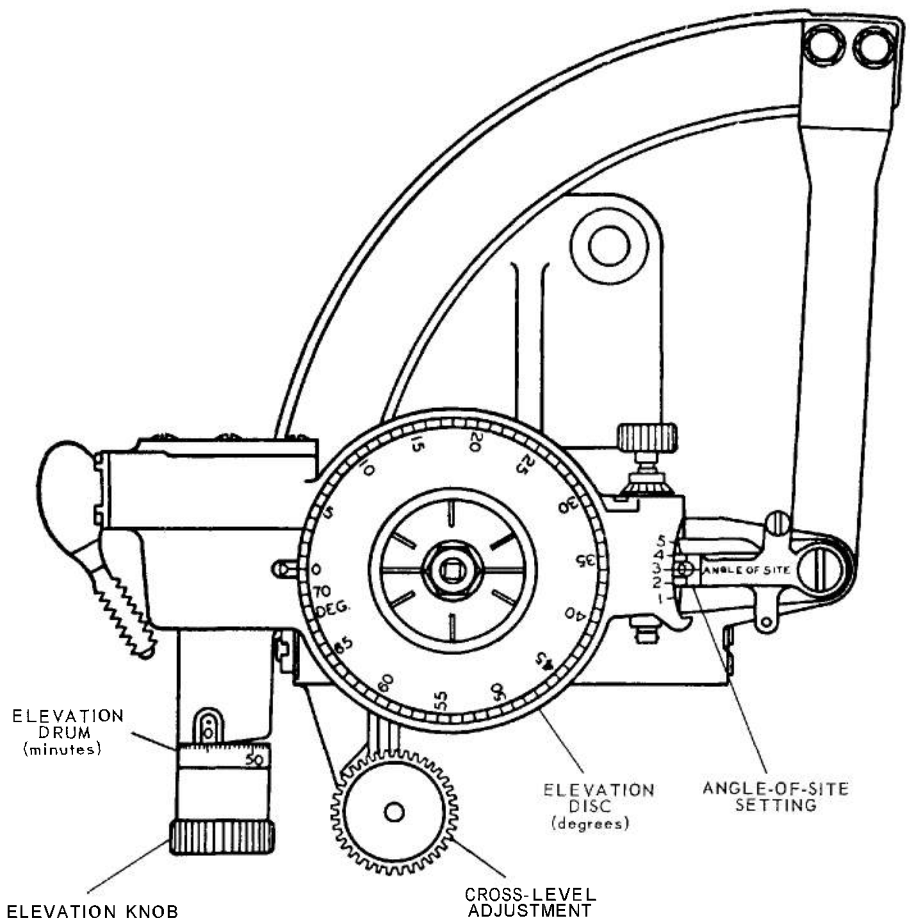


Fig. 1-6. Elevation Quadrant M 1917.

one type of ammunition; for other types, firing tables were used and the elevation angle obtained was set on the left-hand drum and scale by means of the elevation knob.

1-2. 4. 3 Optical Sighting Equipment

When long-range artillery, automatic weapons, powered vehicles, and aircraft entered the military field during the early part of the 20th century, battle ranges and target speeds increased enormously. Improved apparatus for sighting moving targets was required, both to cope with these targets and to make maximum use of the increased range capabilities of the new weapons. Many target-observation devices were developed by ordnance engineers. World War I witnessed the application of special sighting telescopes, range finders, binoculars, and observation telescopes to Army weapons.

The special telescopes developed by the Army included (1) the straight-tube telescope, (2) the elbow telescope, (3) the panoramic telescope, and (4) the battery-commander's telescope.

Efforts at standardization were thwarted in sighting and aiming equipment because each type of weapon presented special design problems. A special telescope and telescope mount were usually required for each type of weapon to match its particular mechanical design. In fact, ordnance engineers and designers spent as many man-hours in the design of mounts as in the design of telescopes, in an intense effort to devise stable mounts that would hold the telescopes in the proper position with respect to the gun tube and control gears, and that would at the same time provide for carefully adjusted movement of

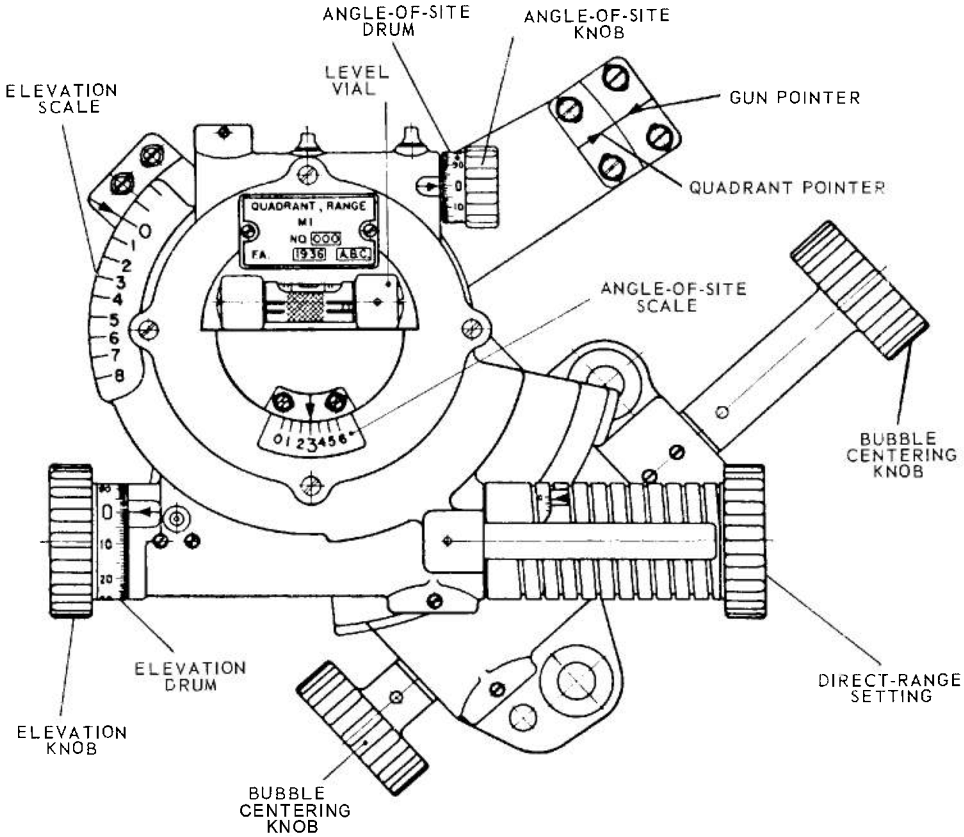


Fig. 1-7. Range Quadrant M1.

the telescope over a wide field of vision.

1-2.4.3.1 <u>The Straight-Tube Telescope</u>

The straight-tube telescope has been one of the most widely used instruments in direct fire control. Its principal advantages are (1) simplicity, with lower costs of manufacture and maintenance, and (2) good optical characteristics. Its chief disadvantage is that it recoils toward the observer--a severe problem with all but the lightest-recoil weapons whenever repetitive fire is involved.

Two basic types of straight-tube telescopes have been employed:

1. Telescopes employing a lens erecting system (see Fig. 1-8).
2. Telescopes employing a Porro prism erecting system (see Fig. 1-9).

The lens erecting system keeps the diameter of the telescope tube to a minimum, consistent with a given field of view. Hence, it is used in telescopic rifle sights. The Porro prism erecting system, on the other hand, is superior when tube length rather than diameter must be limited. This system was used extensively in the sighting systems for seacoast and railway weapons.

Figure 1-10 shows a typical straight-tube telescopic sight used in the early part of the 20th century as direct-fire sighting equipment for fixed seacoast weapons, to which it was attached by a cradle mount. By means of the elevation and deflection mechanisms of the cradle, the sight could be offset from the weapon by an amount proportional to the required elevation (obtained from externally measured range and firing tables) and deflection (also obtained from firing tables). (The cradle, with the sight rigidly secured to it, could be moved vertically and horizontally about its front end by means of the elevation- and deflection-setting mechanisms.) The telescope was then kept on the

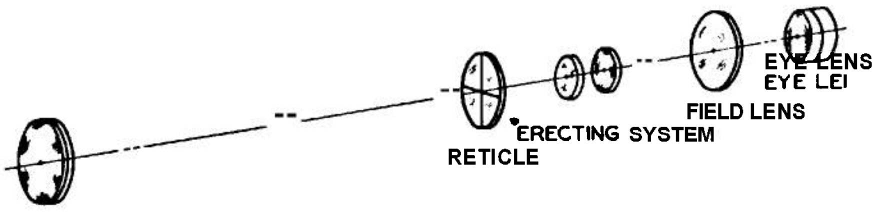


Fig. 1-8. The optical system of a straight-tube telescope based on the use of a lens erecting system.

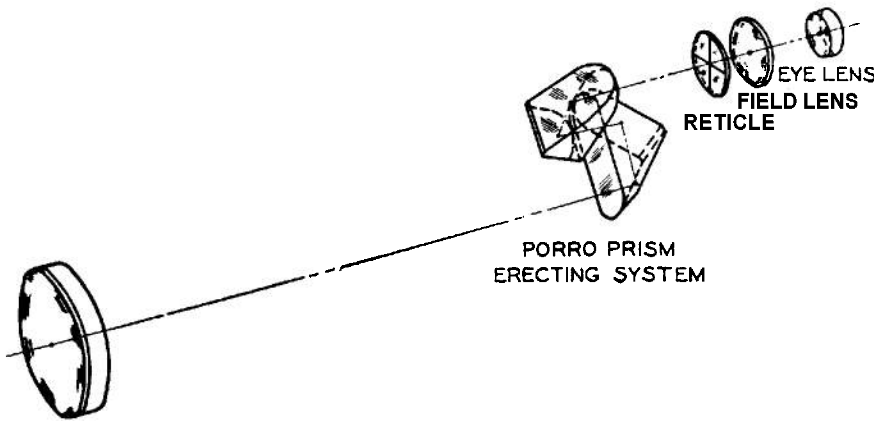


Fig. 1-9. The optical system of a straight-tube telescope based on the use of a Porro prism erecting system.

target by adjusting the gun in elevation and deflection. The spring bolt kept the cradle firmly against the elevating screw and prevented accidental motion of the sight relative to the weapon.

1-2.4.3.2 The Elbow Telescope

The elbow telescope was slightly more complicated than the straight-tube telescope but it ended eyeward recoil by permitting the gunner to stand to one side of the gun. The elbow telescope's optical system was like the straight-tube telescope's except that it Included a roof prism which both bent the optical axis and erected the image.

Figure 1-11 shows the optical system of a right-angle elbow telescope. A disadvantage was the difficulty of manufacturing the roof prism with the required accuracy. Any variation of the roof angle from 90° produces secondary images which makes the definition less distinct.

1-2.4.3.3 The Panoramic Telescope

The panoramic telescope combined the functions of a telescope and periscope; it permitted the observer to see in any direction without changing his position. Its main purpose was to permit aiming a weapon in any direction during indirect fire; in fact, it was the most convenient telescope for indirect fire. The optical system required to achieve this advantage, however, made it more complex and light absorbent than straight-tube

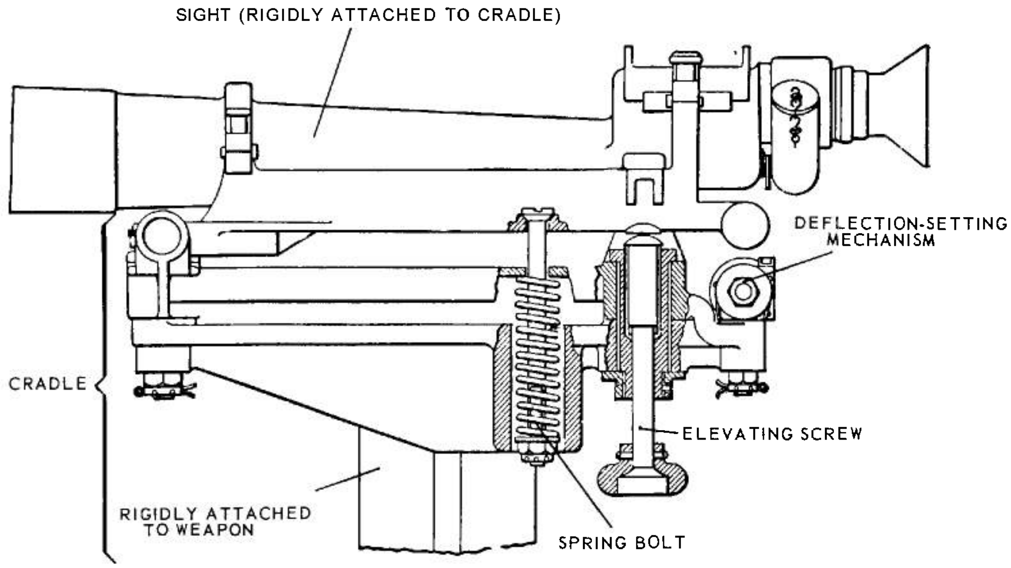


Fig. 1-10. Telescopic Sight, Model of 1912, together with its cradle mount.

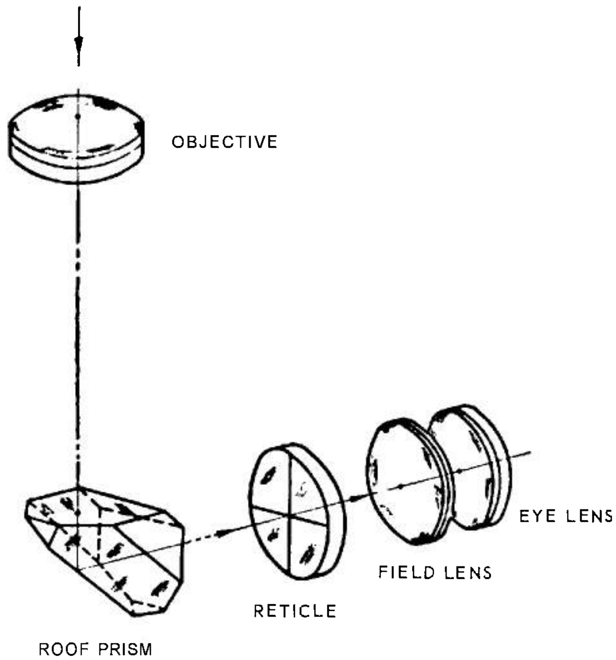


Fig. 1-11. The optical system of a right-angle telescope.

and elbow telescopes.

The panoramic telescope was used to control direct fire as well as indirect on weapons used for both, to avoid installing a second telescope, but it was not as well suited for direct fire.

The optical system of a panoramic telescope (see Fig. 1-12) was like that of a right-angle telescope but with rotating and objective prisms added. The objective prism could be rotated about the vertical axis, through any desired deflection angle, by means of an attached azimuth control with a scale. The rotating prism kept the image as seen by the viewer in a vertical position as the objective prism rotated. This was accomplished by gearing the prisms so that the azimuth control turned the rotating prism through one-half the angle of rotation of the objective prism about the vertical axis. If such an arrangement were not included, the image seen by the eye as the objective prism rotated would itself rotate through a corresponding angle about the axis of the eye and field lenses.

Figure 1-13 shows a typical panoramic telescope that was used as the sighting equipment for the 75 mm Pack Howitzer Carriage M1. Here, the telescope was attached to and remained integral with the Telescope Mount M3, also shown in Fig. 1-13. The mount, in turn, was attached to the left-hand side of the howitzer carriage.

The mount and telescope permitted (1) deflecting the weapon by any amount from the fixed point of aim in indirect fire, or by the

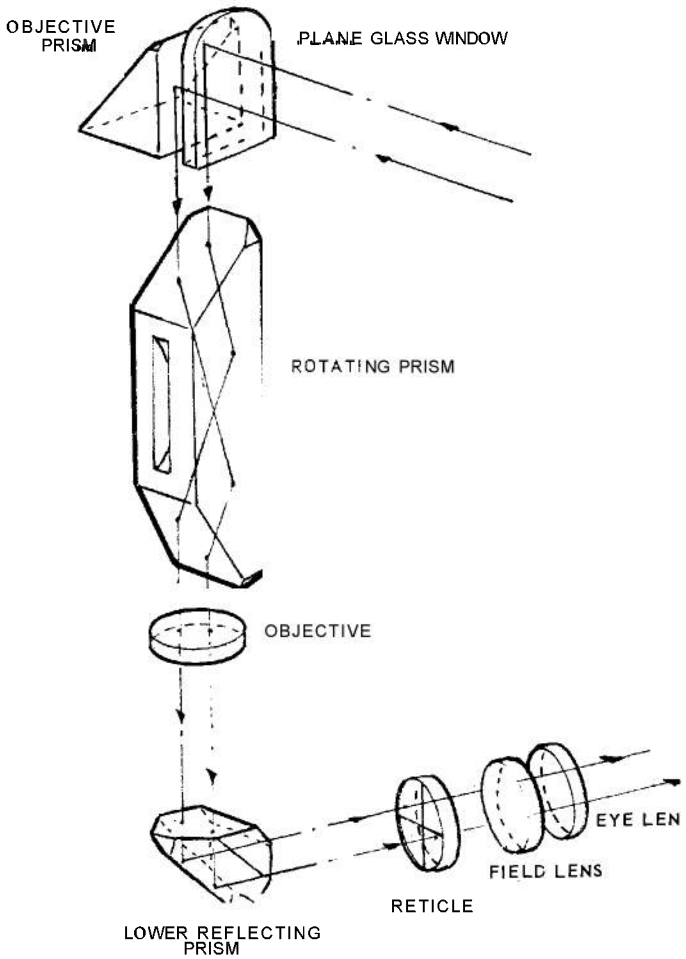


Fig. 1-12. The optical system of a panoramic telescope.

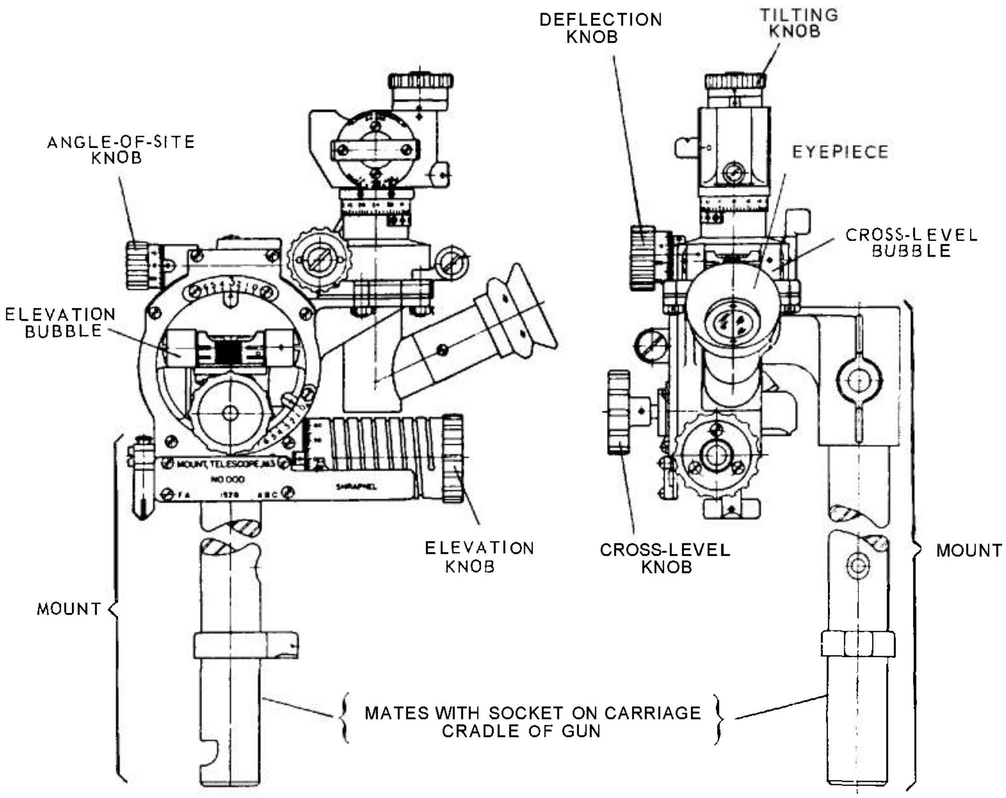


Fig. 1-13. Panoramic Telescope M1 and Telescope Mount M3.

amount determined from the firing tables in direct fire; (2) setting the angle-of-site component and the range component of elevation, or (for a specific type of ammunition) setting the range directly; and (3) cross-leveling the telescope and mount. They thus combined the features of an all-around telescopic sight with those of the weapon-laying devices described in par. 1-2.4.2. As in some of those devices, elevation and angle of site were set individually, each with its own knob, scale (for degrees) and drum (for minutes); the howitzer was then moved in elevation until the elevation level vial bubble was centered. The deflection-setting mechanism rotated the objective and rotating prisms of the optical system (see Fig. 1-12), and the tilting knob enabled the panoramic head to be tilted as required to keep the aiming point in view during indirect fire.

Note that the eyepiece for this telescope was inclined 25° from the horizontal (see Fig. 1-13) to give the operator a more comfortable position. In addition, it could be rotated about the vertical axis so that he could get his head out of the path of carriage motion when sighting to the rear.

Panoramic Telescope M1 was used with the Telescope Mount M3 to lay the 75 mm Pack Howitzer Carriage M1 as follows:

In indirect fire, a fixed point of aim would be selected to give a geographical reference line. Weapon azimuth would then be established with respect to this line on the basis of target information obtained at another observation station. To lay the weapon, (1) the required azimuth was set on the deflection mechanism, (2) the required elevation was set either in terms of range directly or by means of an elevation angle obtained from firing tables, (3) the mount was cross-leveled, and (4) the telescope and mount were moved in elevation and azimuth by elevating and training the howitzer until (a) the elevation level bubble was centered, and (b) the vertical line of the telescope was on the aiming point.

In direct fire, (1) the deflection, as ob-

tained from firing tables, was set on the deflection mechanism, (2) elevation or range (as appropriate) and angle of site were set as in indirect fire, (3) the mount was cross-leveled, and (4) the howitzer was trained and elevated until the telescope cross-hairs were on the target.

This panoramic telescope was also used in combination with other instruments. On a later-model howitzer, for instance, it was used for laying in azimuth only; its mount (see Fig. 1-14) had no provisions for elevation, angle-of-site, or range settings. Instead, these were set on Range Quadrant M3 (see Fig. 1-15) which was similar to Range Quadrant M1 (see par. 1-2.4.2.2). This arrangement was used for indirect fire; for direct fire, an elbow telescope was clamped to the top of the quadrant.

The standard panoramic telescope in general use from the latter part of World War I until 1940 was the M1917. At the start of World War II, the M12 was adopted to replace it. The M12 was used primarily for aiming the gun in azimuth in indirect fire against a distant stationary target (a quadrant was used for aiming the gun in elevation). Included on the horizontal crossline of the telescope reticle was a mil scale for use against moving targets under direct fire. This arrangement was unsuitable with rapidly moving targets, however, so this reticle was supplanted in subsequent models by a grid-type reticle having vertical lines to measure lead and horizontal lines to give elevation as a function of range. When used on medium-range artillery weapons, the new reticle made possible the rapid aiming of a gun at a moving target by a single gunner.

1-2.4.3.4 The Battery Commander's Telescope

The battery commander's telescope, which was developed during World War I, is a binocular observation instrument designed for the measurement of vertical and horizontal angles. It was used primarily for spotting and observing the effect of medium and light mobile artillery fire; it was also frequently used for range and position finding (defined as the determination of the actual range and direction of the target from the directing point of a battery).

Figure 1-16 depicts the optical system of the battery commander's telescope. It actually comprises two telescopes with similar optical systems, except that the right-hand telescope has an additional retical that incorporates horizontal and vertical cross lines and a deflection scale. Each telescope is basically the same as a panoramic tele-

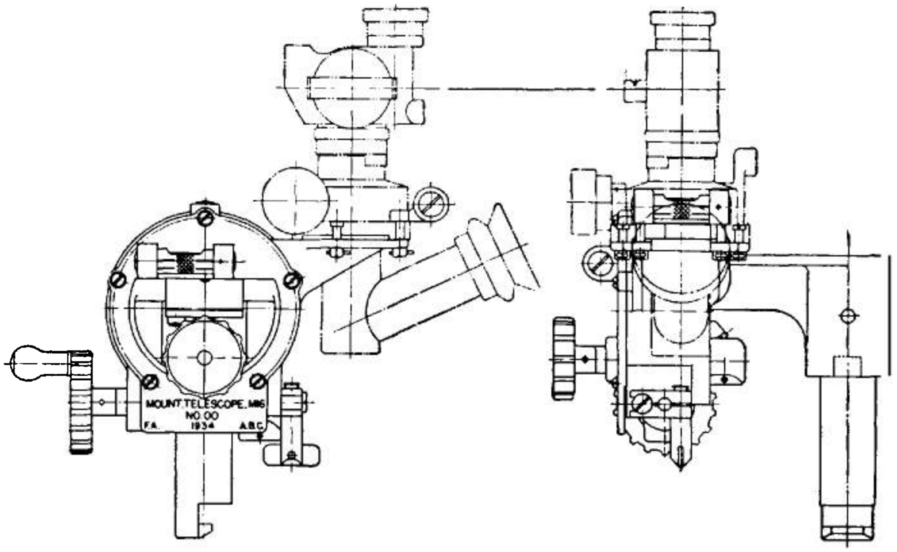


Fig. 1-14. Panoramic Telescope M1 mounted on Telescope Mount M16.

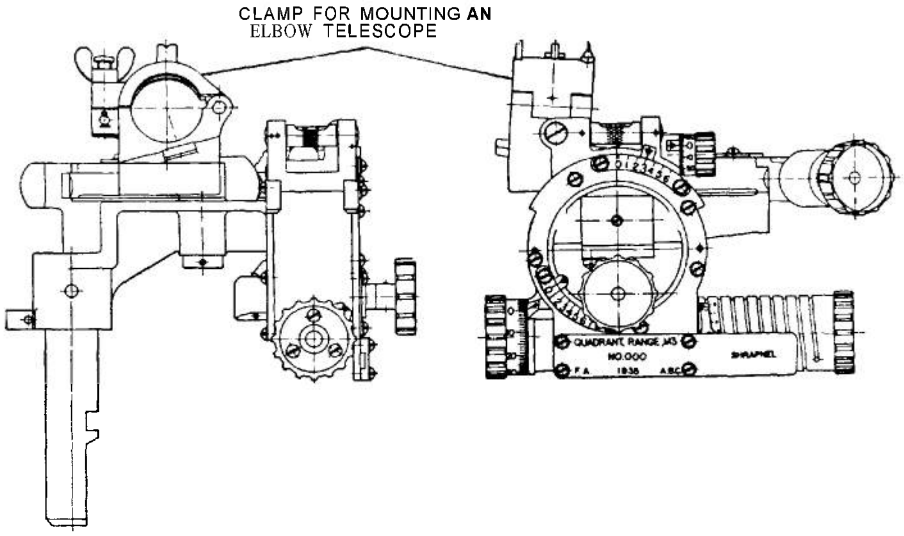


Fig. 1-15. Range Quadrant M3; used in conjunction with Panoramic Telescope M1.

scope (see Fig. 1-12) with the rotating prism omitted.

The Battery Commander's Telescope M1915 was the standard battery commander's telescope during World War I. The major features of this instrument are:

1. The two telescopes can be rotated laterally from a vertical to a horizontal position. In their vertical position, the objective prisms are approximately one foot above the eyepieces, thereby facilitating concealment of the observer.

2. The angle-of-site mechanism permits the measurement of vertical angles over the range -300 mils to +300 mils. The angle-of-site scale is graduated every 100 mils. The angle-of-site micrometer allows readings of 1 mil.

3. The azimuth mechanism provides complete rotation of the telescopic system about a vertical axis. This rotation is measured on an azimuth scale that is graduated every 100 mils. An azimuth micrometer allows azimuth readings of 1 mil.

4. The telescope mount is attached to its tripod by a ball-and-socket joint. With the spherical level, this joint provides means for leveling the telescope.

The Battery Commander's Telescope M1915 was primarily a 10-power binocular with a rather narrow field of view (4° 15'). After World War I the instrument was modified to widen its field of view, to provide reticle illumination, and to improve the telescope in general. These improvements proved of minimal value. A telescope of a new, superior design was tested during World War II and was adopted as standard equipment in 1943. This instrument, the Battery Commander's Telescope M65, provided a wider field of view, better lighting qualities, and a more advanced reticle design than had been possible with the improved M1915.

1-2.4.3.5 Sights for Recoilless Rifles and Bazookas

Sights for the new types of weapons introduced during World War II presented few fire control problems for engineers. Both the recoilless rifle (introduced in 1945) and the shoulder-fired bazooka rocket launcher (1943) required relatively simple sighting equipment.

Recoilless rifles, produced in 57 mm and 75 mm calibers, used straight-tube telescopic sights (see par 1-2.4.3.1) calibrated in yards for direct fire. They could also use panoramic sights (see par 1-2.4.3.3) for

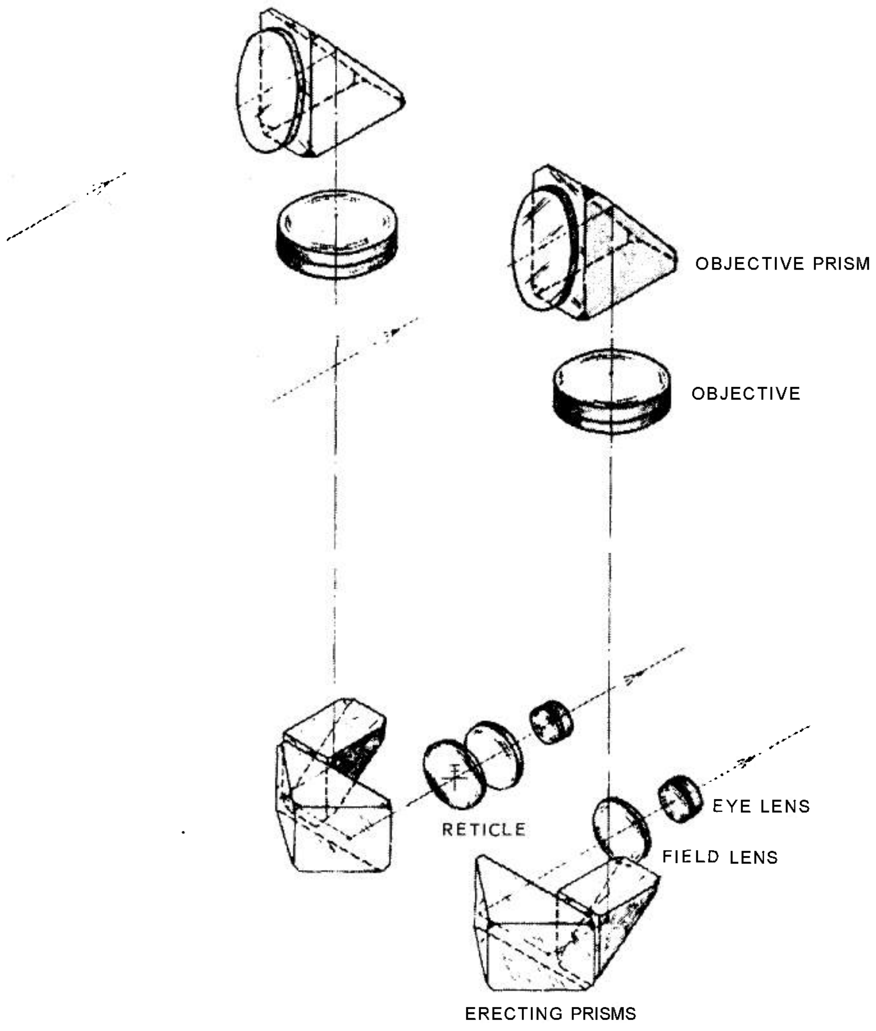


Fig. 1-16. The optical system of the battery commander's telescope.

indirect fire.

Bazookas were first equipped with simple but inaccurate ring sights. Optical devices were later developed but war shortages of critical optical materials forced their abandonment in favor of a hinged-bar sight. However, in combat this sight was subject to alignment inaccuracies and was replaced in turn by a unique reflecting sight. It consisted of a disc-shaped eyepiece in which a small reticle was positioned opposite a concave transparent mirror. The mirror partly reflected the reticle pattern and partly transmitted light. By 1945, all bazookas were equipped with reflecting sights of this type.

1-2.4.4 Devices for Night and Extreme Environments

1-2.4.4.1 Instrument Illumination

During World War I, many of the fire control instruments lacked adequate night-lighting devices for illuminating the scales and levels. The most useful device was the ordinary commercial flashlight which, however, had to be shielded to prevent exposure of weapon position to the enemy.

In the postwar years, the Instrument Light M1 was developed and standardized for the Battery Commander's Telescope M1915.

A lamp was clamped to the telescope that could be pointed at the scale and level. A short flexible cable connected the lamp to a battery case containing a dry cell.

Complicated lighting systems powered by storage batteries were also tried but were abandoned because of an excess of exposed cable, difficulties in installation and servicing, and high costs. Instead, flashlight-type lighting systems using standard dry cells were adopted for all fire control instruments.

Luminous material was also tried; reticles were designed with luminous lines and dots that glowed in the dark. Just prior to World War II, luminous material was used successfully in the level vials of gunner's quadrants--a significant achievement, since the quadrant was normally used in exposed positions where the gunner was in danger of being seen by the enemy if conventional illumination was used.

1-2.4.4.2 Light Collimation

A more sophisticated approach to the problem of directing night fire, suggested shortly after Pearl Harbor, involved the use of infrared illumination. Engineers and scientists turned their attention to the development of an electron telescope capable of penetrating darkness, fog, and smoke. By late 1943, an illuminated collimator was developed that was attachable to a rifle or carbine, yet easily removable for day service. The new collimator performed satisfactorily up to fifty yards during night firing tests. Further development effort in this general area of fire-control sighting equipment was halted at this time, however, because of the lack of a significant need by the Army.

Since World War II, however, the ability to conduct combat operations with armored vehicles at night has assumed increasing importance (see par 1-3.2).

1-2.4.4.3 Antiglare Filters and Protective Lens Coatings

Considerable research was conducted on antiglare filters and protective lens coatings during World War II. Tests made by the Desert Warfare Board in 1942 indicated some advantages to the use of red, amber, and neutral filters for sighting equipment but none justified adoption. Better results were obtained with nonreflecting coating on glass surfaces and the substitution of solid-glass prisms in telescopes for mirrors. Further work expanded into the development of anti-rain and antifog coatings, hoods for protection against sun and rain, as well as mechanical modifications to sighting equipments in order to simplify and facilitate operation.

1-2.4.4.4 Environmental Protection

The effect of extreme cold on the performance of all types of fire control equipment was investigated by the Army at Fort Churchill in Canada during the winter of 1943-44 and yielded valuable design and maintenance-engineering information. The use of fire control instruments in tropical theatres of warfare soon revealed the ravaging effects of fungus growth and other types of deterioration. In June 1944, a committee was formed at Frankford Arsenal to study the protection of fire control instruments. The committee's efforts were directed toward the use of protective coatings, the development of moisture-proof sealing, the incorporation of silica-gel desiccants, and the employment of a volatile fungicide with the instruments.

1-2.4.4.5 Target Illumination and Sound Location

Before the advent of radar during World War II, both light and sound were used to locate aircraft targets approaching under the cover of darkness, fog, or smoke. Systems comprising searchlights, control stations for the searchlights, sound locators, and associated power equipment were utilized. A typical arrangement is shown in Fig. 1-17. The purpose of the sound locator was to provide initial information on the general position of the target. The control station was located about 200 feet from the searchlight so that the controlling observer's view of the target would not be obscured by the diffused light within the beam.

A 60-inch barrel-type high-intensity-arc searchlight was standard in the U. S. Army for antiaircraft artillery fire. The searchlight was usually located in advance of the battery so that a typical aircraft target

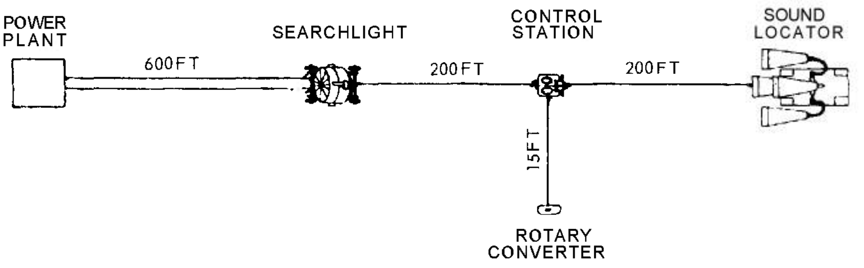


Fig. 1-17. A typical arrangement for a target-illumination system.

would be illuminated at a visual angle* of about 40 degrees just prior to entering the effective zone of gun fire. This advanced position was more favorable to target illumination than a position at the battery itself because at a given slant range an aircraft flying over a searchlight at altitude presents several times more reflective surface than it does coming in head-on in a dive toward the searchlight.

A typical searchlight control station used sound to initially place the narrow pencil beam of the searchlight on or near the target. The station was equipped with two pairs of concentric dials, one pair for azimuth and the other for elevation. In each pair, the inner dial indicated corrected sound-locator data which were received by self-synchronous transmission (see par 1-2.4. 12) from an acoustic corrector. (Prior to World War II, the a-c self-synchronous data-transmission system had been adopted as standard for the U. S. Army.) The outer dial indicated the position of the searchlight. The control-station operator directed the positioning of the searchlight until both of the concentric dials were matched, indicating that the searchlight had been positioned in accordance with the corrected sound locator data. He accomplished this with handwheels connected to two d-c brush-shifting, step-by-step, transmitter motors, which in turn actuated receiving motors at the searchlight that moved appropriately in elevation and azimuth.

The sound locator was constructed on the principle that if one is not facing directly toward a sound source, the sound waves reach his two ears slightly out of phase since they strike one ear before the other. This phase effect was magnified, and the operator's binaural sense sharpened, by the use of four horns spaced several feet apart in a diamond-shaped pattern. The horns all moved together in elevation and azimuth. In effect, they increased the baseline between the ears to the distance between the horns in much the same way that the stereoscopic range finder (see par 1-2. 4. 5. 2) effectively increases the interpupillary distance of the operator's eyes. In addition, the horns served as sound collectors and acoustic amplifiers. One pair of horns located sound in azimuth and one pair located sound in elevation. By means of handwheels, the operator moved the horn assembly in elevation and azimuth until it was so directed that the sound waves were reaching the operator's ears exactly in phase.

Large sound-lag corrections had to be made to the azimuth and elevation angles observed at the sound locator. since sound travels at only about 1100 feet per second (in still air at sea level). Thus, a 300-knot aircraft at a slant range of 11,000 feet would advance 5000 feet during the 10 seconds required for its noise to reach the sound locator. Furthermore, the lag varied with atmoshperic con-

* Angle between the longitudinal axis of the aircraft and the searchlight beam.

ditions.

A typical acoustic corrector was carried on the trailer of the sound locator and computed the corrections as follows:

1. It divided the estimated slant range (which was manually set into the device) by 1100 feet per second to obtain approximate sound lag.

2. It measured the angular change of the target position in azimuth and elevation (as supplied by the sound locator) during a fixed period to obtain the angular rates in azimuth and elevation,

3. It multiplied the target angular rates by seconds of slant-range sound lag to determine the required azimuth and elevation corrections.

4. It combined these corrections with corrections for wind and other atmospheric conditions.

5. It added the combined corrections to the directional observations obtained from the sound locator, thereby providing corrected indications of the present position of the target.

6. It continuously transmitted these corrected data to the control station by self-synchronous motors; the data were used in the manner already noted to place the searchlight beam on the target.

1-2.4.5 Optical Range Finders

From the earliest days of warfare, target ranges for direct-fire weapons were obtained by estimating; errors were corrected by adjustments based on observed fire. As gun accuracy and range increased, the problem of range finding became more acute until a mechanical range-measuring instrument, called a stadimeter, was developed by the Navy in 1898. Coupled with the advent of well-regulated powder charges, this development resulted in relatively accurate control of weapon fire; fire control was transformed overnight from guesswork to science.

The stadimeter was crude, was accurate for only short ranges, and depended on knowing target height or other dimension. Army ordnance engineers, utilizing the principle of the stadimeter, pressed forward in the design of more dependable and accurate range finders. The instruments that evolved were based on the solution of a horizontal right triangle whose apex is the target and whose base is the optical length of the range finder. The base and, of course, the right angle between base and line of site are fixed; range is then proportional to the other, variable base angle, which is measured. The degree of magnification and the length of the range finder controlled, to a great extent, the accuracy and effective range.

Two types of optical range finders were developed: first the coincidence range finder, and later the stereoscopic range finder. The coincidence range finder utilized a single eyepiece and measured the angle required to move a prism *so* that the images of the target picked up at the two ends of the range finder (i.e., the base line) were aligned (see Fig. 1-18). In the stereoscopic range finder, two eyepieces were arranged to utilize the observer's sense of stereoscopic vision in determining the range to the target.

1-2.4.5.1 Coincidence Range Finder

Figure 1-19 represents schematically the optical system of the coincidence range finder which comprises the following ele-

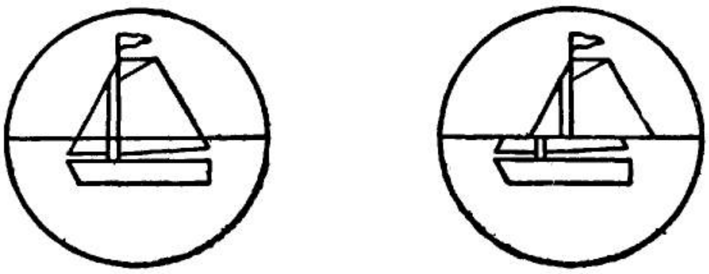


Fig. 1-18. Partial images in and out of coincidence in a coincidence range finder.

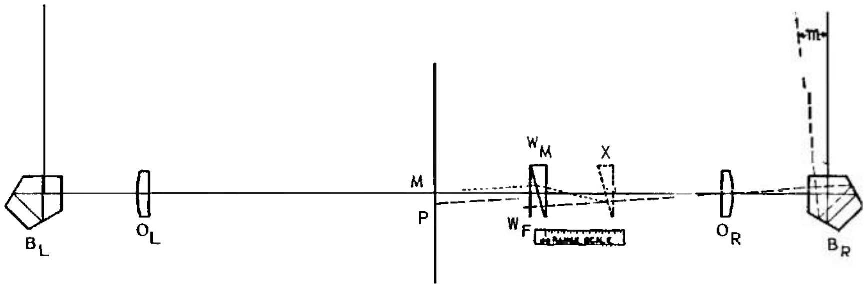


Fig. 1-19. The optical system of a coincidence range finder.

ments:

1. Penta prisms B_L and B_R at the left- and right-hand ends of a horizontal base line.
2. The associated objectives O_L and O_R.
3. Two identical wedge prisms of very small angle W_F and W_M, mounted in inverted positions, as shown. W_F is fixed, while W_M is movable.
4. An eyepiece and an associated ocular prism (not shown). The optical axis of the eyepiece is perpendicular to the base line at its center M so that its focal plane is parallel to the base line.

The rays of light entering prisms B_R and B_L from a target at an infinite distance will be parallel and will be reflected by B_R and B_L at right angles, so that they will pass along the base-line axis of the instrument. The images formed by the rays from B_R and B_L will be in coincidence at point M in the focal plane, provided wedge prisms W_F and W_M are in the closed or infinity position; in this position, the light rays will pass through without being refracted since the exterior surfaces of W_F and W_M are parallel.

If the range finder is directed at a target within its working range, however, the rays of light entering B_L and B_R from that target will not be parallel. Instead, if the range finder is directed so that the target image through B_L is at M, as before, then the image formed by the ray through B_R will no longer be at M, but at some other position P, and the two images will not be coincident. The displacement MP is a function of the angle m, which is inversely proportional to the target range.

The image formed by the ray through B_R can be brought into coincidence with the other image at point M by moving the wedge prism W_M to the position X. Each of the two wedge prisms in this open position refracts the light ray through an equal angle, W_M refracting upward and W_F refracting downward. The distance through which W_M is moved from its infinity position—in order to achieve coincidence of the two images—is proportional to the angle m and is thus a measure of the range which may be read from a scale affixed to the range finder.

After passing through W_F and O_L, the light rays are reflected by the specially constructed ocular prism to the focal plane of the single eyepiece. The field in the focal plane is divided by a sharp horizontal line, as shown in Figure 1-18. The partial image in the lower half of the field is produced by rays passing through B_L, while the partial image in the upper half is that produced by the rays passing through B_R. In some coincidence range finders, the images are inverted, as shown in Fig. 1-20. This arrangement is desirable for targets that are clearly defined against the sky.

A typical early coincidence-type range finder was the Range Finder M1916. It had a 1-meter base, 15-power magnification, inverted images, and a 3°10' field of view. The range scale was graduated from 400 to 20,000 yards (a range that far exceeded the instrument's capabilities, however). An elevation mechanism permitted rotation of the plane of sight 18° above and below the horizontal. Also provided, with scales and micrometers, were angle-of-site and azimuth mechanisms.

The Range Finder M1916 was used by the Infantry, the Field Artillery, and the Cavalry. Accordingly, it was designed to be easily

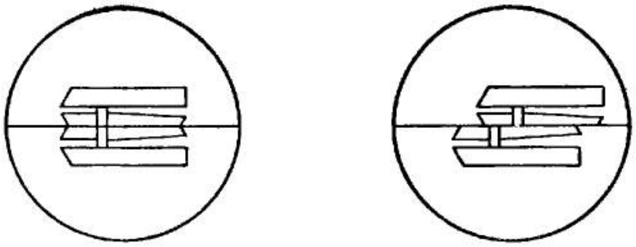


Fig. 1-20. Partial images in and out of coincidence in a coincidence range finder that employs inverted images.

transportable and ruggedly constructed to permit its precision optical equipment to withstand the abuses of rough handling; it became the standard range finder for field artillery during World War I and, with minor modifications, for the two decades following. The Army tried to design an entirely new instrument that was more compact, rugged, and lightweight and that could be produced more quickly and rapidly. While Germany was marching over Europe in the late 1930's, experimental models were introduced but failed to match the proven superiority of the M1916. In 1942, the Army acquired, for test purposes, a 1-meter Canadian-built range finder that was being produced for the British. The Fire Control Laboratory at Frankford Arsenal found it more accurate and lighter than the M1916, and easier to manufacture in a period when materials were in short supply. It was accordingly adopted in December of 1942 as fire control equipment for field artillery (with the designation M7) and for infantry use (with the designation M9).

From World War I until early World War 11, several other models of coincidence range finders were used, ranging in length from 80 cm to 30 ft. (The larger models were used on fixed bases, principally for coast artillery.) Coincidence range finders were satisfactory for fixed or slowly moving targets; however, coincidence on fast-moving targets is difficult to achieve. Stereoscopic range finders are more effective on such targets.

1-2.4.5.2 The Stereoscopic Range Finder

The stereoscopic range finder differs from the coincidence range finder in that (1) it is a binocular device and (2) its operation depends upon stereoscopic vision, i.e., the capability of seeing objects in three dimensions. In a human being, this capability is due to the spacing between the eyes (the so-called interpupillary distance), which causes different images to be formed on the retinas of the eyes when an object is viewed. The brain then converts this image difference into an estimate of the distance of %he object from the observer; the ability to estimate distance in this way is called <u>depth perception</u>. It is important to note that, when two objects are observed simultaneously, stereoscopic vision enables the observer to judge with considerable accuracy the <u>relative</u> distances to the two objects. This relative-distance estimation, rather than the estimation of actual distances, is the basis of stereoscopic range finding.

With the unaided eyes, the absolute limit of stereoscopic vision is approximately 480 yd. In a stereoscopic range finder this range is increased by magnification and by increasing the base length of the instrument, which effectively becomes an increased spacing of the operator's eyes.

The stereoscopic range finder operates as described below (see Figs. 1-21 and 1-22). A system of reticle marks and optical wedges are used to achieve the range-measurement objective, as shown graphically in Fig. 1-21. With the reticles directly in front of the observer's eyes, as shown, the incoming rays are parallel and the reticle marks appear to be at infinity as is the case when one is gazing at a star. Suppose the right-hand reticle can be moved along the axis of the range finder to positions 1, 2, and 3. Then the two images in the observer's eyes will be combined so that he will sense the positions of the reticle marks in space to be at corresponding distances I, II, and III. The amount of move-

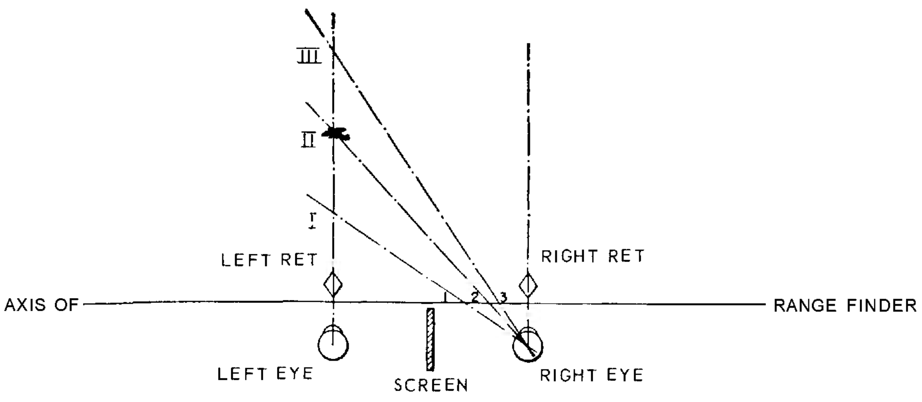


Fig. 1-21. Graphical illustration of the function of reticle marks in a stereoscopic range finder.

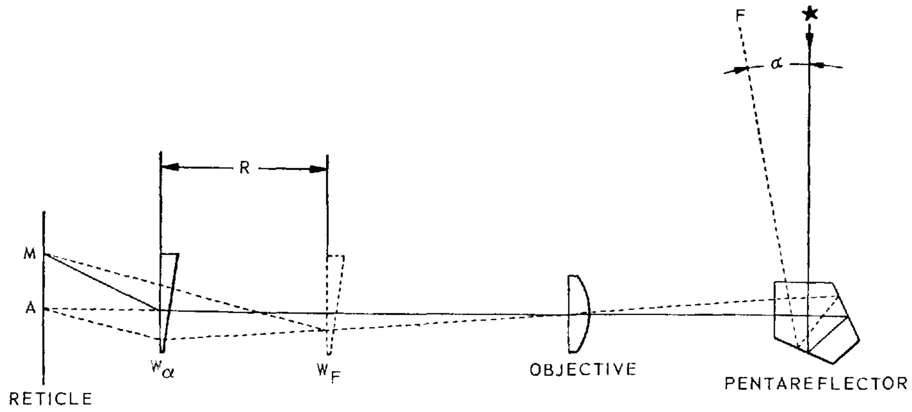


Fig. 1-22. Schematic representation of the right-hand side of the optical system of a stereoscopic range finder.

ment that the reticle undergoes in translating along the axis from point 1 to point 3 is thus a measure of the distance in space between points I and 111. Therefore, if the right-hand reticle is moved so that its central mark appears to be at the same distance in space as an observed object at point II—e.g., an aircraft target—then the movement of the reticle can be used as a measurement of the target range.

In the practical design of a stereoscopic range finder, the reticles are held in fixed positions for greater accuracy, but the same result is obtained as in Fig. 1-21 by moving the right-hand image of the target as shown in Fig. 1-22. The effect on the observer is the same as before: as range changes, the target will appear to remain stationary while the reticle mark will appear to move. An optical ray from an infinitely distant object such as a star would be deflected 90" by the pentareflector at the right-hand end of the range finder and then travel along the range-finder axis until it struck the optical wedge W„ where it would be deflected to the point M on the fixed reticle. The subscript α on wedge W, indicates that the position of the wedge can be made to vary with the angle α of an incoming ray in the same way as with the coincidence type of range finder. With the wedge at the infinity position on the range scale, the point M – where the optical rays from an infinitely distant object hit the reticle – would coincide with the central mark on the reticle.

If a target F at a finite distance were under observation, the optical ray entering the right-hand side of the range finder would

be at an angle α with respect to the perpendicular to the base line and would follow the dotted-line path shown on Fig. 1-22, ending up at point **A** on the reticle. In order to make points A and M coincide, the optical wedge W_α would have to be moved a distance R to the new position designated W_F. The distance R necessary to move the wedge to obtain coincidence is a function of the angle α which, in turn, is inversely proportional to the range. Thus, as in the case of the coincidence type of range finder, the distance the wedge has to be moved provides a measure of the range to the target.

In practice, the observer rotates a pair of wedges instead of translating a single wedge. The effect sensed by the observer, however, is the same. The motion of the wedge-pair is transmitted by gearing to a visible scale, thereby enabling the target range to be read directly.

Stereoscopic range finders found early use in both seacoast artillery and field artillery. The fixed installations at seacoast batteries permitted the use of large, accurate stereoscopic range finders with bases 9 to 30 feet long. For field artillery use, however, smaller range finders were required that were easy to transport and rugged, so that their precision optical equipment could withstand rough handling. In response to this problem, the Army developed the 1-meter M1916 coincidence-type range finder (see par 1-2.4. 5. 1) that remained the standard range-measurement equipment for two decades. Although few stereoscopic range finders remained in use by World War II, that type of instrument has subsequently been favored in the development of tank fire control systems (see par 1-3. 2); however, the present trend is toward coincidence-type range finders.

Just before World War 11, stereoscopic height finders were used extensively by antiaircraft artillery. (These instruments were made obsolete by radar during World War 11.) This type of height finder was, in effect, a stereoscopic range finder, with an additional optical wedge system (comprised of two optical wedges and associated mechanical parts) for measuring aircraft altitudes. The additional components solved the right triangle in which the angle of elevation and measured slant range were known quantities.

The chief advantages of the stereoscopic type of range finder over the coincidence type are (1) improved accuracy, (2) superior ability to function under conditioris of poor visibility, and (3) adaptability to operation against small, fast-moving targets. The chief disadvantage is that, despite the utmost care, changes in optical alignment occur in the range finder, so that it must be frequently checked and adjusted. There is also a belief prevalent in the U. S. that the training of operators of stereoscopic range finders is difficult.

1-2.4. 6 Tank Fire Control Equipment

The British built and used the first tank during World War I in an effort to break the stalemate of trench warfare that prevailed during the first phases of that war. Subsequently, in the period between the two World Wars, much effort was devoted to tank development, particularly by the Germans. World events during the 1930's emphasized the importance of the tank in modern warfare and U. S. Army ordnance engineers also directed considerable attention toward the end of that decade to the problems of tank warfare.

The use of rapidly moving tanks and armored vehicles confronted ordnance engineers with two basic requirements:

1. To design fire-control sighting equipment that would enable guns to be aimed more rapidly at swiftly moving tank targets.
2. To develop observing and sighting devices for tanks themselves.

As a result of the first requirement, antitank reticles were devised that allowed the gunner to accommodate proper target lead and range adjustment at the same time. The M6 telescopic sight was adopted as standard in 1938 for use with the 37 mm antitank gun. Later, other sighting telescopes employing antitank reticles were developed that permitted antiaircraft weapons to be brought to bear on ground targets.

The second requirement — improved means of observation by tank crews — posed serious design problems for Army ordnance designers. Prior to 1940, targets were sighted through narrow openings in the turret. These direct-vision slots weakened tank armor and increased the danger to the tank crew

from projectile fragments; also, their limited visibility forced the crew to open the turret to make observations, exposing them to enemy fire. (Despite the hazards, most crews reportedly preferred this technique.)

To solve these problems, experimentation with many sighting devices based on the principles of the periscope was undertaken. At first these devices were unsuitable because the observer had so little room to move his head in the narrow confines of the tank interior. Late in 1940, however, Army weapons design engineers integrated the periscope with a telescope in an effort to give both the instrument and observer some degree of protection; two experimental tank periscopes, the T1 and the T2, that incorporated a straight-tube telescope for gun sighting were designed. A linkage mechanism to the gun enabled the gunner to aim the weapon for direct fire simply by centering the proper telescope reticles on the target without moving his head since the line of sight moved with the gun. The optical line of sight was adjustable in deflection and elevation for boresighting the weapon but adjustment proved difficult. The two periscopes were standardized in 1941; the M1 for the 75 mm gun and the M2 for the 37 mm gun.

Early in 1942, a more complex and expensive but also more accurate periscope, the TC, was developed as a major improvement over the M1 and M2 units. This periscope utilized a high-powered telescope on the right-hand side for sighting distant targets; the periscope itself, on the left-hand side, employed a reflex reticle for sighting nearby targets. Despite the high costs of manufacture inherent in the optical and mechanical features of the design, the evident superiority of the instrument warranted its acceptance and standardization as the M10 in 1944.

The upper end of telescopic periscopes projected above the armor plate of tanks; accordingly, a chance hit would shatter the exposed window, mirror, and body of the unit. Therefore, additional direct-sighting capability was subsequently provided the tank crew by means of a small, straight-tube telescope that permitted sighting through a tiny opening in the turret. This instrument, the M70, was standardized in 1943 on the basis of having acceptable optical characteristics (adequate magnification and a wide field of view) and a size small enough to allow accommodation to the limitations of space inside a tank. The uniquely small aperture in the tank turret used with this sighting instrument minimized danger to the tank crew from enemy fire. Later improvements increased telescopic power from 3-power to 5-power in the M71 (see Fig. 1-23), which became standard equipment on most tanks by 1945. The M71 manifested a wider field of view and better light-gathering power than the M70.

A variable-power telescope was later developed that could be readily adjusted to provide either 4-power magnification with a relatively wide field of view or 8-power magnification with a much narrower field. This major innovation, the M83, was adopted near the end of World War II. It was uniquely adaptable for aiming at close-by targets, using its 4-power capability, and for sighting on distant targets, with its 8-power adjustment.

During World War II, the fire-control capabilities of the tank were patently limited by the lack of satisfactory range-finding equipment. The M71 used a ballistic reticle, like that shown in Fig. 1-24. The tank gunner first estimated range by eye and then elevated the weapon until the proper range graduation of the reticle was placed on the target. The deflection pattern of the ballistic reticle permitted the gunner to make allowance for target motion. The same principle was used in tank periscopes such as the M4A1 (see Fig. 1-25).

Accordingly, ordnance engineers in 1944 and 1945 applied themselves to the task of developing an integrated tank fire control system that would properly combine ranging, computing, and aiming functions. The end of the hostilities found the project still in the development stage. However, optical range finders were later devised that constitute the primary sighting system for our current medium tanks.

For recent developments in tank fire control, see par 1-3.2.

1-2.4.7 Tank Stabilization Systems

The African Campaign in World War II revealed the need for a stabilized tank gun

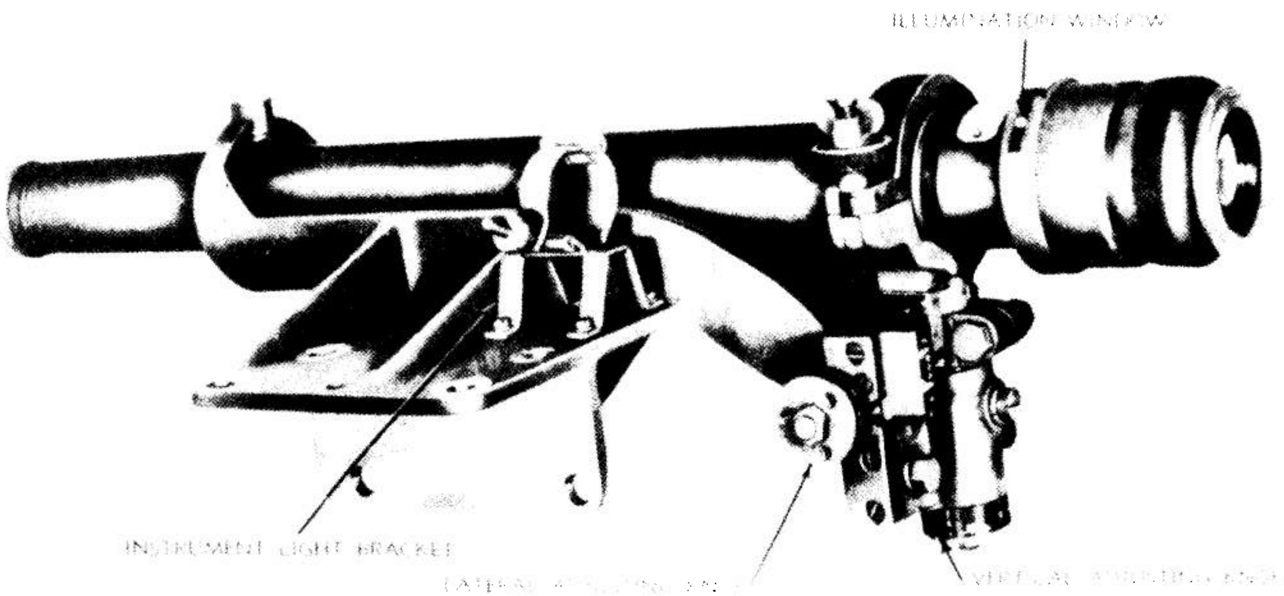
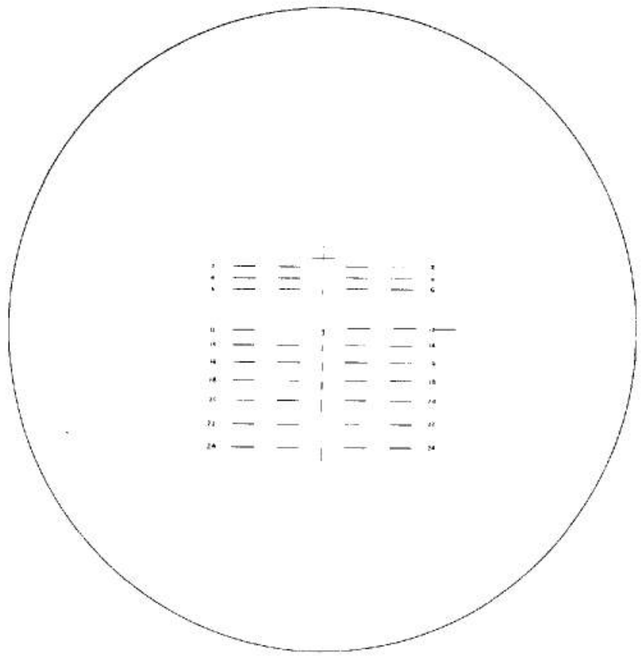


Fig. 1-23. Telescope M71D and Telescope Mount M57.



Notes:

1. Each vertical line and space equals 100 meters.
2. Each horizontal line and space equals 5 mils.
3. There are 1200 meters on the optical axis.
4. The field presentation is shown as viewed 10 inches from the eye.

Fig. 1-24. The ballistic reticle for the 152 mm round.

for accurate fire from a tank moving over rough terrain. Crews were forced to stop the tanks momentarily to aim accurately, thereby providing enemy artillery with convenient sitting targets. Shortly thereafter the Westinghouse elevation stabilizer was placed in the Medium Tank M4, the series of tanks that included the Light Tank M5 and the Medium Tank M26.

Maximizing the advantages of high-powered tank sighting systems for gun laying while a tank was in motion became a paramount objective of Army weapon design engineers during World War II. The ultimate goal was a stable platform for completely stabilizing tank weapons during travel over rough terrain. The gyrostabilizer, the stable element employed by the Navy to lay ship's guns in accordance with computed orders (obviating the need to fire only in the middle of a ship's roll), provided the Army with the logical answer to the tank fire control problem.

The gyrostabilizer is based upon the spinning gyroscope's tendency to resist displacement away from its axis of spin. Applying the gyrostabilizer to vehicles, ordnance engineers mounted a gyroscope on the gun cradle with its spin axis parallel to the gun axis. Displacement, however small, of the gyro control mechanism due to the vertical pitching of the tank as it lumbered over rough terrain produced gyroscopic forces that returned both gun and control mechanism to the original aiming position. In effect, the tank weapon remained fixed in space, pointed at its target, while the vehicle rocked about the weapon. However, lack of intensive training of gun crews limited the usefulness of stabilized tank guns in combat; gun crews still preferred to stop their tanks to obtain accuracy of fire. As a result, preliminary steps were taken by the Army to abandon the use of these expensive stabilizers but these steps were later rescinded. Instead, a program of intensive training was undertaken;

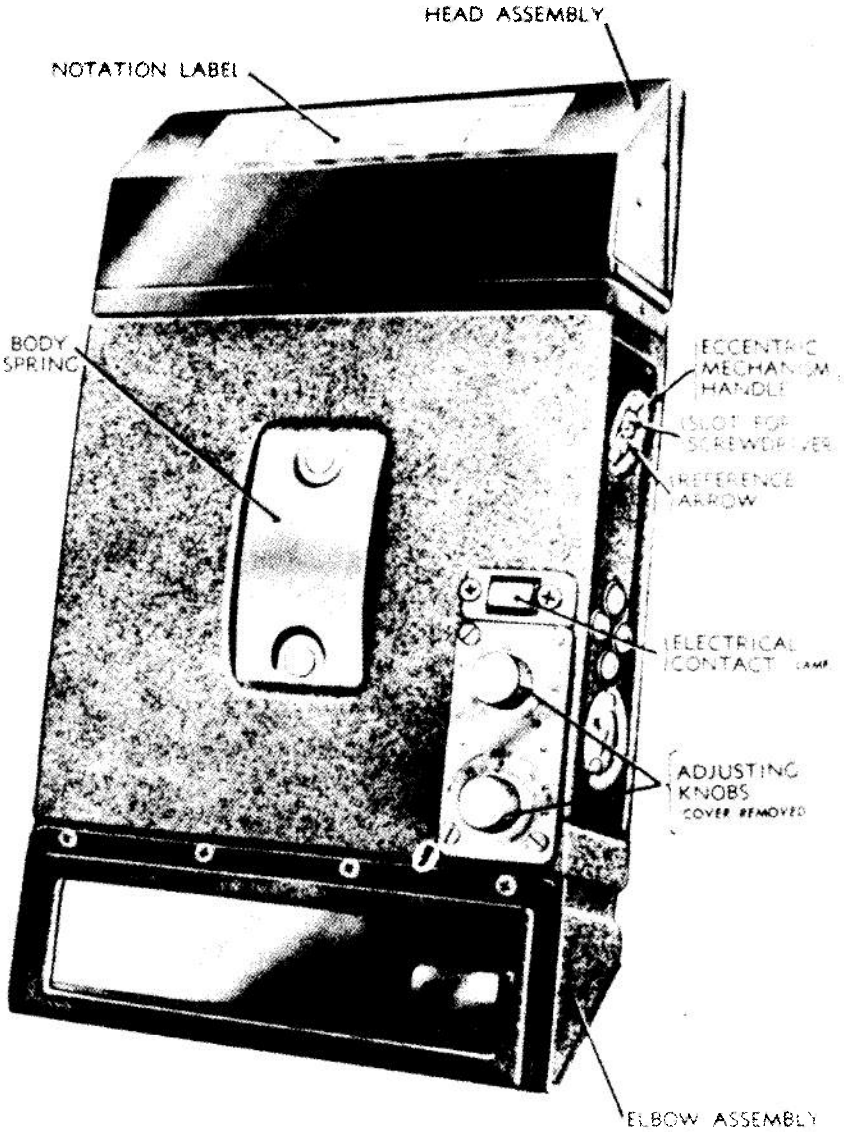


Fig. 1-25. Periscope M4A1 (rear view).

by the end of World War II, effective use was being made of gyro control equipment.

Work on tank stabilization systems has continued with strong emphasis in the years subsequent to World War II (see par 1-3.2).

1-2.4.8 Coast Artillery and Antiaircraft Fire Control Equipment

The defense of United States territory by seacoast gun batteries (established in the latter part of the 19th century) and antiaircraft gun batteries (established in the early part of the 20th century) emphasized the need for developing fire control equipment and systems for these weapons. The evolution of faster moving, more heavily armored vessels was paralleled by formidable improvements in the speed, range, and accuracy of Coast Artillery weapons. The problems of seacoast artillery were unique: targets were often below gun elevation; ships maneuvered at sea, presenting a field of fire that offered no reference points; and because of the techniques then current for acquiring accurate gun-fir-

ing data, gun batteries were separated from fire-direction centers, which complicated the transmission of firing data and delayed the time of fire. Antiaircraft artillery introduced further complications since the targets moved in three dimensions and were much faster and more maneuverable than ships.

The paragraphs which follow highlight the development of the various kinds of equipment — primarily computational — that were devised to supplement the sighting and range-finding equipment already described, as required to form effective fire control systems for seacoast artillery and antiaircraft artillery.

1-2.4.8.1 Plotting and Correction Devices

Early in the 1900's, ordnance engineers resolved the various problems associated principally with seacoast artillery fire by devising specially instrumented fire-control aids. These problems and the devices that were developed to solve them can be summarized as follows:

1. Range and position finding: the determination of the actual range and direction of the target from the directing point of an artillery battery. For moving targets, this includes the prediction of the set forward point which is the predicted future position of the target at the instant the projectile is expected to arrive at the target. The various devices developed to solve these problems can be classified as either plotting boards or plotting and relocating boards.

2. Range and deflection correction: the determination and application of position, materiel, and weather corrections for deviations of existing conditions from those assumed in the firing tables; once the actual range and azimuth of a stationary target – or the set forward point of a moving target – have been established, it is still necessary to consider the various factors that influence the flight of a projectile through the air before applying standard firing data to the problem. (As discussed in Chapter 2, these factors include pre-determined variations from the normal in such items as the density of the air, the direction and velocity of the local wind, the rotation of the earth, and perhaps the weapon itself or the propellant used.) While such corrections could conceivably have been calculated from the firing tables, this procedure was not practical with moving targets or under other circumstances requiring rapid application of firing data. The various devices that were developed to determine and apply the necessary range and deflection corrections include range correction boards, deflection boards, and percentage correctors.

3. Spotting and adjustment: Spotting is the process of locating the points of projectile impact with respect to the target or some adjusting point. Adjustment is the calculation and application of the range and azimuth corrections required to place the center of impact on the target. The necessity for spotting and adjustment stems from the fact that, in spite of (1) the care taken in the range-and-position-finding phase and (2) the refinements employed in determining and applying range and deflection corrections to the firing data, it is virtually impossible to place the center of the projectile impact exactly on the target. Spotting was accomplished by either bilateral spotting, in which two stations some distance apart were used, or single-station spotting. With the latter method, only sensings were generally attempted, i.e., determination of whether the points of impact were over or short and right or left of the target. Spotting by aerial observation was a typical means of single-station sensing. In bilaterial spotting, which was much more accurate, the angular deviation of the impact was read on an instrument at each of the two observation stations. From these two angular deviations, the range and direction deviations were determined graphically by means of a spotting board. This completed the spotting phase. In the adjustment phase, the range and direction deviations were used in the calculation of the required range and azimuth corrections. Adjustment boards and charts were commonly used to facilitate the determination of these corrections.

The paragraphs below describe briefly the systems and devices used to find range and position, correct, spot, and adjust.

Range and position were determined by the "horizontal base system". This system, which was standard for the seacoast artillery, required two observing stations, each equipped with an azimuth reading instrument. A

typical 1918 instrument was mounted on a tripod and consisted of an elbow telescope mounted on a yoke as well as a vertical shaft, so that it could measure elevation as well as azimuth. Azimuth and elevation micrometers permitted fine readings, and a throw-out lever was provided for swinging the instrument rapidly on target. The observing stations were located at the ends of a measured horizontal baseline. They furnished data to a plotting board for use in the manner already described. Another range-and-position-finding system, known as the "self-contained horizontal base system", required only a single observing station equipped with a horizontal-base range finder (such as those described in pars 1-2.4.5.1 and 1-2.4.5.2) that was also capable of reading azimuths. This system was used normally by the smaller, rapid-fire seacoast batteries, and by the larger seacoast batteries as an emergency standby. Other applications were for field artillery, cavalry, troops equipped with infantry weapons, and — subsequently — tank fire control systems.

The essential geometry of the horizontal base system is shown in Fig. 1-26. AB represents the measured base line, C the target, X the direction point of the battery, and the angles CAB and ABC the azimuth angles determined by measurement. While it is evident that the system of triangles in Fig. 1-26 could be solved by trigonometric calculation to determine the length and orientation of the line XC, the need for speed precluded such a slow process. Therefore, a plotting board was used.

Plotting boards located accurately, to scale, the field of fire of the battery and all the elements of the range-and-position-finding systems in their correct relative positions. An early plotting board, used primarily for mortar fire, was the M1904 plotting board. Modifications to this hoard led to the development of the M1915 plotting board during World War I. Later, the more intricate M1923 (Cloke) and M1 plotting and relocating boards were devised for use with all types of mobile seacoast artillery.

The principle of operation of a plotting board is illustrated in Fig. 1-27. The top sketch shows a typical seacoast artillery fire control situation, which includes (1) a ship target, (2) a base line (B'B'') whose length and direction are accurately known, and (3) a directing point (D. P.) of the battery which is located accurately by means of coordinates with reference to the base line. The bottom sketch shows the basic elements of the plotting board, which consists of the B' arm, the B'' arm, the gun arm, and an azimuth circle which is graduated and oriented so that azimuth angles actually determined at points B' and B'' at the ends of the base line can be duplicated on the plotting board. The plotting board was used as follows:

1. The operator initially set the pivot points of the three plotting board arms so that the field of fire of the battery and all elements of the range-and-position-finding system were located accurately to scale on the plotting board in their correct relative positions.

2. He then set the ends of the B' and B''

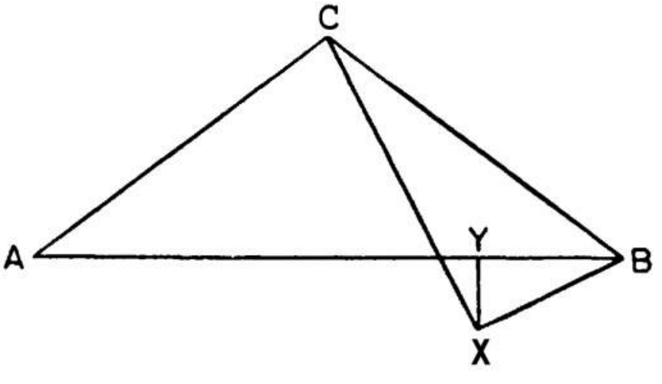


Fig. 1-26. The essential geometry of the horizontal base system.

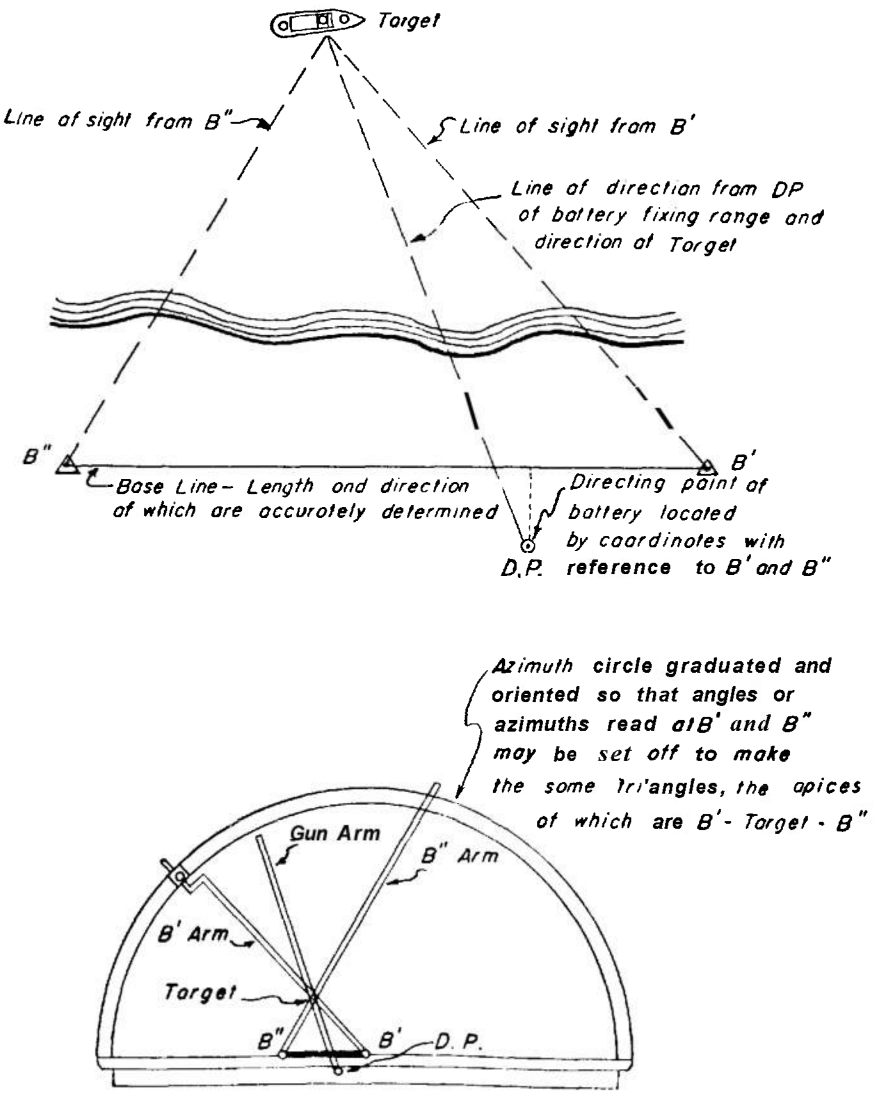


Fig. 1-27. A typical application of a plotting board.

arms of the plotting board on the azimuth circle in accordance with actual azimuth angles determined at and transmitted from points B' and B".

3. He next plotted the intersection of these two arms. This located the instantaneous position of the target.

4. He then placed the gun arm over the plotted target point. Since the gun arm carried a graduated range scale and had an associated azimuth scale, the operator could directly read the range and the azimuth of the target from the directing point.

5. For the general case of a moving target, the operator repeated the plotting process at predetermined intervals. This procedure yielded a plot of the target's course, from which its speed could also be determined. The location of the set forward point on the plot could then be predicted by extrapolation of the plotted course (taking into account the time of flight of the projectile), and the range and azimuth of this point from the directing point could be read by means of suitable scales.

Plotting and relocating boards, developed at a later date, permitted greater flexibility in shifting from one base line to another

and relocating the target, by assuming the target to be stationary and plotting successive relative positions of the directing point with respect to the target. The mechanical equipment used to accomplish this is shown in Fig. 1-28. It was more complex than the plotting board described above; but with the built-in adjustments plus replaceable scales, it could be adapted for use under almost any set of conditions.

Range and deflection corrections. It was recognized early that range must be corrected for nonstandard ballistic conditions and that the corrected range must be translated into suitable data for pointing guns in elevation. A mechanical device, Range Correction Board M1, was adopted as standard for Coast Artillery (see Fig. 1-29). It mechanically computed and combined range corrections required for the prevailing nonstan-

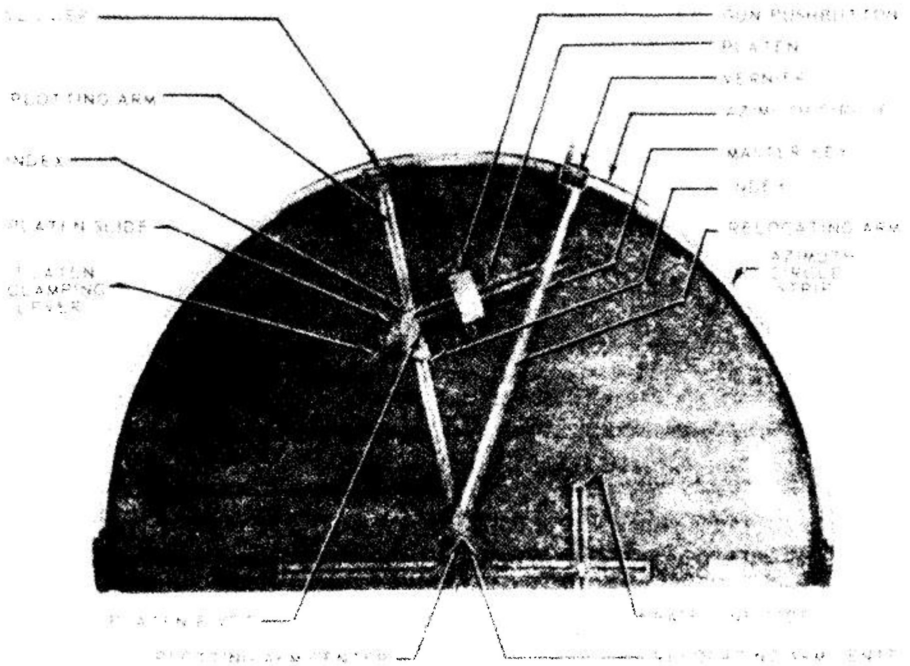


Fig. 1-28. The mechanical configuration of a plotting and relocating board.

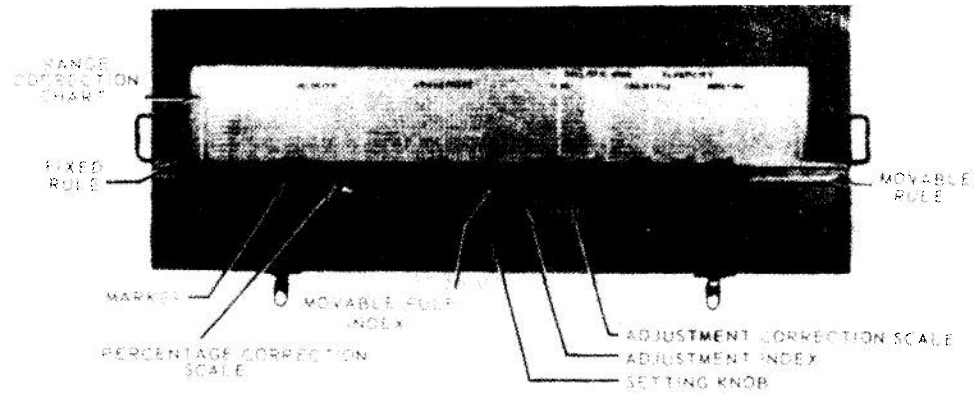


Fig. 1-29. Range Correction Board M1.

dard conditions, such as variations in effects of wind and tide, projectile weight, muzzle velocity, air density, temperature, and rotation of the earth. The result was a net correction as a percentage of the range. Percentage Corrector M1 (see Fig. 1-30) was used with this board. It was a mechanical instrument for applying ballistic corrections, as well as fire-adjustment corrections, to the actual range of the target to achieve the corrected range or elevation.

The deflection board was developed for the mechanical computation and application of corrected azimuths. One of the earliest, developed for guns, was the M1905. An improved model, Deflection Board M1 (see Fig. 1-31), introduced the principle of applying wind, drift, earth-rotation, and fire-adjustment corrections in a manner similar to that used in the Range Correction Board M1-Percentage Corrector M1 combination.

Spotting boards were employed in seacoast artillery to determine the range and lateral deviations of the point of impact from the target. One of the earlier types developed by the Army was the Gray spotting board. This was later modified and followed by the Cole spotting board, an improved instrument that featured an adjustable spotting base line. Spotting Board M2 (see Fig. 1-32) was finally adopted as a standard instrument for determining range and lateral deviations by means of bilateral observation. These deviations were applied as range and deflection fire-adjustment corrections by means of a percentage corrector and a deflection board, respectively, such as those already discussed.

1-2.4.8.2 Mechanical Computers

The advantages of the plotting and correction devices described in par 1-2.4.8.1 were not fully realized since some of them required as many as four operators to perform the required tasks. The resulting human-error factor and the time lag between error-data acquisition and correction setting induced the Coast Artillery to draw up specifications for computers that would automatically produce firing data.

One of the earliest firing-data computers was the Mechanical Computer M1917, developed by the French for antiaircraft purposes and adopted as standard during World War I by the United States. It represented an initial approach to the complex gunnery fire-control problems that were beginning to extend beyond the reach of human performance capabilities, and was considered one of the best of its kind in 1917.

However, it did not allow for nonstandard conditions, and worse, it required time to transmit and apply the firing data to the gun, since at that time electrical transmission of data to guns had not yet been achieved. Instead, firing data were telephoned to the gun, often from remote locations. The concept of

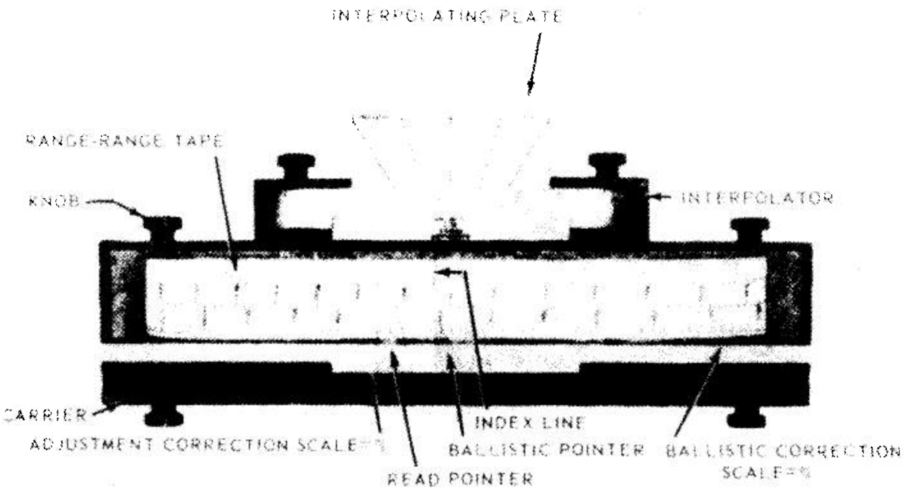


Fig. 1-30. Percentage Corrector M1.

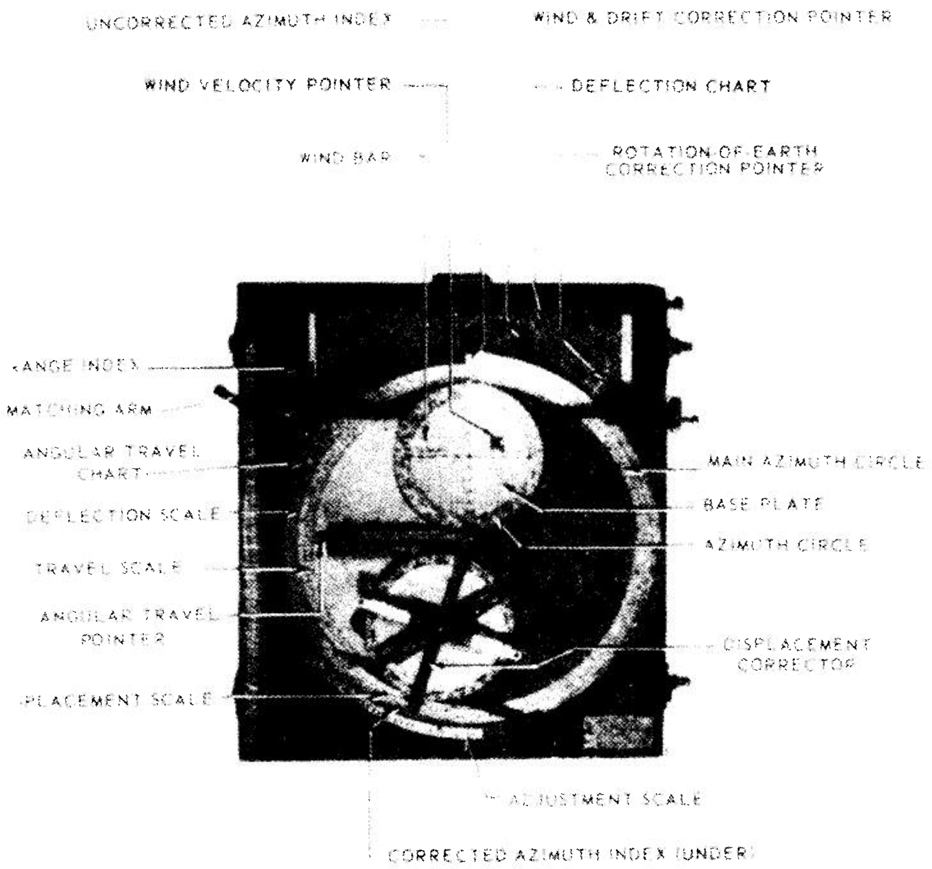


Fig. 1-31. Deflection Board M 1.

instantaneous and continuous calculation of data and its application to the weapon was in large measure invalidated by these disadvantages.

1-2.4.8. 3 <u>Early Mechanical Directors</u>

Before and duriiig World War I, the British Admiralty mastered the principle of director fire, by which a battery of ship's guns could be positioned for firing from some advantageously remote location. Satisfactory gun directors based on those principles were designed and built by the English Vickers Corporation for the British Navy. Soon afterwards, other directors became available to British military ordnance. U. S. Army ordnance, borrowing a leaf from its Navy counterpart, adopted as standard a Vickers-designed director designated M 1, The design of this director was based on the target angular-rate-of-travel method (see par 2-3. 3. 2 of Chapter 2) for determining lead, It was of the semi-ballistic type, i.e., there was a partial correction for the nonstandard conditions involved in projectile Flight.

For the next two decades, the search by ordnance engineers for automatic computing devices that would eliminate human error and save on time and manpower culminated in the development of the standard M2, M3, and M4 Directors, which were fully corrected for nonstandard ballistic conditions. These equipments were designed to use the target linear-speed method for determining lead (see Chapter 2, par 2-3. 3. 2) and to compute ballistic data by means of three-dimensional cams. They were classified as universal directors – because they could be used against air, land, and sea-going targets – and their field of operation included 360° of traverse, 10° of depression, and 80° of elevation. They

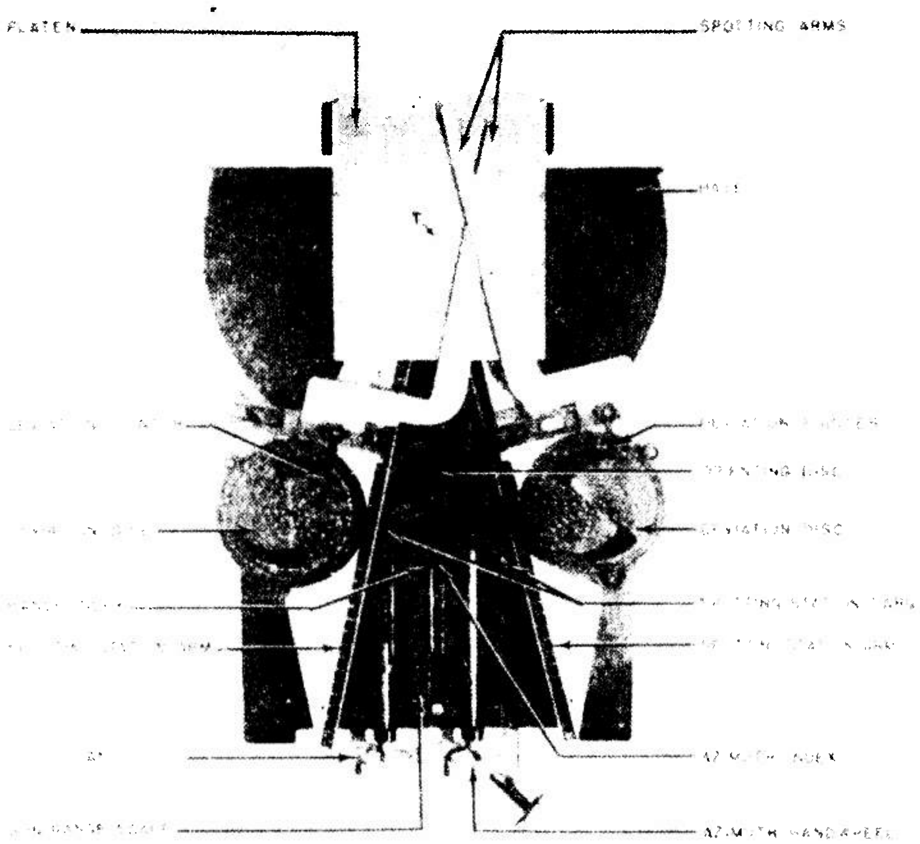


Fig. 1-32. Spotting Board M2.

were particularly usefulagainst aircraft targets. Because such targets are small and move rapidly in three dimensions, a system of automatic computation and transmission of firing data to the gun battery had become necessary to the satisfactory solution of the antiaircraft fire control problem. (The development of data-transmission equipment is covered in par 1-2.4.8.4.)

The Computing Directors M1, M2, M3, and M4 each formed the heart of its associated antiaircraft defense system. A typical complete system included (1) two tracking telescopes mounted on the director and a separately located height finder (see par 1-2.4. 5. 2) to continuously determine the present azimuth, elevation, and height of the target for use by the director; (2) a data-computing director to automatically digest this information and, taking into account its stored ballistic information, procude the angle of traverse, the quadrant elevation, and the fuze setting for transmission to the gun battery; (3) a battery of guns, complete with such on-target fire control equipment as sights, drums, quadrants, and other aiming and laying devices (see pars 1-2.4. 2 through 1-2. 4. 3.4) to utilize the firing data generated by the director, *so* that the guns could be aimed to hit the target; and (4) an automatic data-transmission system to provide adequate information transfer frorn the data-gathering elements of the system to the director and from the director to the gun battery. For operation at night, the system also included a searchlight, together with a control station, a sound locator and a sound-lag corrector (see par 1-2.4.4. 5).

In carrying out its part in the antiaircraft fire control system, the director performed three essential functions:

1. It continuously determined the present position of the target in rectilinear earth-reference coordinates aligned with the local

north-south and east-west axes by combining the input information on target azimuth, elevation, and height.

2. It predicted the future position of the target, to account for the travel of the target during the time of flight of the projectile (and thus the lead component of the total prediction angle).

3. It computed ballistic data to (a) enable the projectile trajectory to be placed so as to intercept the predicted future position of the target and (b) enable the projectile to burst at that point.

The paragraphs below describe briefly the mechanisms by which the directors performed these functions.

Present Position Determination Mechanism. Present position of the target in terms of azimuth, elevation, and height was converted into rectilinear coordinates in two steps:

1. The magnitude of the present-range vector's projection onto the ground horizontal plane that is tangent to the earth's surface directly below the target and also at the director (the earth's curvature between the point of tangency and the director was insignificant for the problem at hand) was determined by the relationship (see Fig. 1-33)

$$R_o = H \cot E_o \tag{1-1}$$

where

R_o = magnitude of the present horizontal range
H = target height
E_o = present target elevation.

2. The horizontal-range magnitude was then converted into the corresponding horizontal-range vector in accordance with the relationship (see Fig. 1-34)

$$\vec{R}_o = R_o \angle A_o \tag{1-2}$$

where A_o is the present azimuth angle of the target.

3. This horizontal-plane projection of the present-range vector was then resolved into components X_o and Y_o along the E-W and N-S axes, respectively, by utilizing the simple relationships between R_o, A_o, X_o, and Y_o that are represented in Fig. 1-34:

$$\phi = 270° - A_o \tag{1-3}$$

$$X_o = R_o \cos \phi = -R_o \sin A_o \tag{1-4}$$

$$Y_o = R_o \sin \phi = -R_o \cos A_o \tag{1-5}$$

Note that the third item required for the representation of target position in rectilinear coordinates is target altitude which was pro-

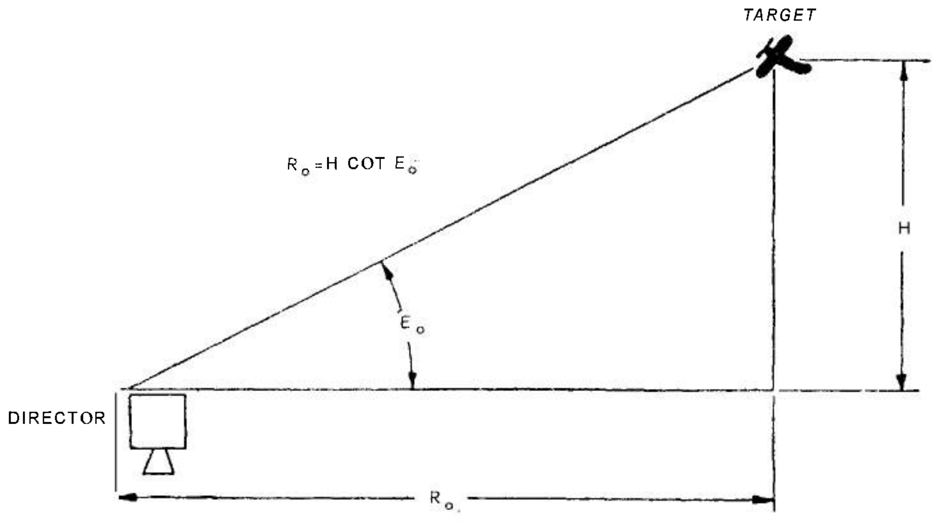


Fig. 1-33. The geometry associated with the determining of present horizontal range.

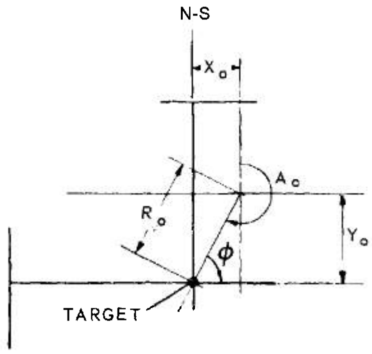


Fig. 1-34. The geometry associated with the conversion of the horizontal-range magnitude into the horizontal-range vector and the subsequent resolution of this vector into its X_o and Y_o components.

vided directly by the height finder by means of electrical data transmission.

In order to determine the required numerical value of the present horizontal range R_o, the $R_o = H \cot E_o$ relationship was solved mechanically inside the director by a pin riding on a three-dimensional cam (see Fig. 1-35). The cam was designed to rotate about its longitudinal axis (the cam axis) with respect to the pin in proportion to the altitude of the target H, and to translate along its longitudinal axis with respect to the pin in proportion to the present horizontal range R_o. The amount of lift provided to the pin by the cam with this arrangement was proportional to the target elevation E_o for given values of H and R_o. The shape of the three-dimensional cam was accordingly the element that related R_o, H, and E_o in accordance with the relationship expressed by Eq. 1-1. That is, for a given value of present

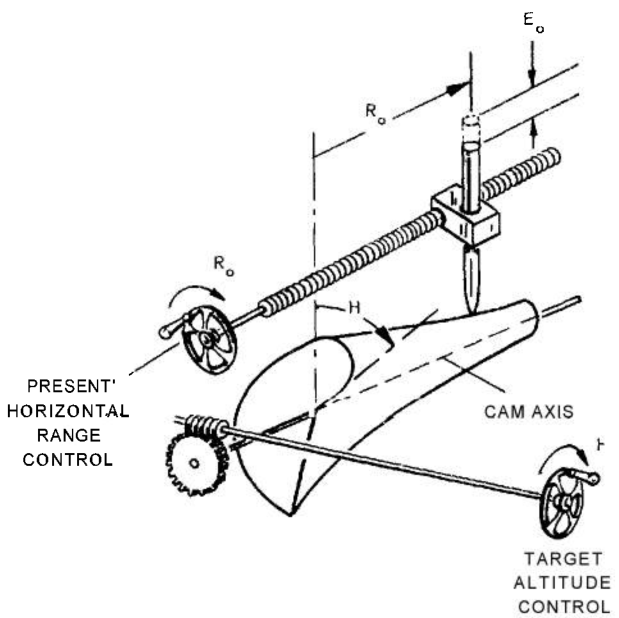


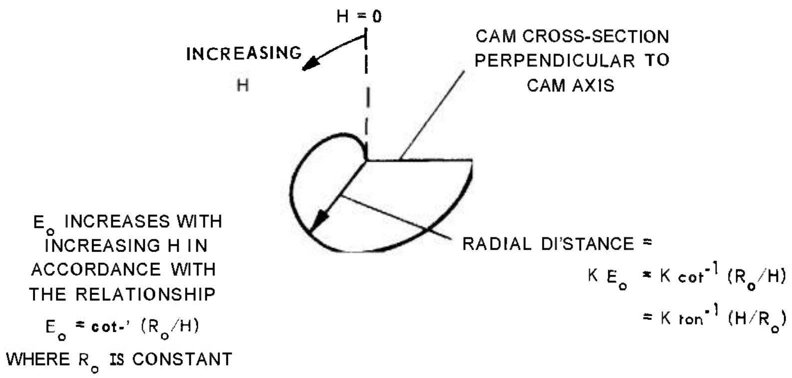
Fig. 1-35. The three-dimensional cam-arid-pin arrangement used for the determination of present horizontal range.

target range R_o, the cross section of the cam perpendicular to the cam axis (see Fig. 1-36) would have a contour that varied with the amount of rotation (proportional to the target altitude H) in accordance with the relationship

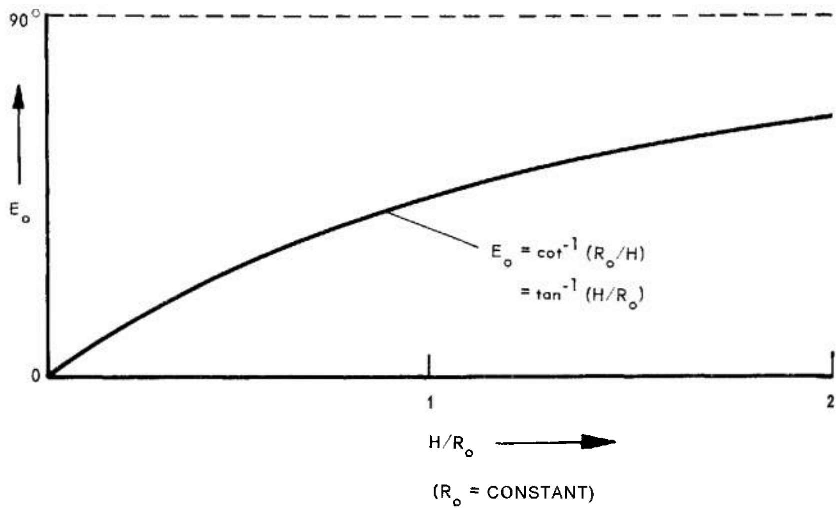
Radial vector of contour =

$$K E_o = K \cot^{-1}(R_o/H) = K \tan^{-1}(H/R_o) \qquad (1\text{-}6)$$

where K is a constant of proportionality relating the radial distance of the contour and the present target elevation and R_o is constant at its given value for all values of H. Similarly, for a given value of the target height H, the cross-section of the cam passing through the cam axis (see Fig. 1-37) would have a contour that varied with the distance along the cam axis (proportional to present

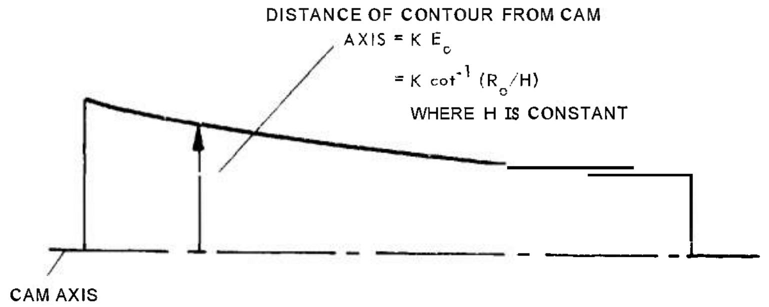


(A) Cam cross-section for a constant value of R_o

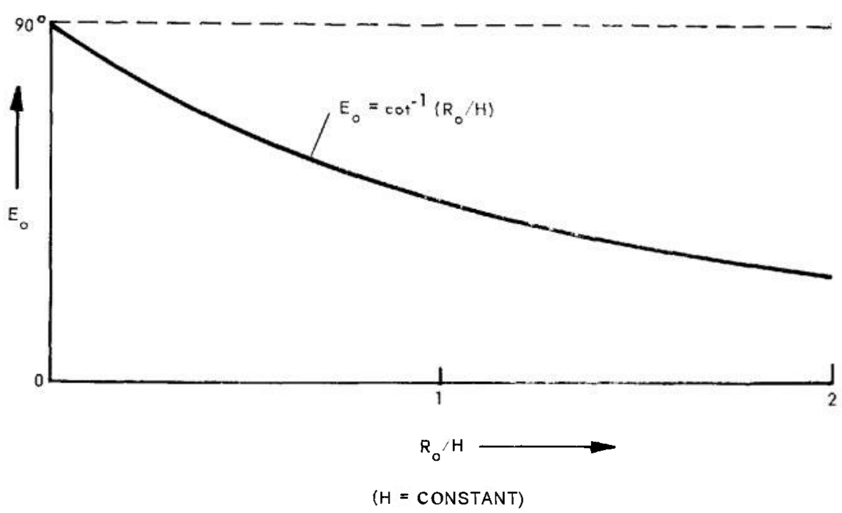


(B) Mathematical basis of contour

Fig. 1-36. The contour of the present-horizontal-range cam for a cross section perpendicular to the cam axis.



(A) Cam cross-section for a constant value of H



(B) Mathematical basis of contour

Fig. 1-37. The contour of the present-horizontal-range cam for a cross section passing through the cam axis.

horizontal range R_o) in accordance with the relationship.

Distance of contour from cam axis =

$$K E_o = K \cot^{-1}(R_o/H) \qquad (1\text{-}7)$$

where H is constant at its given value for all values of R_o and K is the same constant of proportionality that applies in Eq. 1-6. With the known target altitude (as transmitted electrically to the director from the height finder) set into the cam-and-pin arrangement of Fig. 1-35, the horizontal-range control was used to translate the cam until the target elevation represented by the lift of the pin matched the known target elevation obtained from the elevation tracking telescope.

The required horizontal-range magnitude, which corresponded to the horizontal range setting thus established, was then con-

verted into the horizontal-range vector $\bar{R}_0$, and this vector was then resolved into its X_0 and Y_0 components. These two steps were accomplished by means of the mechanical arrangement depicted in Fig. 1-38, whose relationship with the geometry shown in Fig. 1-34 is self-evident. This mechanism continuously located the horizontal projection of the target (represented by the movable pin) with respect to the director (represented by the vertical center line drawn through the mechanism). The lower disc rotated in accordance with the magnitude of the present horizontal range as determined and set into the director by the cam arrangement just described. Through the use of equiangular spiral grooves, rotation of the lower disc thus moved the pin along the radial groove of the upper disc in accordance with the predetermined magnitude of the present horizontal range. The upper disc, on the other hand, was positioned angularly in accordance with the azimuth orientation of the azimuth tracking telescope so that the movable pin became positioned in azimuth as well as range. By virtue of its engagement of the two target-position slides, as shown in Fig. 1-38, the pin in turn positioned the slides and thereby established the displacement X_0 and Y_0 of the target from the E-W and N-S lines passing through the origin of the coordinate system associated with the mechanism. Thus, the horizontal range vector defined by the vector distance from the center line of the mechanism to the pin was resolved into its X_0 and Y_0 components and the target was continuously located in space, with respect to the selected earth-reference coordinates. As the target was tracked, the mechanism described continuously served to locate the target with reference to this coordinate frame.

Future Position Prediction Mechanism. To obtain the future position of the target, the present rate of target movement was first determined and then mulitplied by the projectile time of flight, which was computed on the basis of time of flight versus range data for the particular weapon and ammunition concerned. (Because the projectile velocities were so great compared with the target velocities involved, the error resulting from use of the time of flight to the present target position did not produce any significant error.)

The solution was obtained by one or the other of two mathematical approaches. One approach, used in the Vickers-designed M1 Director, was the so-called angular travel method which was based on the use of angular rates; the future position of the target was predicted by solving the mathematical relationships through the use of mechanical link-

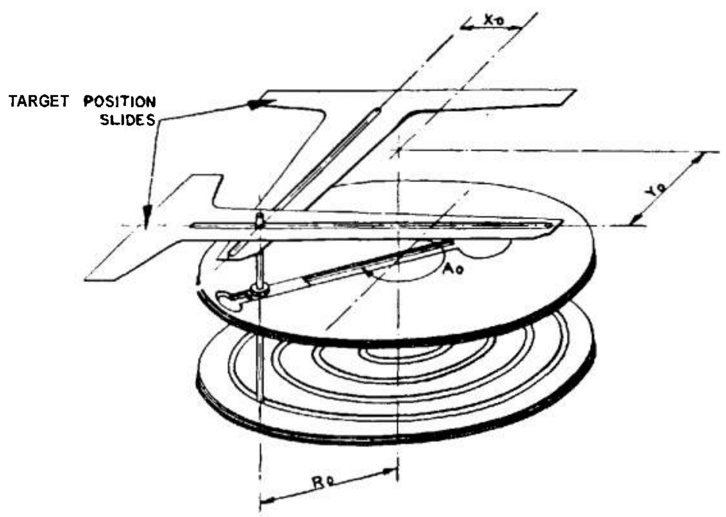


Fig. 1-38. The mechanical arrangement used for locating the horizontal-range vector and resolving it into its X_0 and Y_0 components.

ages. As mechanized in the M1 Director, this method turned out to have the following disadvantages:

1. Approximations were required to permit mechanical solution of the mathematical equations.

2. The mechanism would not function in certain areas.

3. Parallax corrections were complicated.

The second mathematical approach — which was employed with the M2, M3, and M4 Directors of the U.S. Army — used a plan-prediction method based on linear rates, Rates of target travel along perpendicular axes in the horizontal plane were determined by various means (described below). For convenience in orientation and application of wind corrections, these axes were aligned with the N-S and E-W directions. The rate of target movement along each of these axes was multiplied by the computed time of flight of the projectile to determine the horizontal projection of the future target position.* The complete prediction of the future target position was made by applying the target height as obtained from the height finder. Any corrections required to account for variations from a constant flight altitude were made by spotting controls located on the director. The plan-prediction method for solving the future position prediction problem satisfied the objections to the angular-travel method. It did, however, retain the intrinsic difficulty of accurately representing large spatial distances on a small mechanical scale.

Several means were used for determining the target rates along the N-S and E-W axes of the reference coordinate frame employed; the M2, M3, and M4 Directors differed chiefly in the means of determining these rates. For example, in the M2 Director, the target rates were set in by hand. This was accomplished by the operator's matching (1) the rotation of a dial actuated by a constant-speed spring motor operating through a variable-speed drive with (2) the rotation of another dial, concentric with the first, that was driven by one of the two target position slides in the present position determination mechanism (see Fig. 1-38). The setting of the variable-speed drive to obtain the match between the two concentric dials was thus a measure of the target component rate concerned. This rate was then multiplied in a linkage by the projectile time of flight to obtain the predicted component of future target position.

Disadvantages associated with the use of the spring motor led to the use of tachometers for rate-measurement in the M3 Director. The movements of the two target position slides in the present position determination mechanism were geared respectively to the stems of two tachometers. (See Fig. 1-39, which shows part of the future-position-prediction mechanism that applied to the N-S axis.) These tachometers, in effect, acted as reversible-reading stop watches. They were actuated by a trip lever that caused them to measure the component target rates for exactly three seconds. This duration was considered sufficient to allow any irregularities in the tracking data caused by the tracking operators to be smoothed out. The dial of each tachometer was graduated in target yards per second along the associated component axis. The rate thus measured along each axis was then matched with a pointer on the tachometer by a handwheel that at the same time coupled the rate information into the multiplying linkage of the predictive mechanism. As Fig. 1-39 shows, the operator, by simply matching the pointers on the N-S tachometer with his handwheel, displaced the pivoted lever of the multiplying mechanism by a horizontal distance proportional to the N-S target-travel rate; the slide on this lever was displaced along the lever by a distance whose vertical component was proportional to the projectile time of flight. This was accomplished by the constraint placed on the sleeve by the time-of-flight arm, which was free to move vertically as a function of the position of the time-of-flight cam. This cam, which was shaped to suit the time of flight versus range and altitude data for the particular weapon and ammunition, was positioned in accordance with future horizontal range and altitude, com-

*The assumption was employed that during the time of flight of the projectile the most probable course for enemy aircraft, especially massed-formation flights, would be a continuation of a linear course at constant speed. In particular, an enemy bomber had to be aligned in a definite direction prior to bomb release, which necessitated flying on a linear course for a limited period of time.

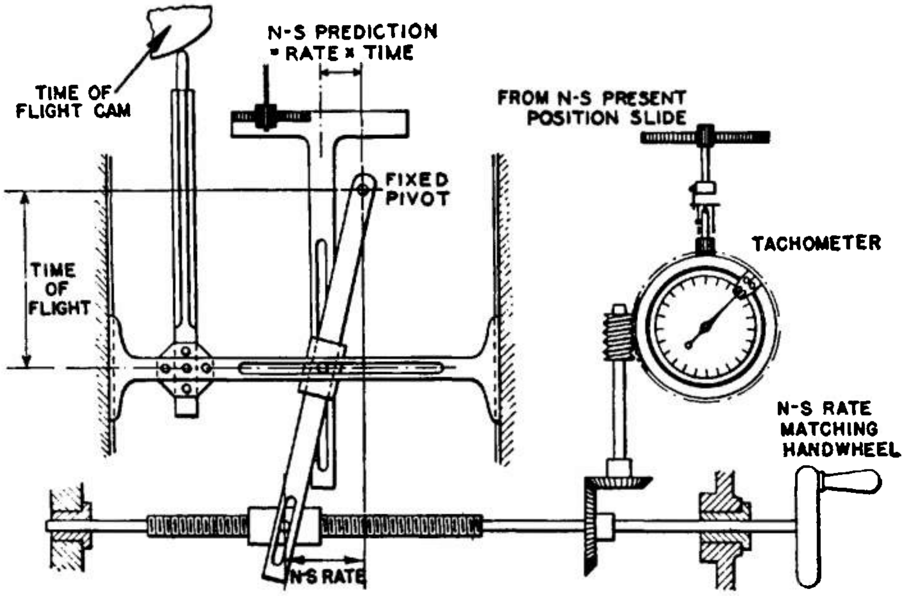


Fig. 1-39. The mechanical arrangement used for determining the N-S component of target travel during the projectile time of flight.

puted on a ballistic data mechanism (described below). The resultant horizontal displacement of the slide, which carried with it the N-S prediction arm, was proportional to the product of the N-S travel rate and the time of flight of the projectile. The E-W component of target travel was primarily measured in terms of a shaft rotation. The two components were then combined mechanically with the present position components X_0 and Y_0, as determined by the present position determination mechanism, to establish the horizontal components of the future target position in the selected coordinate frame. Provision was also made for parallax corrections to the two future target position components since director and gun battery were sometimes widely separated.

The N-S and E-W horizontal components of future target position with respect to the gun battery were then converted to future horizontal range and future azimuth by a system of discs and slides similar to that used to determine the components of present target position (see Fig. 1-38). This was accomplished continuously and automatically by means of an electro-mechanical follow-up system.

Future target height was not computed in the early mechanical directors; the computing mechanism was based on a constant target altitude. However, corrections for altitude changes could be made with spotting controls on the director, as described below.

Ballistic Data Computation Mechanisms. The computation of ballistic data by the early mechanical directors was based on the future position of the target. The ballistic data computed were of the following types:

1. The quadrant elevation, obtained by adding the normal superelevation for future range and altitude to the line of site from the gun battery to the predicted future target position.

2. The time of flight which was used in the prediction of future target position, as discussed in connection with Fig. 1-39.

3. Corrections to the normal quadrant elevation to account for the effect of (a) variations in muzzle velocity from the standard, (b) variations in air density from the standard, and (c) the direction and magnitude of the ballistic wind (see Chapter 2).

4. Corrections to the future azimuth to account for the drift of the projectile and the effect of the crosswind (see Chapter 2).

5. The fuze setting which was based on the corrected quadrant elevation.

The mechanisms for computing these data are described in the paragraphs below.

In the standard Directors M2, M3, and M4 individual three-dimensional cams were used to determine respectively the quadrant elevation, the time of flight of the projectile to the future target position, and the fuze setting. Each of these cams, which were similar to the cam in the present position determination mechanism (see Fig. 1-35), incorporated translation parallel to the cam axis that was proportional to the predicted future height of the target aircraft H_p, and rotation about the cam axis that was proportional to the predicted future horizontal range of the target aircraft R_p. As in the cam-pin arrangement in Fig. 1-35, the output was a function of the radial distance of the cam follower pin from the cam axis. The assembly of ballistic cams was readily replaceable *so* as to provide for any change in weapon or ammunition that might affect the ballistics.

Each three-dimensional cam was simply a range table represented in physical dimensions and each plane section through a cam axis represented a specific firing table condition. For example, the output of the quadrant elevation cam (see Fig. 1-40) included the normal superelevation – i. e. , the superelevation associated with standard firing conditions – associated with given values of the predicted horizontal range R_p and the predicted altitude H_p. During the operation of the mechanism represented pictorially in Fig. 1-40, the cam was rotated about its axis by an angle proportional to R_p and the follower pin was translated along the cam axis by a distance proportional to H_p. The resulting radial displacement of the follower pin from the cam axis was proportional to the quadrant elevation Q. E. Thus the unit mechanized the mathematical relationship

$$Q.E. = f(R_p, H_p) \tag{1-8}$$

in which the relationship of Q. E. as a function of R_p and H_p was provided by the standard firing table data for the ammunition and weapon concerned.

Ballistic corrections to account for off-normal conditions – i. e. , to account for variations between the standard conditions on

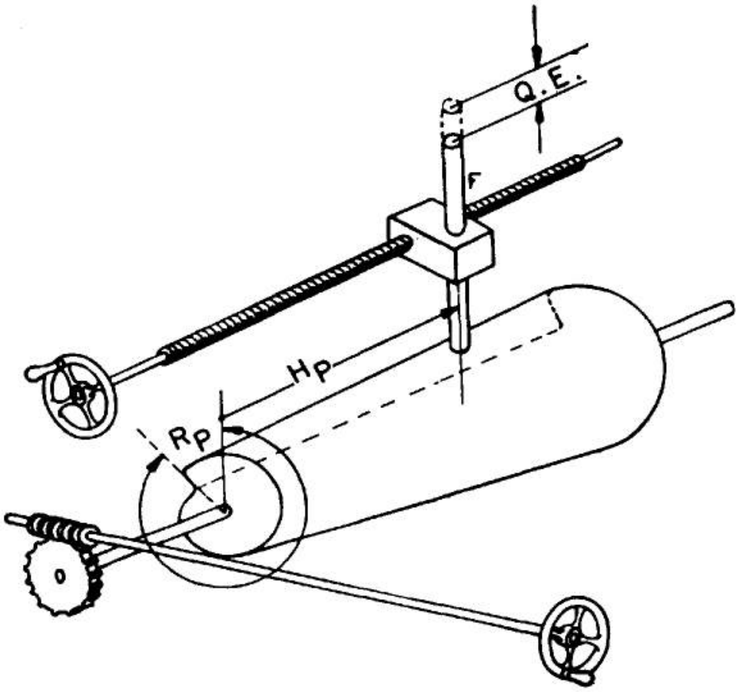


Fig. 1-40. A pictorial representation of the three-dimensional quadrant-elevation cam used in the ballistic mechanism.

which the firing table data were based and the conditions actually existing at the time of fire – were introduced by the following means:

1. A drift correction, to account for the drift of the projectile, was made as a horizontal angular correction to the predicted future azimuth of the target.

2. A correction for muzzle-velocity variation from normal was introduced by changing the value of target height H_p, used in the computations.

3. A correction for density variation from normal was made by introducing the equivalent muzzle-velocity correction since

$$\text{Muzzle-Velocity Correction} = f(\text{Density Correction}) \tag{1-9}$$

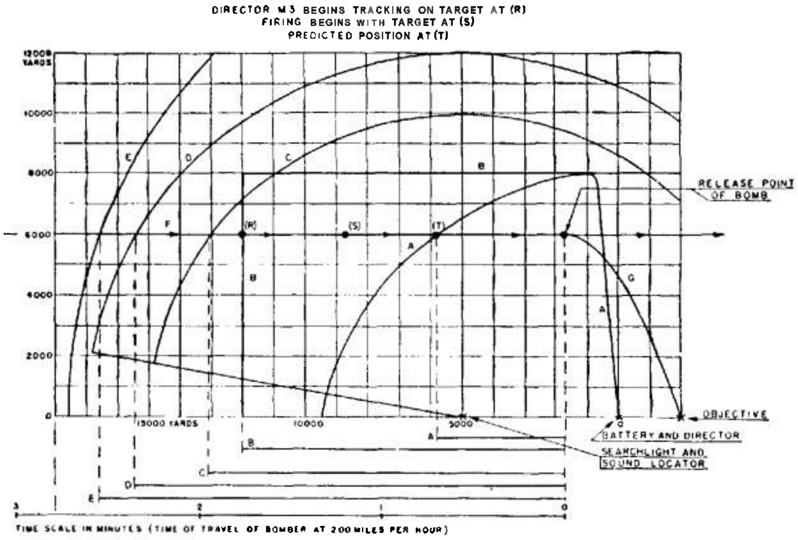
4. Wind corrections were introduced as corrections to the target rates employed in the future position prediction mechanism.

A quite different means of computing ballistic data was used in some other models of directors; in these, charts were mounted on cylindrical drums that revolved under movable pointers. Normal superelevation was plotted as a function of predicted target height H_p. As a drum rotated in accordance with H_p (the abscissas), the ordinate of the superelevation could be determined by matching the proper height curve with the pointer which moved along a vertical screw. Corrections for abnormal conditions were made by substituting another chart that had height-versus-superelevation curves to suit the conditions at hand. A nest of charts was furnished with each director to provide for the spread of abnormal conditions that would normally be encountered.

Spot-Correction Mechanisms. The early mechanical directors were an improvement over mechanical computers of the type described in par 1-2.4. 8. 2. Still, the firing data provided to the gun battery by the director were seldom perfect. Errors arose from such varied causes as erroneous metereological data, target maneuvers (since linear target motion was assumed in the method of *so*lution), and inaccurate measurement of target height. Therefore, the director was also equipped to control gun fire by applying spot corrections for observed deviations. For example, in the Director M3, spotting handwheels were grouped on the rear face of the director below the spotting telescope. In order that the spotter could insert the required spot corrections by feel without removing his eye from the spotting telescope, the spotting handwheels were equipped with spring-loaded detents.

1-2.4.8.4 Data Transmission

As work on gun computers and directors progressed, researchers sought to minimize the time consumed and errors committed in transmitting firing data by telephone from observation posts to plotting rooms and finally to gun positions. During the 1930's, direct electrical transmission of data was adopted, permitting effective use of director-type automatic and continuous fire control systems such as those described in par 1-2.4.8. 3. Time was clearly the most essential factor in the application of such systems. For example, in the case of the most probable type of aircraft target, an enemy bomber, the future position of the target had to be accurately and continuously determined, the firing data automatically computed, and the necessary shots fired to destroy the target; all within the brief time interval that commenced shortly before the target came within firing range and ended shortly before the target was in a position to drop its bombs effectively. Figure 1-41 portrays graphically a typical fire-control situation in which the Director M3 determined firing data for the 3-inch antiaircraft gun against an incoming bomber traveling at a rate of 200 mph arid at a height of 6000 yd. The director began tracking the target when it came within the range operating limits of the director's Computing mechanisms, at point R, and firing commenced when the target came within firing range, at point S. The first projectile fired met the target at the predicted future target position T, located on the envelope of the gun trajectory. The distance ST represents the distance traveled by the target during the time of flight of the first projectile from the gun battery (located at the origin O) to the predicted point T. To be effective, the weapon system had to either destroy the target, or turn it back prior to its arrival at the point of bomb release. Application of the time scale at the bottom of Fig. 1-41 shows that the time interval during which the target could be ef-



KEY

A. Envelope, 3-in. Antiaircraft gun.
B. Prediction limits.
C. Range illumination, searchlight.
D. Effective sound locator range.
E. Limit of visibility.
F. Path of bomber at 6000 yd altitude.
G. Bomb trajectory.

Fig. 1-41. Example of the time limitations inherent in antiaircraft fire.

fectively fired upon was less than one minute and that the total time period between the start of effective tracking of the target and the end of effective fire upon the target was only slightly more than one minute.

Two basic types of data-transmission systems were devised to meet the time problem: the direct-current step-by-step system and the alternating-current self-synchronous system which was adopted as standard by the U. S. Army. In both systems, a displacement of the transmitter rotor was compensated automatically by a corresponding displacement of the receiver rotor when an excitation current was supplied. Thus, it became possible to provide the means, sought as far back as 1919 by the Coast Artillery, for the continuous and instantaneous transmission of fire-control data between two or more remote units.

A typical early type of self-synchronous data-transmission system is shown schematically in Fig. 1-42. The transmitting motor (transmitter) and receiving motor (receiver) were identical electrically, each being a small bipolar, three-phase alternator. The rotor windings were not three-phase in the usual sense, however, because as in present-day synchros all induced voltages were in time phase.[24] With the two rotors connected as shown in Fig. 1-42 and with the two stators supplied with excitation from a common 110-volt, 60-cps, single-phase a-c source, the two rotors always took positions so that the voltages induced in the rotor windings were of balanced magnitude and displacement. Because of this, any rotational motion given to the rotor of one of the two motors would be transmitted to and duplicated by the rotor of the other unit.

A single transmitter often drove a number of rotors. One azimuth or elevation transmitter in a director might be connected to four or more receivers at the gun battery. The transmitter, of course, was larger than the receivers and was designed to insure that each receiver would develop its standard torque.

In direct, or "coarse", transmission of data, the standard transmitter of the 1930's controlled the rotor of a companion receiver to a precision of 0.5". For greater accu-

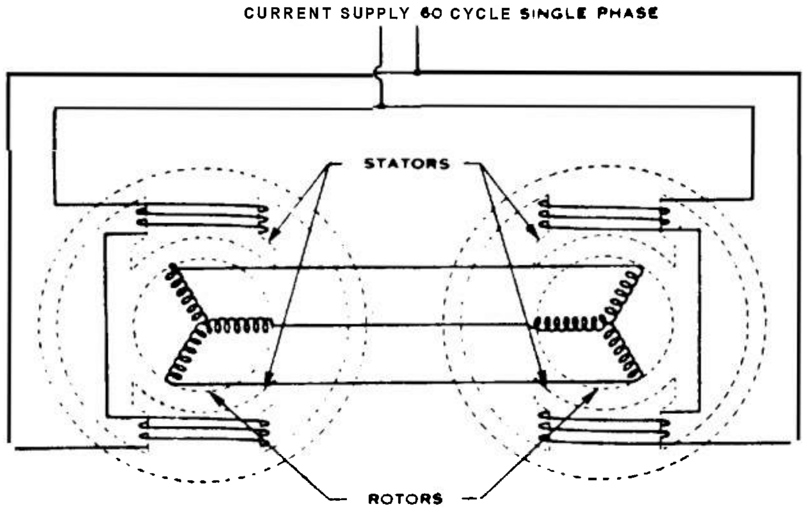


Fig. 1-42. A schematic representation of an early a-c self-synchronous data-transmission system.

racy, "fine" vernier motors were added that increased precision to 1/32° or 0.55 mils. Both elevation and azimuth data were customarily transmitted by separate coarse and fine transmitter-receiver combinations.

A complete data-transmission system between a director and typical four-gun battery included three pairs of fine and coarse transmitters at the director (for azimuth, elevation, and fuze-setting data), the interconnecting cable between the director and the gun battery, a main junction box at the director, four distribution boxes at the gun battery, and twelve pairs of fine and coarse receivers at the gun battery — three pairs at each gun for receiving azimuth, elevation, and fuze-setting data.

To develop the full effectiveness of the computing elements in a fire control system, engineers sought ways of applying these new data-transmission techniques so that a constant flow of input data from remote coast artillery observation stations could be transmitted and processed instantaneously and accurately, and then relayed as output firing data to the weapons under fire control. In 1942, research sponsored by the National Defense Research Committee (NDRC) produced long-range azimuth and elevation transmitting and receiving systems that met these requirements; they were standardized in 1943 as components of the fire control computing systems. By this time, the synchro units and data-transmission systems had changed from those shown in Fig. 1-42 to something approaching the modern configuration of Fig. 1-43. The stators in the generator (transmitter) and motor (receiver) had three windings displaced 120° from each other in space, and the rotors were of two-pole construction.

1-2.4.8.5 Refinements in Mechanical Directors

Refinements in complex mechanical directors continued during the 1930's in the face of rapidly increasing speeds of aircraft and development of heavier, more powerful weapons. The introduction of low-flying strafing planes and high-speed dive bombers in the 1930's reduced the accuracy of existing mechanical directors for close-in fire and created monumental design problems. Fortunately, a British mechanical director, the Kerrison Predictor, had been developed that automatically elevated and trained an antiaircraft gun through direct gearing as the director transmitted its computation. It was adapted to American weapons as the M5 Director under the aegis of NDRC. It was based on the target angular-rate-of-travel method, had an effective range of 2000 yards, and was used with 40 mm guns.

Director accuracy was improved and crew training time reduced in later modifications, notably the M5A2, by the addition of

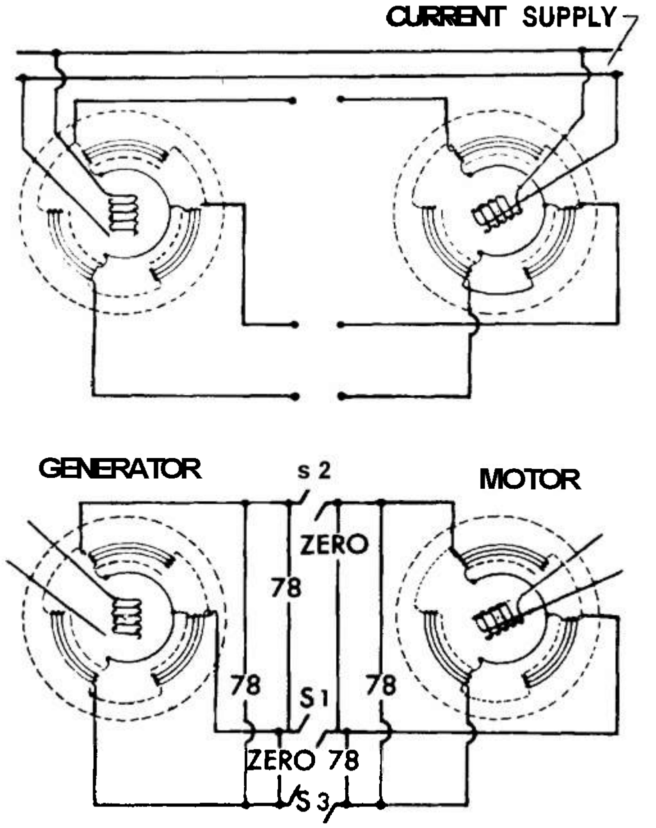


Figure 1-43. A schematic representation of a present-day synchro data-transmission system.

an altitude converter, a stereoscopic range finder, and an electrical arrangement for introducing slant range into the multiplying mechanism of the director. The Army then attempted to upgrade director performance with its M6 design, which was based on the target linear-rate-of-travel method.

A subsequent mechanical Director, the M7, more adequately answered the requirements of an antiaircraft data computer. This director incorporated a device for correcting transmitted fuze data for "dead time". Fuze setting was transmitted from the director to a fuze setter at the gun, which set the proper fuze time on the projectile. However, there was a time lapse, termed "dead time", between the setting of the fuze on a projectile and the firing of that projectile, in which the correct setting would usually change from the director-computed value. Without a means of correcting for dead time, projectiles would probably burst at a point far from the target because of incorrect fuze settings. Combat experience with this director indicated, however, that still further refinements were required. Accordingly, the development of mechanical directors was dropped in favor of electrical directors.

1-2.4.8.6 Transition to Electrical Directors

The inception of the 90 mm AA gun in 1938 and the standardization of the 120 mm high-velocity AA gun, the M 1, in 1944 created an array of design problems. The 90 mm gun was initially designed without the automatic controls required for rapid elevation and traverse of the gun. This factor precluded the use of adequate gun-director control. Later, in 1940, a power control servo system for the gun was worked out by the Sperry Gyroscope Company. It was a complex of electrical, mechanical, and hydraulic

units that, despite minor defects, provided the gun with relatively accurate means of aiming at targets moving at high angular speeds. It had the outstanding feature of operating effectively on targets at short ranges, where manual tracking was difficult.

Prior to the simplification and standardization of this Sperry servo system in 1941, Army efforts were concentrated on the design and development of a gun-data Computer, the M1, which embodied much of the engineering know-how and design skills accumulated over the previous two decades. It dwarfed all previous computers in size. This enormously complicated mechanical brain was nearly seven feet long, three feet high, and three feet wide, weighed 5, 000 pounds, and cost approximately $100,000. Used by the Coast Artillery for long-range guns, this computer stored within itself a target-position generator, wind-component indicator, ballistic-correction unit, predictor, range-to-elevation converter, parallax unit, and three triangular resolvers, all interconnected. The M1 Computer became standard in 1940.

Despite its size and complexity, the M1 still was unable to overcome the vexing errors inherent in mechanical systems. These errors resulted in improper gun firing data, particularly when the target was a considerable distance away. In fact, as the vertical range of high-altitude bombers increased, the mechanical directors generally became unsuitable for antiaircraft defense since the yardage error increased directly with the range.

As work on the 90 mm and 120 mm AA guns progressed in 1940, scientists at the Bell Telephone Laboratories proposed an electrical gun director. Enthusiasm over the plan mounted when, later in the year, it appeared feasible to incorporate the newly developed principles of radar for tracking purposes.

Army ordnance designers, Bell Laboratories, and NDRC collaborated to develop, design, construct, and standardize the mathematically complex M9 electrical Director early in 1942. Manufacture was simplified by the use of standard components. However, the M9 Director, which was developed for the 90 mm AA gun and later the M10 electrical Director which was developed for the 120 mm AA gun represenred formidable and extremely complicated devices that were suitable only for large AA guns. Becauseof their weight (about 3, 500 pounds) they were installed in separate trailers. Yet they manifested many distinct advantages over the mechanical directors:

1. They eliminated many of the inherent errors of mechanical prediction.
2. They provided complete solutions forthe nonstandard ballistic conditions prevailing.
3. They effected a shorter minimum slant range and an increase in maximum horizontal range.
4. They improved target tracking.

Each electrical director consisted of a tracker, a computer, an altitude converter, and power elements that were all interconnected by a cable system. For visible targets, the tracker provided the computer with range, elevation, and azimuth data. The radar system was used when the target was not visible. The raw data defining the target's position in polar coordinates (range, azimuth, and elevation) were converted into rectangular coordinates in the computer. The computer also determined the target velocity in order to account for the time element and thereby provide for lead. It then searched its ballistic references for firing {data and corrected for nonstandard conditions. It continuously computed all firing data automatically and electrically; these data were transmitted to the gun continuously and almost instantaneously.

1-2.4.9 Fire Control System Development During World War II

A unique pattern of weapon system development emerged during World War II. Before that conflict, the responsibility in the United States for the development of new weapons and fire control systems rested with the Army and Navy. Commencing in June 1940, that responsibility was shared with a civilian agency of the Government, the National Defense Research Committee (NDRC) (which became part of the larger Office of Scientific Research and Development (OSRD) a year later) – though the Army and Navy carried on a great deal of work on fire control apart from NDRC-OSRD. (See Refer-

ences 25 through 30 for an extensive documentation of the work of OSRD and its subdivision, NDRC, during World War II on the development of new means for controlling the behavior of projectiles and other missiles. These means included new fire-control devices and systems as well as proximity fuzes* and guided missiles.)

NDRC found many agencies and individuals concerned with fire control. It also found that, within the limits imposed by their budgets, both the Army and Navy had endeavored to improve methods and equipment. This effort was supplemented by investigations carried out in private industry and by the inventive effort of individuals. However, the total of these efforts was much too little and, although fire control was a long-established subject, it was by no means a well-established one. (For example, the performance of antiaircraft artillery in the Battle of Britain during 1940 was both expensive and ineffective.) With this background, NDRC elected to concentrate its major initial activity in the field of ground-to-air fire control. Activity started immediately in conjunction with the Army.

One important outcome was the development of the M9 electric Director (see par 1-2.4.8.6). Successfully used through much of World War II, this director, with two other OSRD fire-control developments – the SCR-584 radar for locating targets and the proximity fuze for exploding the projectile at the target – solved the great menace of the buzz bomb (V-1) attacks against London during the last year of the war. The combination of high-quality radar ranging, fast and accurate computation, and proximity fuzing made it possible to shoot the bombs down as they came in over the coast. (For security reasons, the proximity fuze was initially restricted to antiaircraft defense of the U. S. fleet, reaching combat use for that purpose early in 1943. Following its release for use against the buzz bombs, its general use by field artillery against ground troops was permitted in December 1844. Projectiles fitted with proximity fuzes helped stop the last great German drive in the war.) Unlike time-fuzed fire, proximity-fuze operation is based on a satisfactory trajectory-to-target distance. Without accurate fire control, a proximity fuze is worthless.

Simultaneously with the crash program of antiaircraft director development, NDRC initiated a project for research at the Massachusetts Institute of Technology in the general field of servomechanisms. This marked the beginning of interest in writing and distributing of mathematical treatments of servo theory.

During this servomechanisms-development period, several other problems vital to the progress of fire control required solution:

1. The lack of adequate instruments and procedures for the measurement, analysis, and assessment of the performance of fire-control components and systems. At the start of World War 11, it was literally impossible to make a decision regarding any fire control equipment on the basis of realistic, quantitative data. This situation was finally corrected but only with intensive effort. As noted in Chapter XIX of Reference 25, "If any common factor of success can be found in the various technical developments described in this volume, it is the factor of quantitative tests. This is particularly true in the complicated dynamical situations found in fire control systems." The principle that measurement is the basis of knowledge is as fundamental to new weapons development as to any branch of science and engineering; and a corollary is that simulating devices are essential to rapid development. (See Chapter 4 for the use of simulation in present-day fire-control design.)

2. The lack of a systems engineering approach. Systems engineering was often overlooked in the development of military devices. All too often, research and design of the various components of a system were assigned to independent groups in the vain hope that the components so developed would function properly together. Time and money were spent correcting this fallacious concept, and a remedy was imperative. (See Chapter 4 for the application of systems engineering to modern fire-control design.)

3. The need for understanding and improving the interactions between men and

*A fuze wherein primary initiation occurs by sensing the presence, distance, and/or direction of a target through the characteristics of the target itself or its environment.

machines. Various educational institutions (mainly Princeton and Tufts Universities, and Iowa State College) undertook studies of the psychological and physiological factors that should be considered in the design of fire control equipment. Human engineering is now, of course, a recognized and established field, and consideration of its principles is a requisite for weapon system development,

4. The need for an adequate theoretical understanding of the fire control problems involved. As fire control developed during World War II, much effort was devoted to the mathematical analysis of fire control problems. The Applied Mathematics Panel set up by OSRD to provide mathematical analyses by the Army, Navy, and other groups at OSRD proved invaluable. The work included theoretical analysis of predicting mechanisms, analysis of the dynamic behavior of various types of sights, analysis of devices for smoothing input data, improvement of existing computers, creation of new computers, and performance of a variety of probability studies. One such study, for example, was made late in the war when AA was becoming more effective and enemy planes were taking more evasive action and avoiding the straight-line flights for which the standard directors were designed. Three different projects to compute data for curved flight were carried out, but probability studies showed that while some gain was obtained by such computations, the increase in the probability of obtaining hits under conditions likely to occur in practice did not make a convincing case for adoption.

Although emphasis was initially on the development of heavy AA fire control equipment during World War 11, other fire-control developments proceeded concurrently; for example:

1. Tank fire control equipment
2. Light AA fire control equipment
3. Rocket fire control equipment
4. Antitank sights
5. Improvements in optical range finders, gyro sights, and gun-data transmission systems
6. Digital computers

In particular, the development of tank fire control equipment gained impetus when, early in 1942, a change in emphasis from defensive to offensive warfare took place, and OSRD set about adapting range finders and other fire-control devices to new offensive situations. Up until 1943, U. S. Army tank crews had merely employed visual. estimation of range and the usual artillery bracketing technique. Since the use of accurate range finders and computers would enable tanks to fire for effect with the first round, intensive effort was applied to the development of this equipment. At the end of World War 11, this work was still in the development stage but an important foundation was laid for the development of integrated tank fire control systems that properly combined ranging, computing, and aiming functions.

The digital computer development program was undertaken to obtain a means of analyzing projectile trajectories (see Chapter 2). It was a joint effort between the Aberdeen Proving Ground and the University of Pennsylvania, with assistance from the Institute of Advanced Studies at Princeton, New Jersey. The ENIAC cornputer was the result.

Guided missiles progressed remarkably during World War II but they were relatively crude and limited to air-to-ground fire.

The civilian responsibility (via OSRD) for the development of new weapons and the associated fire control equipment terminated with the end of World War 11.

1-3 RECENT AND CURRENT DEVELOPMENTS IN ARMY FIRE CONTROL

1-3.1 GENERAL

By the end of World War II, a sort of law of diminishing returns was beginning to take effect in the methods of weapon and fire control technology that had achieved such dramatic results earlier in the war. Sophisticated engineering designs, achieved often at great expense and effort, were now producing only minor improvements. Clearly, fresh approaches, based on new concepts in technology, were needed to extend the capability of striking strategic targets with minimum risk to U. S. personnel.

Accordingly, efforts in the late 1940's and the 1950's concentrated on developing new weapons that would markedly increase striking range, reduce susceptibility to countermeasures, and achieve greater de-

structive effect. These efforts resulted in the development of guided missiles, capable of ranges that vary from a few miles to intercontinental spans, as a means for defense or attack, and of striking a stationary or rapidly maneuvering, high-speed target with predictable accuracy and probability of kill.

Missiles that could be guided in flight — that could be altered in their course and speed to match target maneuvers and compensate for initial errors — opened up new fire control approaches and new magnitudes of destructive potential. At the same time, of course, the missiles themselves, as well as the new high-performance aircraft developed after World War II, were targets of a new order of speed and maneuverability, and the conventional fire control methods and weapons would not suffice against them. In missile guidance and control systems, all or part of the intelligence and control elements were transposed from the aimingpoint of a weapon to the missile itself.

The treatment of guidance and control used with guided missiles falls outside the scope of the handbooks in the Fire Control Series. Information on this subject may be found in the handbooks of the Surface-to-Air Missile Series.[31]

Even for antiaircraft applications, however, the need for developing conventional weapons, together with their fire control systems, continued after World War II. Examples of such Army weapon systems are the Skysweeper (for firing projectiles), the M33 (forfiring LOKI rockets), and the Stinger (for firing 60-caliber guns). All were for AA applications. For artillery, new computing techniques have been introduced such as the FADAC (Field Artillery Digital Automatic Computer) equipment described in Chapter 13 of Section 3 (Fire Control Computing Systems). And, of course, much of the AA equipment is also suitable for use against ground targets. For tanks, new concepts in range-finding equipment, ballistic-computation equipment, and low-light-level equipment have been introduced.

Some recent programs of special interest are described in the paragraphs which follow.*

1-3.2 TANK FIRE CONTROL SYSTEMS

Since World War II, the U. S. has devoted considerable research and development to overcoming weaknesses revealed in tank fire control systems during that war (see par 1-2.4.6). Chief among these shortcomings were the following:

1. Visual estimation of range.
2. Lack of correction of such secondary (but still significant) effects as wind, range, muzzle-velocity change, cant, etc.
3. Accommodation of only one range-elevation relationship on the ballistic reticle.
4. Lack of effective devices for night acquisition and tracking.

Obviously, with such rudimentary fire control, the probability of a first-round hit on opposing armor was unacceptably low,† time for engagement was too long, and operation was limited to daytime. Because of these deficiencies, a number of studies were made immediately following World War II, with the objective of determining what could and should be done to improve tank fire control systems. One such study, conducted by Frankford Arsenal during 1947, was particularly effective in establishing the desirability of more sophisticated fire control than had previously been provided. Moreover, the study indicated that the greatest single improvement in accuracy could be achieved by providing an instrument capable of measuring range. Top priority was given to developing such a device and that same year Frankford developed the Range Finder T37 (see Fig. 1-44). This instrument is essentially an auto-collimated, 5-foot base, stereoscopic range finder with dual magnification (7.2X and 3.6X), into which is incorporated a ballistic computing mechanism.

Unfortunately, the developmental cycle of tank range finders was not completed before the Korean conflict. Therefore, the first vehicles produced during that emergency could not be equipped with range find-

* Security restrictions prevent any more than a broad coverage here. An excellent source of more-detailed information on recent developments is the series of Technical Information Reports issued by the U. S. Army.[32]

† In modern warfare if a tank does not score with its first shot, it may well be put out of action itself before it can fire again.

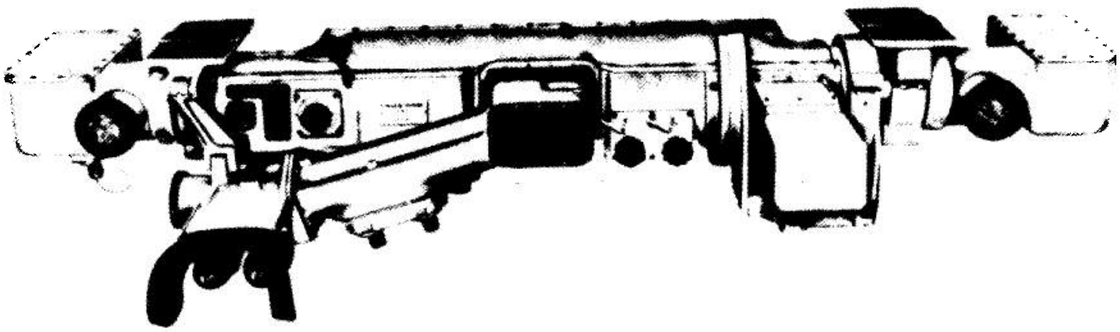


Figure 1-44. Top rear view of Range Finder T37.

ers. For that reason, the fire control for Medium Tank M46 consisted of a direct-fire Telescope T152 and a Periscope of the M10 or M16 series that were not too dissimilar from those used during World War II (see par 1-2.4. 6). However, the telescope reticles, as contrasted with those used with the M4 (see Fig. 1-24), provided for four different range-elevation relationships (see Fig. 1-45) for use with the different weapons and ammunition that might be employed. This was achieved by engraving the ballistic scales on the fixed reticle and providing a movable index line that could be superimposed upon any range graduation of the applicable scale. Azimuth leads were inserted by means of the azimuth deflection scale. The slightly tilted vertical drift line allowed convenient correction for drift. It is of interest to note that a somewhat similar arrangement is currently being utilized in the recently delivered XM114 Telescope for the new U. S. lightweight 105 mm Howitzer.

Another improvement incorporated into Telescope T152 and its mount was a provision for cant correction. This was accomplished by rotating the entire telescope about the boresight mark until the vertical crosshair was truly vertical. Rotational motion was automatically controlled by pendulum-activated contact switches.

It was not until 1952, when the Medium Tank M47 was produced, however, that tank fire control systems became available that were appreciably better than those used during World War 11. The M47 Tank utilized the gunner-operated Range Finder M12 (developed as the T41) as a primary fire control system (see Fig. 1-46). The M12 is similar to the T37. It is a stereoscopic range finder with a 5-foot base length and a 7. 5-power magnification, and contains a number of desirable features such as auto-collimation, stationary eyepieces, boresight controls, and removable end boxes. In addition, it utilizes a ballistic cam to convert range into the required superelevation.

The fire control systems for the M48 Tank (see Fig. 1-47) and the M60 Tank (see Fig. 1-48) use many of the same principles as the M47 system but they also include new features, the chief being ballistic corrections. Whereas the system for M47 Tank converts range into superelevation within the range finder itself, the systems for the M48 and M60 Tanks utilize a separate ballistic computer, the M13A1D, for this function. With separation, six accurately machined ballistic cams can be included in the fire control system and a fire control solution can be selected that matches the particular ammunition being employed. With the single cam of the Range Finder M12 in the M47 Tank, the ballistic solution is only an approximation.

Additional features of the fire control system for the M48 Tank are as follows:

1. The commander-operated Range Finder M13 which was developed as the T46. This range finder, which is also stereoscopic, provides greater accuracy as a result of its improved optical design and longer base length.

2. The Periscope M20A1 which has a fixed-position eyepiece and a rotatable head mirror. This feature is being utilized in all subsequent tank fire control systems.

The fire control system provided for the

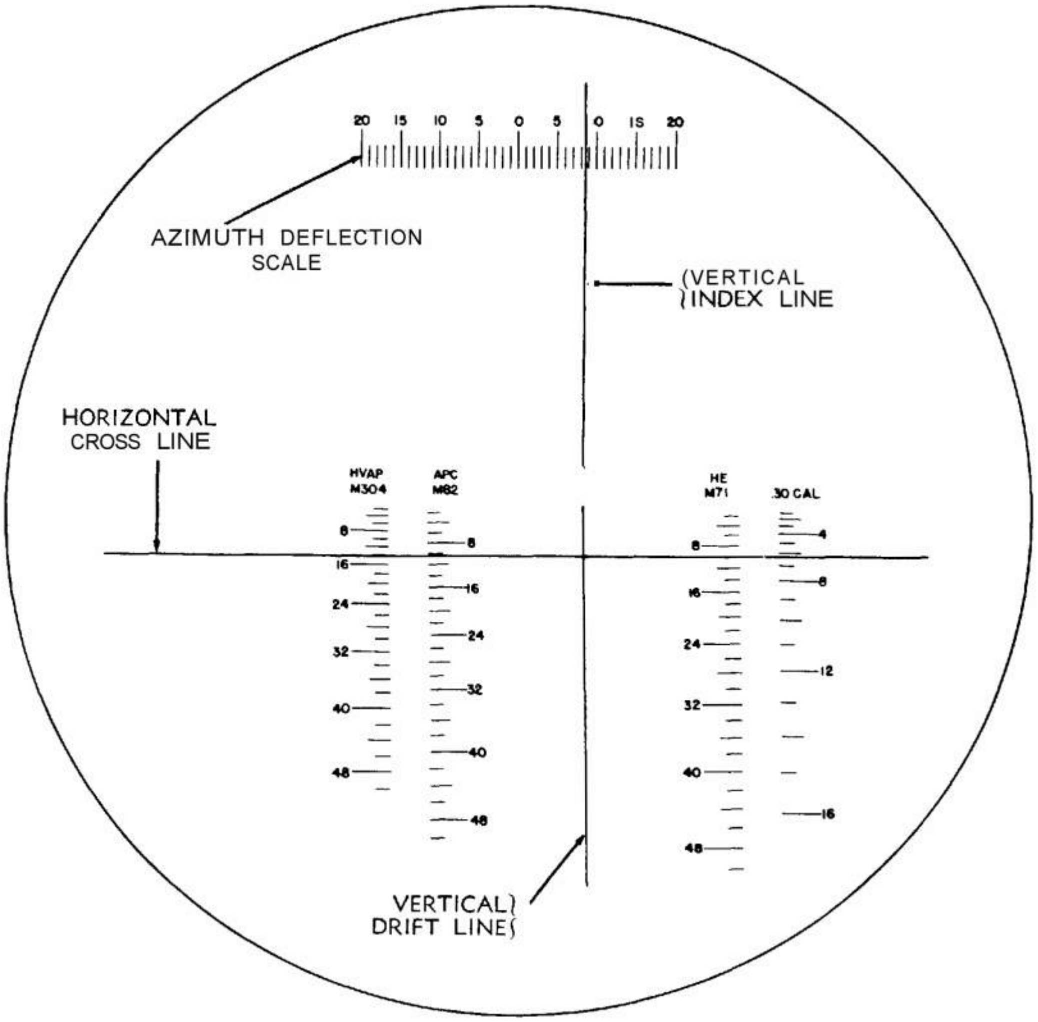


Figure 1-45. The reticle pattern used in Telescope T152.

M60 Tank is the most recent standardization tank fire control system produced for the U. S. Army. Included in this system is the Range Finder M17C. In outward appearance, it closely resembles the M13 but it is a coincidence type of instrument. The coincidence range finder has an inherent instability due to mechanical distortions caused by bending and thermal effects. Heavy construction can minimize these but space limitations in the M60 vehicle ruled out this solution. Instead, a manually operated compensating device is provided that corrects these distortions. The success of this expedient has been demonstrated by user acceptance.

Frankford Arsenal then initiated a study to devise a coincidence range finder that would be insensitive to environmental factors and hence would not require manual adjustment. The new design uses modular principles of construction to keep maintenance and optical adjustment to a minimum. The many advantages of this range-finder design are generally recognized but the program never went beyond the feasibility stage because of the satisfactory performance of the M17C.

In addition to the M13A1D Ballistic Computer and the Range Finder M17C, the fire control system for the M60 Tank also includes the Gunner's Periscope XM32 which

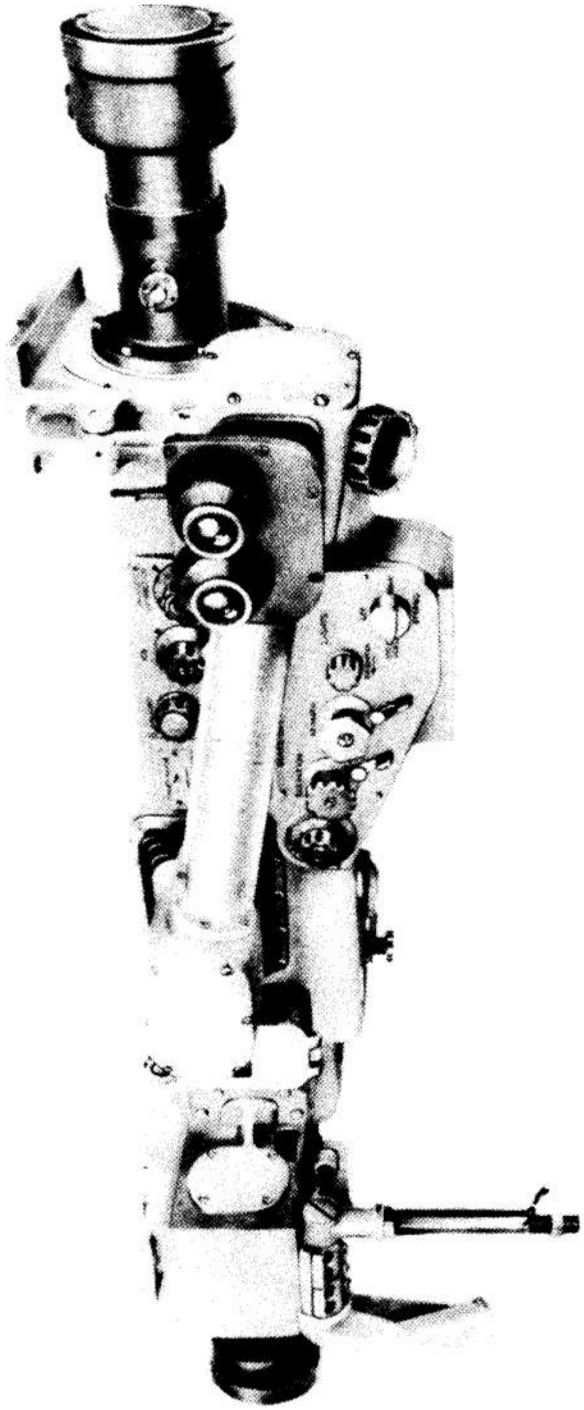


Figure 1-46. Range Finder M12.

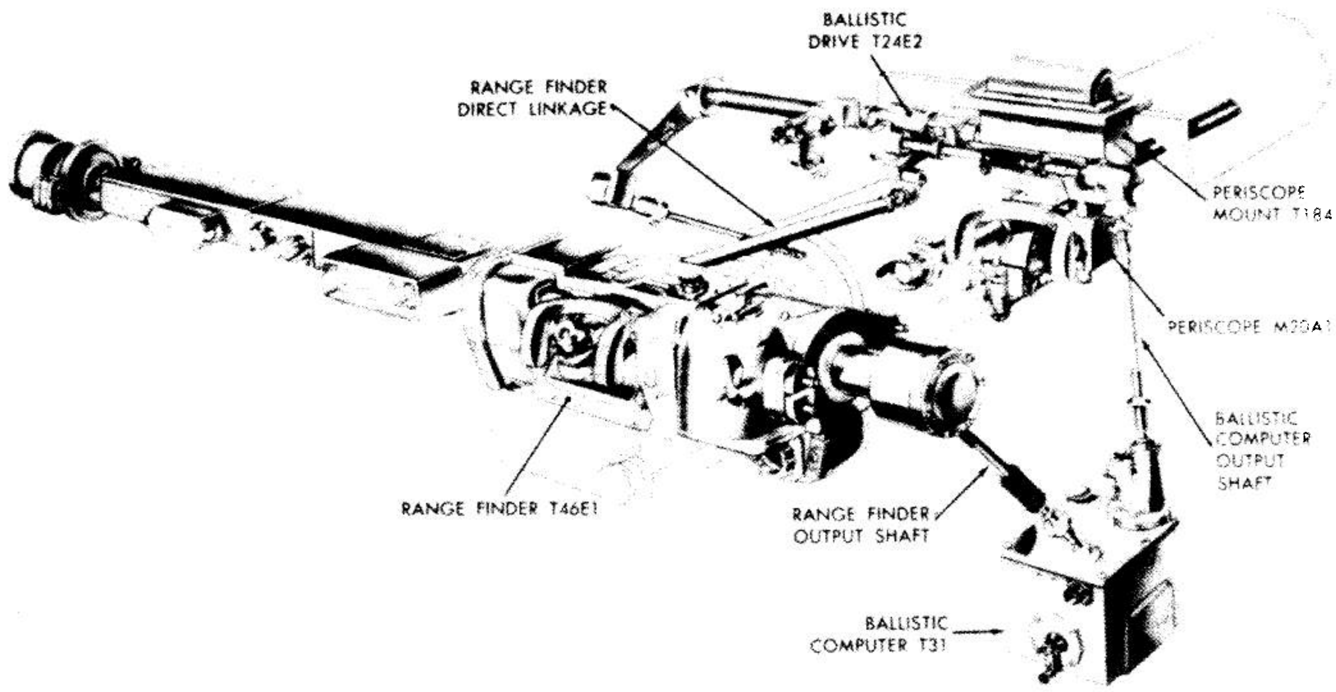


Figure 1-47. The primary fire control system for the 90 mm gun of the M48 tank.

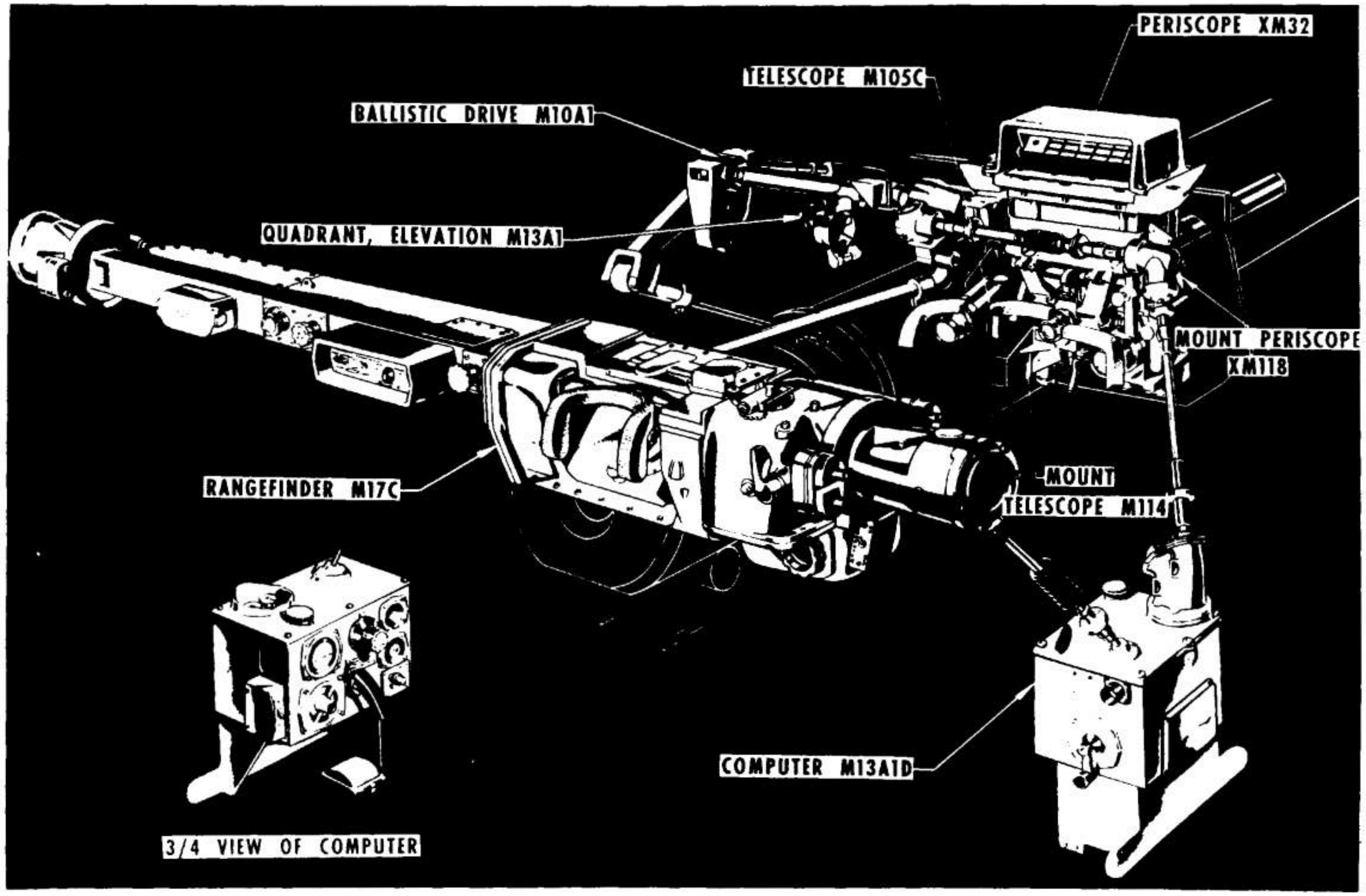


Figure 1-48. The fire control system for the M60 tank

provides alternative 8X visible viewing and sighting, 8X active infrared viewing and sighting, and unity-power visible viewing. Periscope XM35, which also provides for active infrared viewing, may be substituted for the XM32 since, in conjunction with the XM14 Reticle Projector, it incorporates internal projection of reticle data. At the commander's station is the Periscope XM34 which is an 8X, visible, binocular instrument, and the Periscope XM36 which functions like the Periscope XM32.

Obviously, considerably progress has been made during recent years but much remains to be done. For example, the M60's fire control system is far superior to the M48's but it is still deficient in the following respects:

1. Although the best available techniques were utilized in its design, the M17 type of range finder does not completely satisfy the requirements for such a device. One of its principal shortcomings is excessive errors at ranges of 2000 meters and over.

2. Although many factors other than range should enter into the determination of weapon elevation, range alone is used as the basis for generating a solution for weapon elevation. In addition, lateral effects are not compensated for in any existing devices.

3. Although the gunner's and driver's equipment provided in the M60 Tank offer some night-operation capability, they require an active searchlight which is undesirable for security reasons. In addition, the range of operation is not considered adequate.

The solution of these three problems, which exist even in the latest production tank fire control systems, has been the objective of the fire control research and development effort conducted in the U. S. during the past few years, with the following results:

1. Range-finding equipment. Of all the many problems that have been presented to the designers of fire control equipment, the design of equipment to measure range has been the most challenging and, until recently, the most frustrating. In general, two basic techniques are available to the designer of range-measuring devices. The first technique, used in optical range finders, solves a right triangle in which the length of one side (the baseline) is fixed and an angle is measured to determine range. The second technique involves transmission of a pulse of energy (such as a radar pulse) and measuring the time required for it to return after being reflected from a target. An extensive discussion of the various design approaches that can be taken under these two techniques, together with an evaluation of their effectiveness, is reserved for Section 2 of the Fire Control Series (Acquisition and Tracking Systems).

For present purposes, however, suffice to say that while the possibility of improving baseline range finders definitely exists, the problems inherent in this type of design make the time-measurement technique much more attractive. Because it was so successful in microwave radars employed against aerial targets, a program for the development of tank Range Finder T44 on this principle was initiated during the early 1950's. Unfortunately, the reflection of signals from the numerous objects that normally surround ground targets could not be overcome and the program was dropped. The investigations into microwave radar did, however, lead to interest in the so-called pulsed-light type of range finder. For this approach, the recent breakthrough in the field of lasers* has provided a light source of a brilliance that should permit the development of tank range finders of any required range and accuracy. The laser range finder should also prove superior to existing optical range finders in:

(1) Small size (see Fig. 1-49); it will be possible to install range finders in vehicles that were previously too small to accommodate them.

(2) Negligible power requirements.

(3) Night operation; the laser is the only practicable means of ranging at night without using searchlights.

(4) Economy; it is more economical to produce in quantity than the Range Finder M17 (the latest production type of optical range finder).

(5) Ease of use; it will be extremely simple to use compared with existing optical

* Lasers are amplifying devices that work in the infrared and visible light frequencies. The name "laser" is an acronym based on the work that the device performs: light amplification by stimulated emission of radiation.

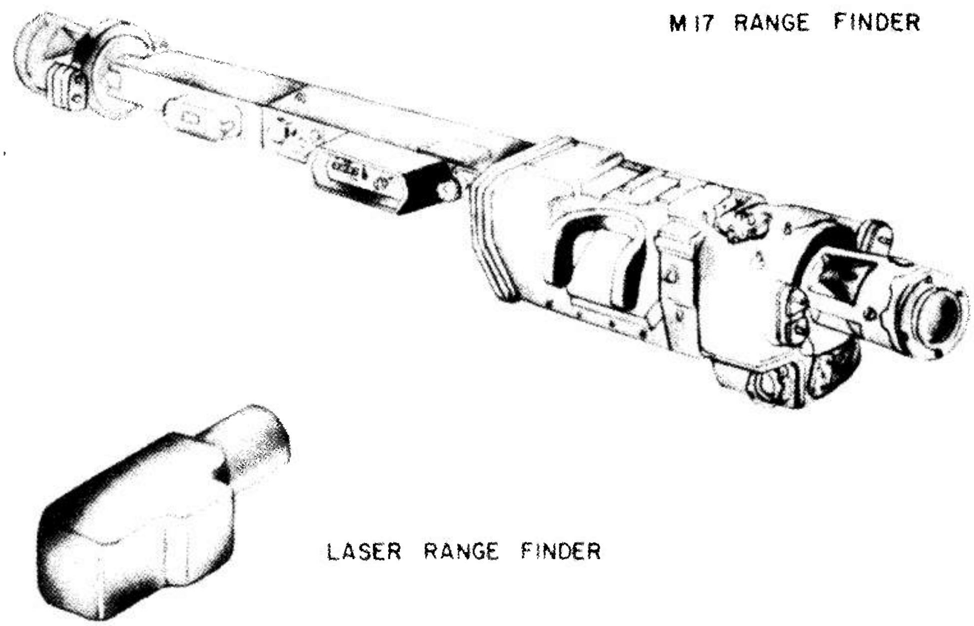


Figure 1-49. A size comparison of the laser and M17 range finders.

range finders. (The operator will only be required to point the range finder accurately at the target and press a button). This will reduce training requirements. However, because of its very short wavelengths, the laser will not penetrate such obscuring agents as fog and smoke, and thus cannot be considered a completely all-weather device.

2. Ballistic-computation equipment. The improvements in tank fire control systems have increased accuracy tremendously. Sources of error that were insignificant in the past have become increasingly important. For example, during World War II, the tank gunner had to estimate range. The resulting error in the fire control solution was of such an order that the relatively small error due to cant could be ignored. When range finders were introduced into tank fire control systems, however, errors due to cant became significant.

Accordingly, in the development of recent computers for tank fire control systems, increased emphasis has been placed on generating corrections for effects that had previously been ignored. An example is the XM16 Computer which was initiated during 1956. This electronic device can generate a solution for gun elevation that is based on many factors not accounted for in the computer provided in the fire control system for the M60 Tank which contains the latest production type of tank fire control equipment. These factors include jump, cant, vertical parallax, gun wear, and gun droop, as well as range and standard ballistics.

The XM16 Computer also corrects for lateral effects such as drift, lateral parallax, gun droop, cant, and lateral jump, none of which were corrected in any previous tank fire control systems. It can also be easily modified to correct for cross wind (a significant factor) if an adequate means of wind measurement is ever devised.

The XM16 produces an elevation output for several ammunitions without specially machined cams or carefully wound nonlinear potentiometers. Instead, the solution utilizes a linear potentiometer provided with 10 tap points. By changing the values of the resistors tapped into these points, the slope of the voltage (which represents range) can be changed. Utilization of this system allows the range-elevation relationship for a given ammunition to be approximated by 10 straightline segments. This principle is illustrated on Fig. 1-50. The resistors needed for any one ammunition are packaged in a plug-in assembly that is somewhat smaller than a package of cigerettes. When

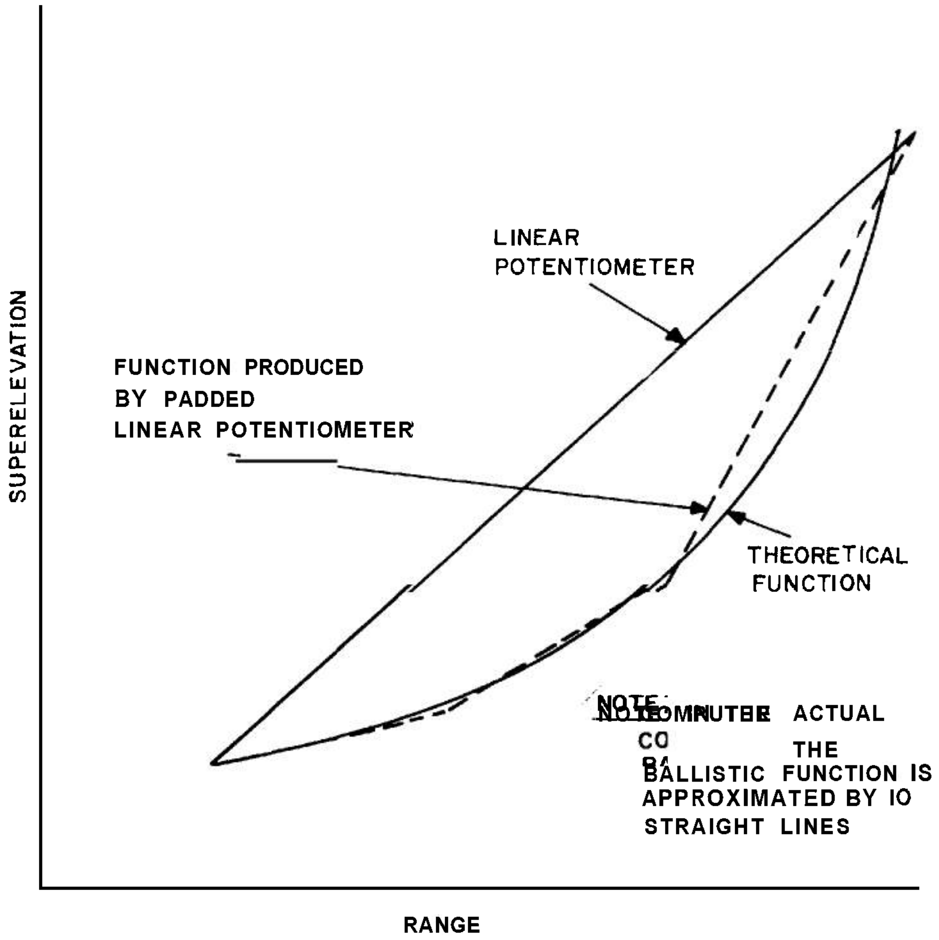


Figure 1-50. The generation of ballistic functions in the XM16 and XM17 computers.

ballistics for a new ammunition are required, a new package can be fabricated and installed with a minimum of effort.

The major design features of the Computer XM16 were incorporated into its successor, the Computer XM17. In the XM17, the outputs were introduced into the direct-fire articulated telescope XM108 instead of into a tilting-mirror periscope as was the case with the XM16. The increased accuracy of computers such as the XM16 and XM17 pays off in a better first-round hit probability and a shorter time requirement to effect a target kill. While the total volume of each of these computers is larger than that of the Computer M13 which is used in the fire control system for the M60 Tank, the computers are comprised of a number of small functional units that may be located at convenient places throughout the vehicle in which they are used.

As previously indicated, muzzle droop and bend have been found to be significant factors affecting firing accuracy. Despite boresighting and "zeroing in", there is currently no assurance at any given time that the tank fire control system is properly aligned with the muzzle of the weapon. This is due to the fact that temperature differentials may arise during firing or during changing conditions of sunlight and wind. These,

in turn, will cause the gun tube to distort in a completely unpredictable manner, thus causing the muzzle of the weapon to change its position relative to the fire control equipment. This movement of the muzzle has been proven by test to be a large source of error in weapon systems utilizing long gun tubes; in rapid firing, the error may be as much as 2.0 mils.

To compensate for these effects with ballistic computers, the magnitude and direction of the bend must first be measured. The U. S. is presently developing muzzle-position indicators for automatically introducing any change in muzzle position into the ballistic computer which, in turn, will compute corrections. Such a device is shown in Fig. 1-51, and Fig. 1-52 shows how it works. When this arrangement is used with long guns, such as the 105 mm, first-round hit probability will be materially improved.

Current ballistic solutions utilized in tank fire control systems generate weapon elevation as a function of range based on the assumption that the target is at the same height as the weapon. In the process of bringing the periscope reticle on the target, the angular height of the target above the horizontal as measured at the gun position (angle of site) is directly added to the elevation generated by the computer. (Figure 1-53 shows the relationships). In other words, current systems assume that ballistic trajectories retain their shape as they are rotated around the point of origin. As shown in Chapter 2 (see par 2-2.3.3.9), this assumption, which is known as the theory of rigidity of the trajectory, can introduce significant error into the fire control solution. Studies recently completed have shown that the errors resulting in tank fire control systems from this assumption, while small, are still significant when compared with other residual system errors. For this reason, it is anticipated that future computers will take into account the difference in height between weapon and target,

Another study in the field of ballistic computation relates to the feasibility and desirability of substituting digital techniques for analog techniques in tank fire control systems. This study will, of course, cover such factors as accuracy, cost, and complexity. Of greater significance, however, will be the determination of whether additional functions, such as tank-to-tank target designation and automatic checkout of vehicular components, can or should be included. This study is still in its preliminary phases.

3. Low-Light-Level Equipment. Since World War II, the ability to conduct combat operations with armored vehicles at night has assumed increasing importance. Starting with the M46 Tank, the driver was provided with an active infrared viewing device (Periscope M19), which permitted limited movement of the tank under black-out conditions. However, the gunner was not equipped with a comparable piece of equipment until the advent of the M50 Tank during 1960. The fire control system of this vehicle (see Fig. 1-54), which is described above, has used Periscope XM32 in connection with a xenon searchlight 18 inches in diameter.

Because of the obvious disadvantages associated with active systems, the U. S. during recent years has devoted considerable effort to research that will result in a passive night sighting system of adequate performance. Passive night sighting equipment, as its name implies, is equipment that may be employed when ambient light levels are low, for vision and fire control without illuminating the target area from the observation point. Night ambient illumination levels vary widely, depending on many factors; representative levels, however, might be full moonlight (10^{-2} foot lamberts) or starlight (10^{-4} foot lamberts). In order to "see" passively under such conditions, an image intensifier must be employed,

One approach to the problem has been the use of television techniques. While the complexity of television was recognized at the outset of the program, the availability of the required components made this approach particularly attractive. As a result, the development of a low-light-level fire control system was initiated during 1959. This system, designated Electronic Viewing Equipment (EVE), utilized an image orthicon tube* as the sensitive element. A representation

* This is a type of tube used for television cameras. By utilizing secondary emission and electron multiplication, it produces the voltages that are subsequently amplified and transmitted as television-picture signals.

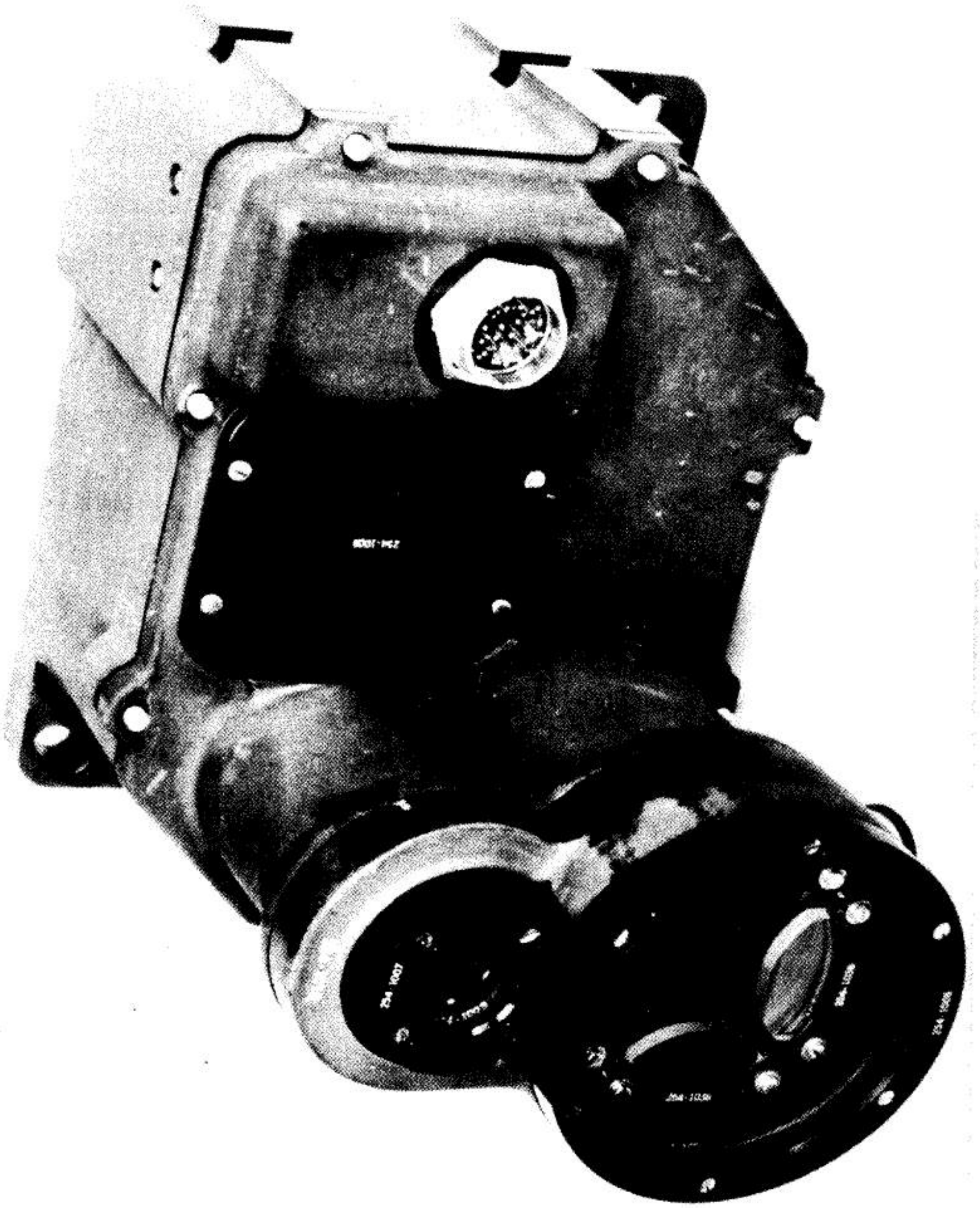


Figure 1-51. An automatic muzzle-position indicator.

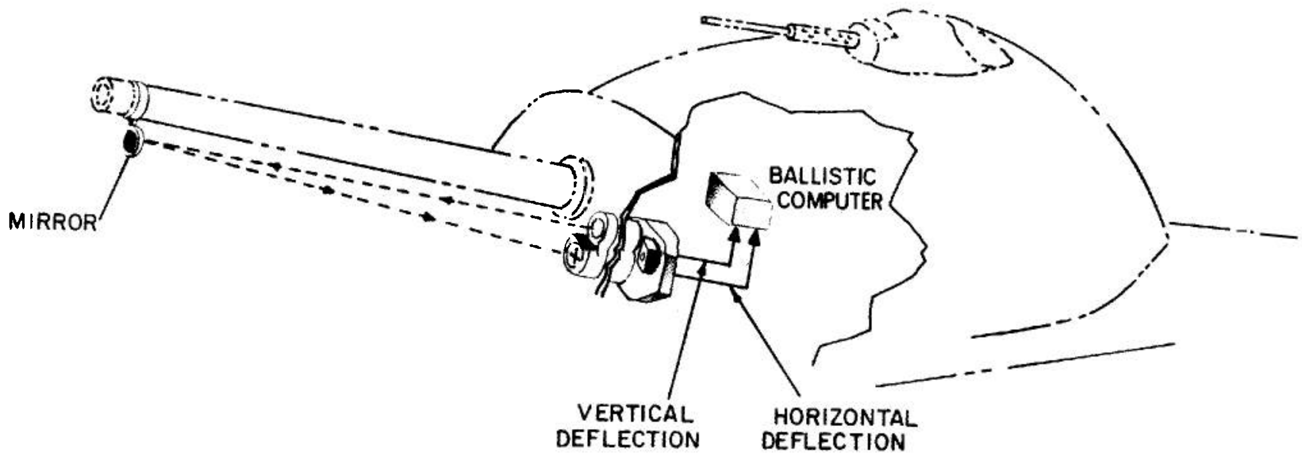
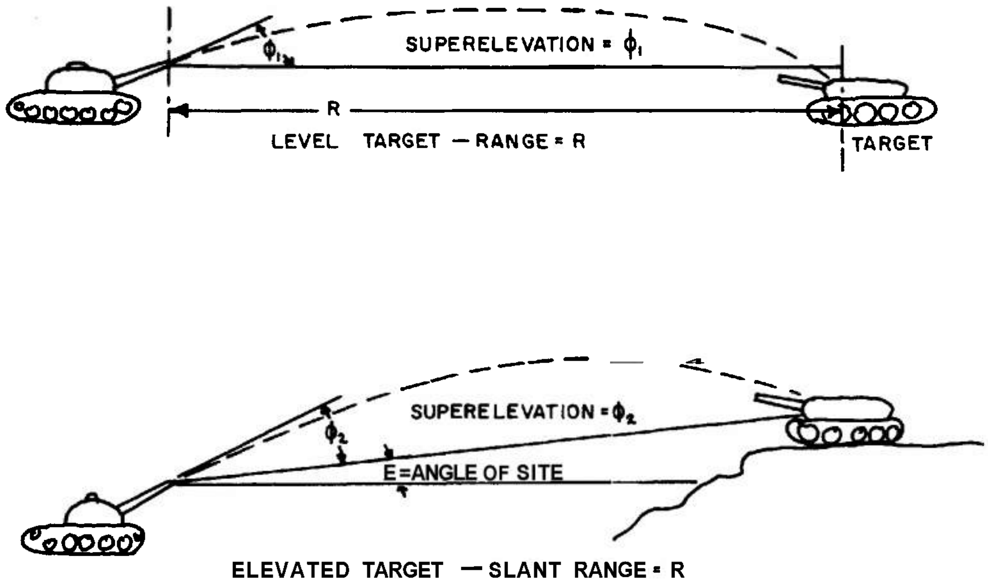


Figure 1-52. Application of an automatic muzzle-position indicator.



NOTE: WHEN TARGET IS NOT AT SAME LEVEL AS WEAPON ϕ_1 DOES NOT EQUAL ϕ_2

Figure 1-53. Change in superelevation due to the elevation of the target above the level of the firing weapon.

of the EVE XM39 installation is shown schematically in Fig. 1-55.

While the XM39 system has progressed well technically, the question arose during 1961 as to its ultimate desirability when compared with potentially simpler equipment then in development. A laboratory evaluation of the imaging components then available or soon to become available showed that (1) the so-called image-intensified orthicon was the most sensitive component against static targets, (2) the three-stage cascaded image converter tube was the next most sensitive component, and (3) the image orthicon (which was used in the XM39) was the least sensitive. The image converter tube also proved simpler and more reliable. On the positive side of the ledger for the television system was its remoting capabilities, which in certain applications is of great importance, and the availability of image orthicon tubes used in the XM39 system.

As a result of the evaluation, practically all current U. S. developmental effort in the field of passive night sighting devices is related to using the three-stage cascaded image converter tubes as the basic element of such devices. For example, at the present time, the 9-power Periscope XM44 using a cascaded image converter tube that is electrostatically focused is being developed for the new U. S. tracked Reconnaissance Vehicle XM551.

In addition to use in the Periscope XM44, the electrostatically focused image converter tube is being considered for use with telescopes; preliminary design of an optical system that can be adapted to the front end of the existing telescope of the M60 Tank is now underway. This system is scheduled to incorporate provision of both 5X and 14.2X IR capability and 1.5X and 8X daylight capability. As may be seen from Fig. 1-56, the existing first-generation device is large (it is much larger and heavier than the IR Periscope XM25). However, elimination of the 18-inch 186-pound xenon lamp, currently included in the M60 Turret, will result in a substantial space and weight saving; Fig. 1-57 illustrates this strikingly. The large searchlight is also,

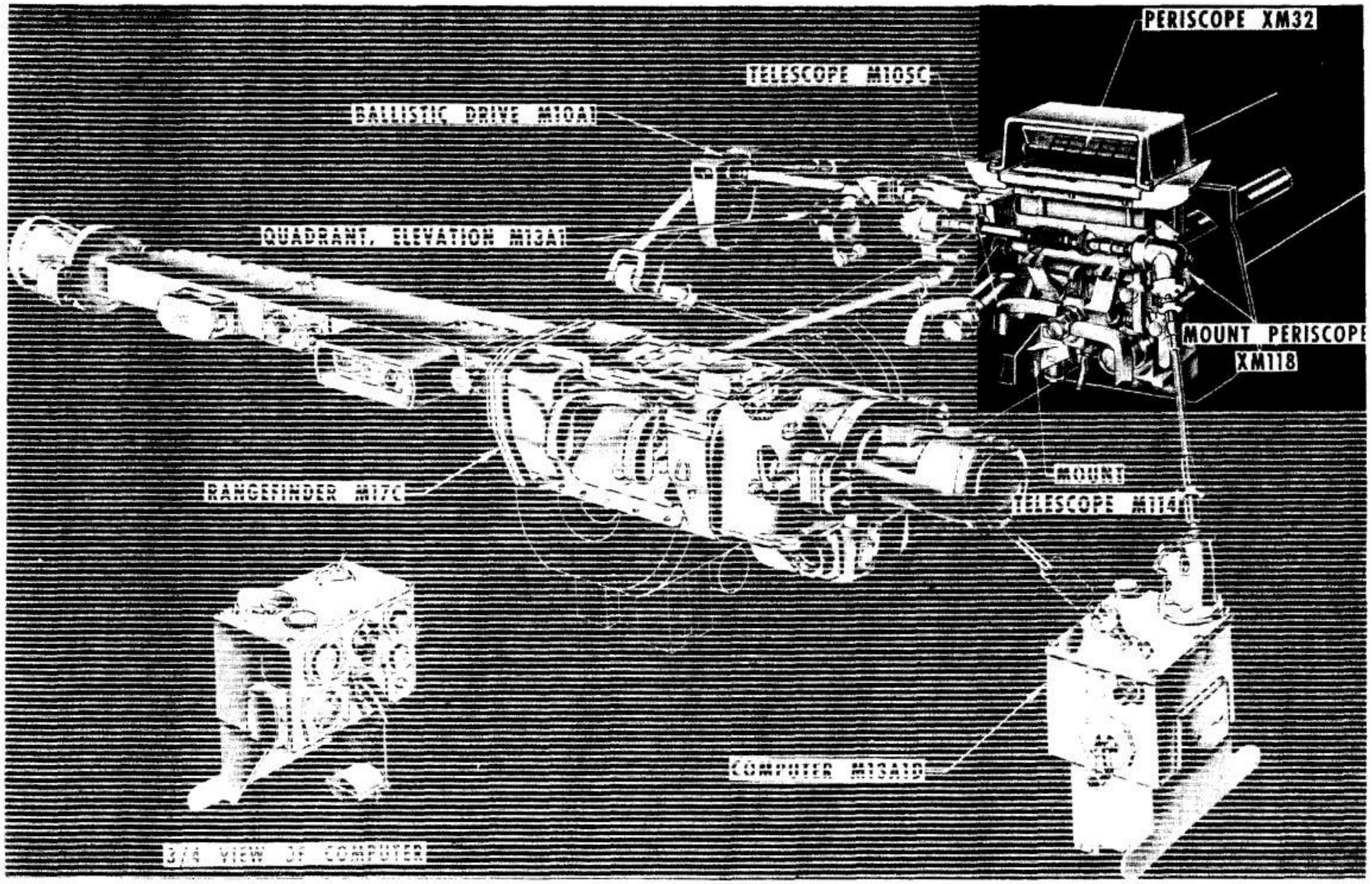


Figure 1-54. Infrared viewing in the fire control system for the M60 Tank by means of the Gunner's Periscope XM32.

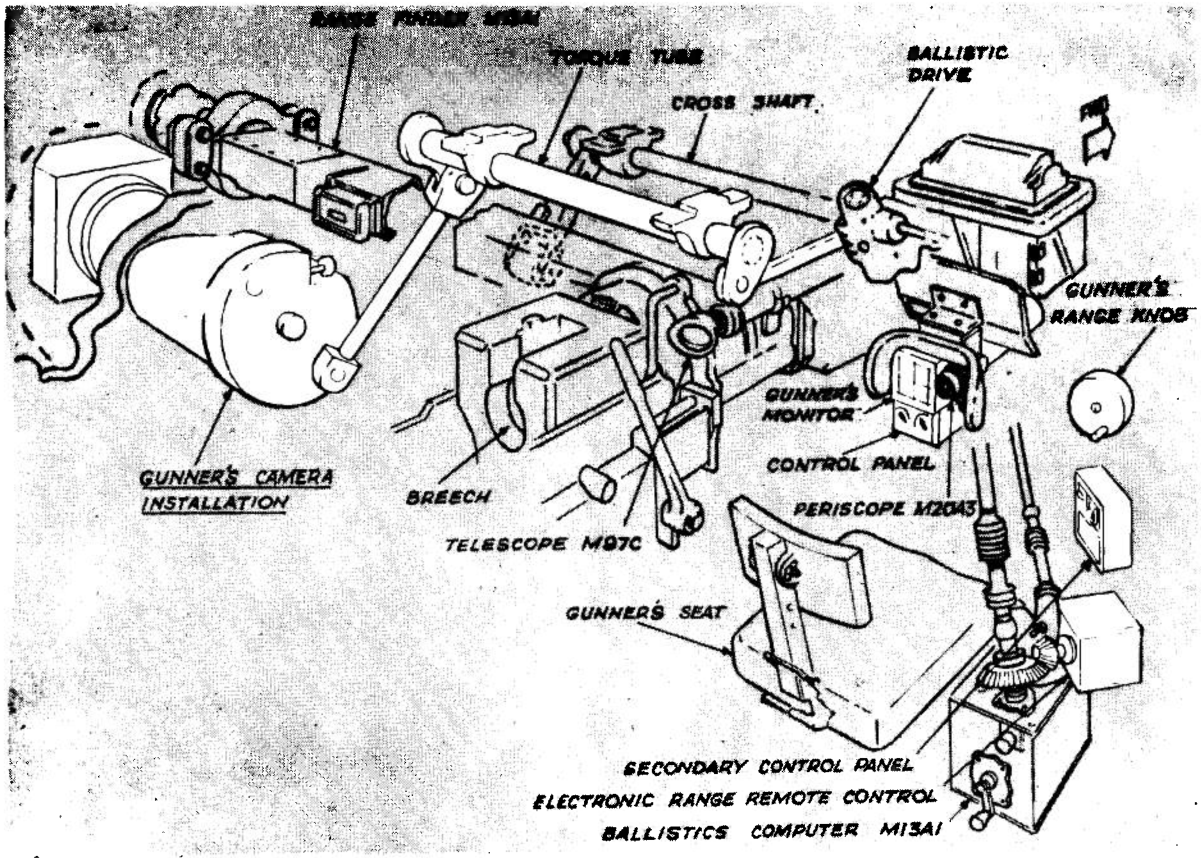


Figure 1-55. Concept of the XM39 Electronic Viewing Equipment (EVE) installation.

of course, very vulnerable to attack.

While the new system is essentially passive, a small searchlight is included, since such a device may be helpful under certain otherwise marginal conditions. At present, the program is restricted to a concept study and the construction of simplified functional mock-ups. Prototype construction is not contemplated in the near future.

The three-stage cascaded image converter is inherently simple and should lead to equipments of low complexities, comparable to the M19 Periscopes. Timely realization of tubes suitable for incorporation into military equipments depends, however, on the successful completion of development programs now underway and perhaps the solution of difficult physical problems. Production in quantity also requires the development and application of special techniques. Concentrated efforts to provide the required production of cascaded tubes are in progress under Army auspices.

The foregoing summary of new range-finding, ballistic-computation, and low-light-level equipment shows that significant advances have been made in the individual components employed in tank fire control systems.

These advances have resulted from both newly achieved technological breakthroughs and less dramatic, but equally important, evolutionary refinements in design. While the embodiment of these advances into equipments has established the feasibility of the principles involved, this is but one aspect of a design developmental program. An equally important aspect is the value of each of these proven components as a part of integrated fire control systems suited to specific weapons and ammunitions.

Typical examples of system integration are provided by fire control systems that would be appropriate for use with two weap-

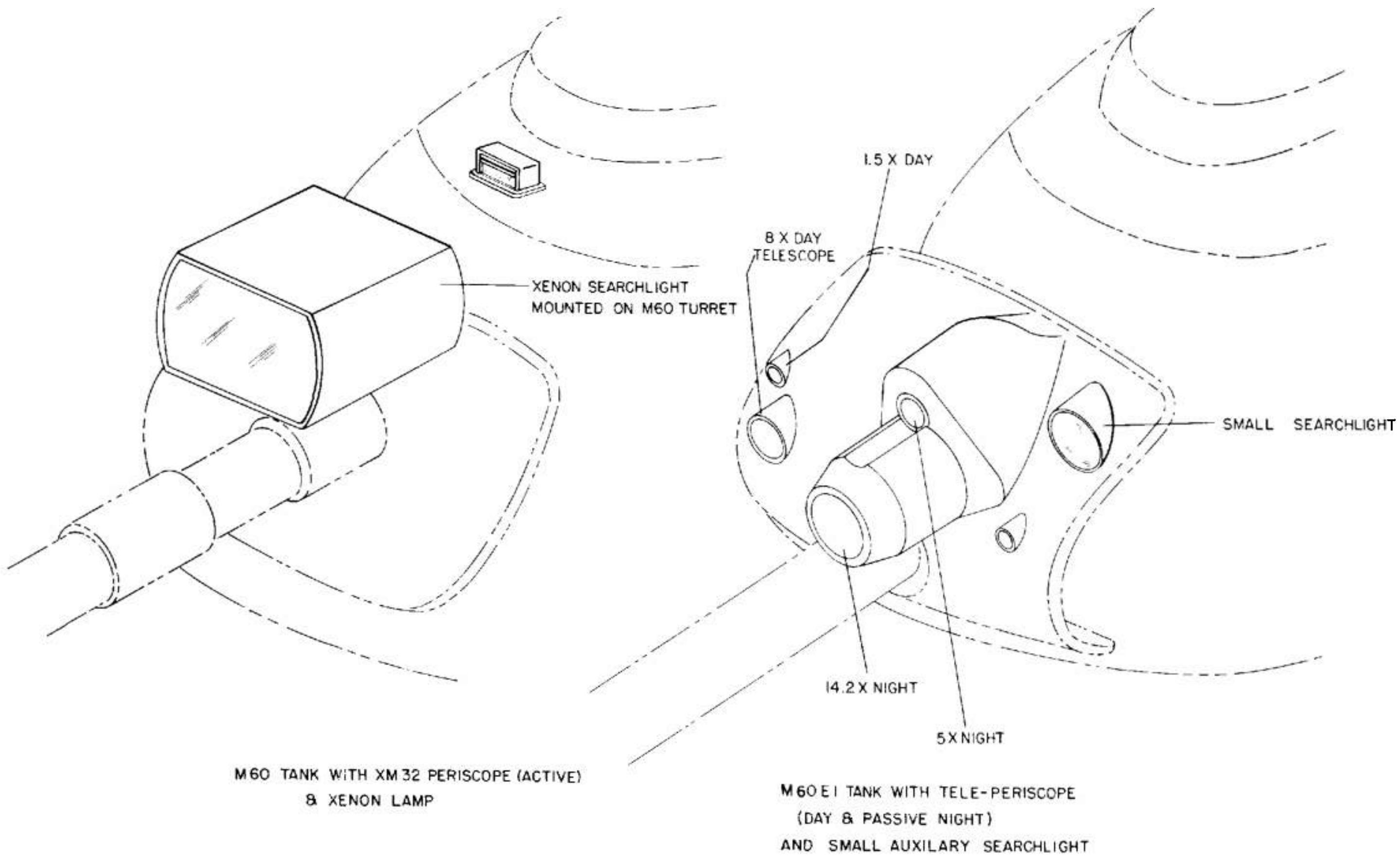


Figure 1-56. A size comparison of the active IR system and the proposed passive IR system for the M60 Tank.

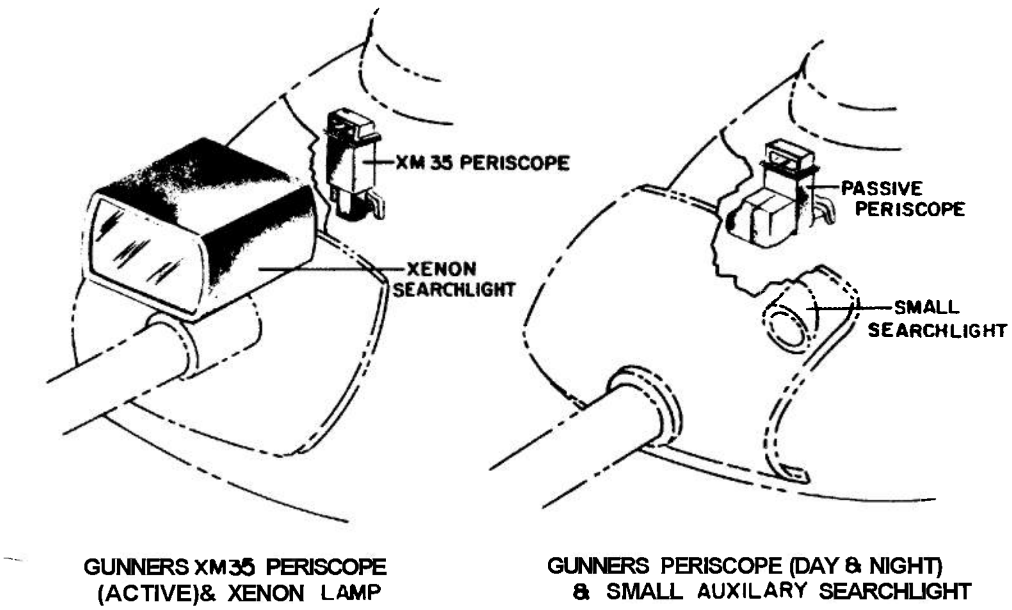


Figure 1-57. Space and weight advantage of a passive night sighting system.

on systems currently being considered for future use in vehicles (see Figs. 1-58 and 1-59). Example 1-1 summarizes system integration considerations in the design of a fire control system for an armored vehicle employing the new low-velocity 152 mm gun as the primary weapon. Example 1-2 examines the fire control system for an armored vehicle employing the comparatively high-velocity 105 mm gun as the primary weapon.

Example 1-1. System-integration aspects associated with a fire control system for an armored vehicle employing the low-velocity 152 mm gun as the primary weapon (see Fig. 1-58).

A. Aspects Associated With the Commander's Station

1. Because of curvature of the ballistic trajectory and the resultant susceptibility to disturbances caused by nonstandard conditions, it is particularly important that low-velocity weapon systems be provided with accurate fire control. While there are many reasons for positioning the laser range finder in the commander's cupola, as shown in Fig. 1-58, this location cannot be finalized until more is known of the user's desires, the space available, etc. In addition to the range finder, the cupola is also provided with a passive day-night periscopic sight. The rotation of the top mirror of the periscope will be mechanically tied to the elevation axis of the range finder and the machine gun *so* that the periscope may be used for aiming both these elements. Because of the passive-night-sighting capability of the periscope, ranging and using the machine gun at night are possible without illuminating the target. Because of the antiaircraft applications of the machine gun, an elevation capability of the line of site of the periscope of at least 60 degrees is required.

2. Currently, the user desires that the commander's sight have optical characteristics tnat are identical with those of the gunner's periscope. While it is the opinion of Frankford Arsenal that the question of power, field of view, etc. of the commander's sight should be given further consideration, it has been assumed for the purpose of this discussion that the commander's and gunner's sight will have identical characteristics.

3. The value of 7-power binocular daylight viewing currently desired by the

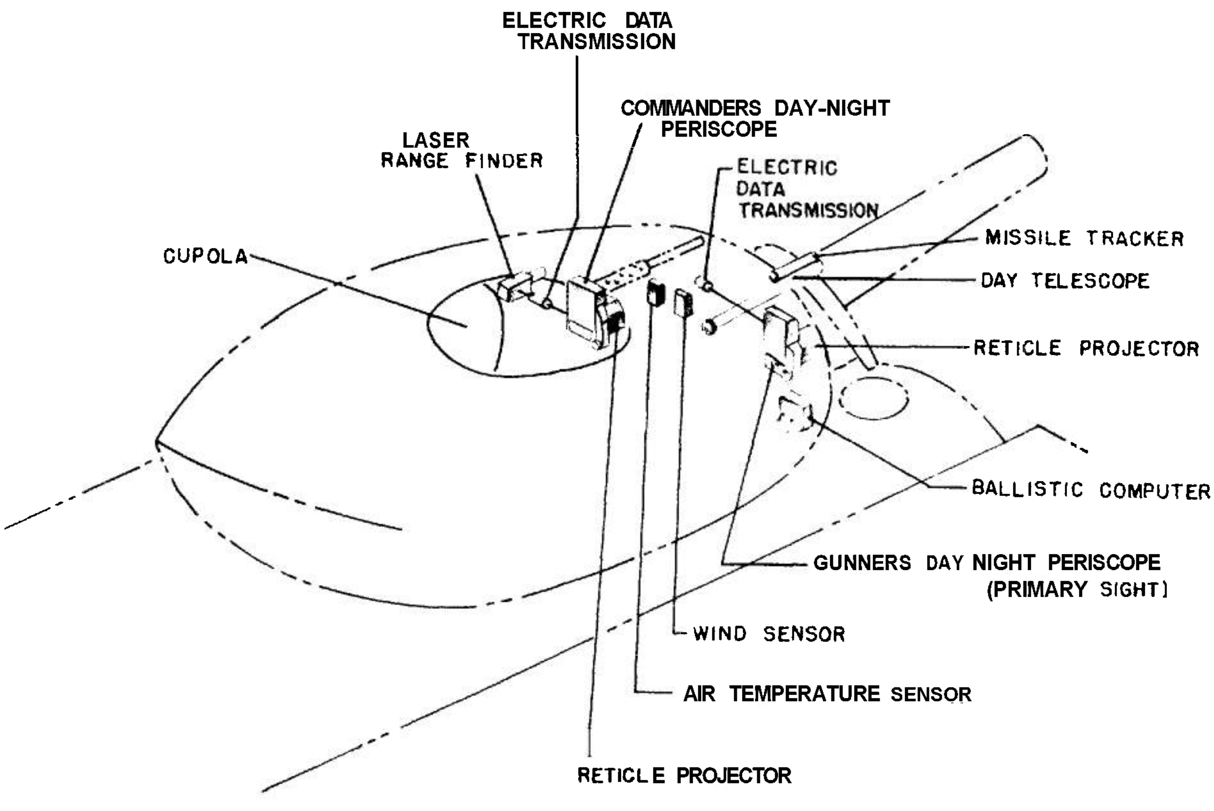


Figure 1-58. Schematic representation of an integrated fire control system for a low-velocity weapon system.

Example 1-1 (Continued)

user is not questioned. There is, however, considerable question as to whether incorporating this capability into a passive night sighting periscope would be practical because of size and weight. One approach to solving this problem would be to provide two interchangeable bodies, one of which would be used for daylight viewing of targets while the other would be for nighttime viewing. The difficulty associated with changing and stowing these bodies may be grasped from the fact that the passive body alone will weigh approximately 50 pounds. Because of the problems noted, it is quite possible that at present it will be necessary to provide a periscope with the characteristics listed in the tabulation below. The ranges of operation shown can, of course, be enhanced by the addition of a small (6- to 8-inch diameter) searchlight that would also be mounted on the cupola.

Possible Characteristics of the Commander's Passive Periscope

Passive Night Viewing

Power	9X
Field of view	6°
Exit pupil diameter	9 mm

Daytime Viewing

High-power monocular	
Power	8X
Field of view	8"
Exit-pupil diameter	7 mm
Unit power	
Power	1X
Horizontal field of view	22"
Vertical field of view	8"

Other Characteristics

Elevation limits: -10° to 60°
Quick-acting shutter to provide protection of operator's eyes from atomic flash
Two alternately viewed recticles:
- Fixed reticle for missile guidance and ranging
- Movable reticle controlled by computer output

4. As combat vehicles must be usable in proximity to atomic blasts, it is necessary that some degree of flash protection be afforded to the gunner. Currently, there are under development a variety of quick-acting shutters that would be activated by the atomic flash. Assuming that the reaction is rapid enough to close the shutter prior to the observer's eyes receiving burns, at least this protection would be available. There are now two approaches to the shutter problem that appear to offer acceptable solutions. It is, therefore, assumed that both the commander's and gunner's sighting equipment discussed herein will be equipped with these "anti-flash" devices.

5. Because it is necessary for the commander to have the capabilities of (1) designating targets, (2) ranging, and (3) emergency gun laying, positioning his sight in the cupola introduces certain complications into the system. These complications are caused by the necessity for orienting the sight in azimuth and elevation together with both the gun and the gunner's sighting equipment. In order to accomplish this orientation in azimuth, the cupola must first be brought to a "zero" position with respect to the turret. The manner in which this is to be accomplished will be the joint responsibility of those designing the fire control system and those designing the armored vehicle. While mechanically complex, the ideal design would permit the commander, once having laid on a target, to bring the turret into the "zero" position with respect to the cupola while holding the azimuth orientation of the cupola fixed in space. It is understood that cupolas providing this capability have been considered.

6. In addition to the problem of orienting the commander's sight in azimuth, it must also be oriented in elevation with respect to other elements of the system. Because of the rotation of the cupola, this orientation cannot be achieved by means of a parallelogram linkage. Instead, it will be necessary to couple the sight to the other elements by means of a servo system utilizing electrical data transmission. (Such a subsystem is currently under development. Errors of less than 0.1 mil are anticipated.)

Example 1-1 (Continued)

B. Aspects Associated With the Gunner's Station

1. The fire control equipment to be provided for the gunner's use will, as for all other elements of the system, be dependent on both the available turret space and the requirements for any auxiliary equipment that may be needed in connection with missile guidance. The configuration shown in Fig. 1-58 is predicated on the assumption that the vehicle will be armed with (1) a low-velocity weapon (such as the 152 mm cannon) that will fire a conventional round plus (2) some type of guided missile (most probably Shillelagh), which, in turn, will require that some appropriate device (such as the Shillelagh tracker) be pointed at the target. This device will be pointed at the target by virtue of the fact that it is closely associated with, and accurately boresighted to, some piece of sighting equipment that will, in turn, be aimed by the gunner. Recent indications are that association of the missile-system tracker with the gunner's day-night passive periscope will permit a more compact vehicle design. Because the existing state-of-art necessitates large components in both the passive periscope and the Shillelagh tracker, however, it appears that a combined instrument may at this time be prohibitively large. For this reason, the system shown in Fig. 1-58 shows the missile-system tracker mounted on an articulated telescope as it is in the AR/AAV vehicle. This telescope would also serve as an emergency means of laying the weapon when firing the complementary guided-missile round.

2. As shown in Fig. 1-58, the gunner would be provided with a passive day-night periscope identical with that of the commander. This periscope would serve as the primary system for laying the complementary guided-missile round. Elevation orientation of the sight with respect to the weapon could be achieved by utilizing either the electrical data transmission or a parallelogram linkage.

3. Because of accuracy and parallax considerations, both the laser range finder and the missile-system tracker should be pointed at the target by utilizing the sight with which it is most closely associated. Thus, in the configuration shown in Fig. 1-58, the range finder would normally be operated by the commander while the missile-system tracker would normally be pointed by the gunner. It should, however, be recognized that inasmuch as the range finder, the missile-system tracker, the gun, the gunner's sighting equipment, and the (commander's periscope are all tied together in azimuth and elevation, it is possible under emergency conditions for the gunner to operate the range finder and for the commander to lay the conventional round and to operate the missile-system tracker.

4. While the elevation orientation of the various elements of the system has the advantage described above, this also is a source of a potential problem. This problem results from the fact that since the range finder will be pointed in the same direction as the gun, neither will be pointed directly at the target when the gun is in firing position. Because both vertical and horizontal corrections are provided by the ballistic computer, this deviation will be in both azimuth and elevation. This condition does not exist in current systems wherein no horizontal firing corrections are computed and wherein the range finder is maintained parallel in elevation to the line of site through the reticle of the gunner's periscope. Provided that the range finder is free to move with respect to the weapon in both azimuth and elevation (as it would be in a cupola mount), the solution to this problem is not technically difficult and will add but little complexity to the system. This is one point in favor of locating the range finder in the cupola. Prior to a final resolution of this problem, it will be necessary to solicit the user's views as to how much the system operation would be degraded by not having the range finder pointed at the target during laying of the weapon.

5. Both the gunner's and commander's periscopes will be provided with projected reticles whose distance away from boresight position will be determined by the horizontal and vertical corrections generated by the new improved ballistic computer. This computer will be similar in concept to Bal-

Example 1-1 (Continued)

listic Computer XM16 except that certain factors previously omitted in the computation of corrections for the system under consideration are as follows:

a. Vertical corrections will be based on:
   Range*
   Vertical jump*
   Parallax*
   Air density
   Air temperature
   Propellant temperature (for its effect on muzzle velocity)

b. Horizontal corrections will be based on:
   Range*
   Cant*
   Drift*
   Horizontal jump*
   Parallax*
   Cross Wind

NOTE: Factors marked (*) are included in the correction computations of the XM16 Computer.

6. There is no need to include corrections for muzzle bend in a fire control system for a short-tube weapon such as the new 152 mm cannon.

7. The feasibility of providing most of the required inputs has been established. The major exception to this is measurement of crosswind. As discussed in detail in Frankford Arsenal Memorandum Report M61-30-1, dated May 1961, there are many problems associated with the measurement of the winds that will influence the path of a projectile during its flight. Among other factors are the time and space variability of low-level winds. As a result of the multiplicity of problems, it has been concluded that providing accurate wind correction is not feasible at present.

8. Studies recently conducted have shown that if a 50-percent reduction of the errors now attributable to neglecting crosswind can be achieved, a significant improvement in hit probability will result. This is particularly true in the case of the low-velocity ammunition under consideration. Thus, despite the fact that a highly precise measurement of the input is impractical, it is advisable to introduce the correction for crosswind into any new full-solution fire control system that may be developed for low-velocity rounds.

9. Recent computations have also shown the importance of correcting the fire of low-velocity rounds for deviations of air temperature, air density, and propellant temperature from standard conditions. As a result, these corrections are also included in the proposed system. At present, the method of obtaining the value of the air-density deviation from standard has not been resolved. It is possible, however, that a satisfactory value may be obtained from the periodic field-artillery metromessages modified by local measurement of temperature. This is a matter that must be studied in more detail.

10. Studies undertaken to determine the anticipated performance of the 152 mm gun firing Projectile XM409, assuming the use of the fire control system described, have shown that:

(a) The increased effectiveness of improved ballistic computers, as compared with the M13, is doubled by the inclusion of corrections for crosswind, air temperature, air density, and propellant temperature. Of this increased effectiveness, crosswind correction is a large contributer despite the fact that the assumption has been made that the correction has an error as large as 50 percent.

(b) In the case of low-velocity ammunition, a fire control system utilizing a computer of the M13 type will not be appreciably more effective even though a laser range finder is substituted for the M17. More-sophisticated fire control systems, however, will benefit to some extent from use of a laser range finder.

Example 1-2. System-integration aspects associated with a fire control system for an armored vehicle employing the high-velocity 105mm gun as the primary weapon (see Fig. 1-59).

1. The fire control system proposed €or use with high-muzzle-velocity cannons, such as the 105 mm cannon used in conjunction with the M60 series Tank, is shown schematically in Fig. 1-59. As in the case of the low-velocity system, a schematic representation has been chosen because of the great dependence of the final appearance on space available in the turret.

2. The equipment provided in the commander's station is identical with that to be utilized with the low-velocity weapon. At the gunner's station, the missile-system tracker is not required and hence the articulated telescope is strictly an emergency gun-laying device. As in the case of the low-velocity system, the day-night passive periscope will be the primary sight. Inasmuch as the effects of air temperature, air density, and propellant temperature on rounds fired from the 105 mm gun are negligible, these factors have been omitted from this system. While the effect of crosswind on the 105 mm rounds is considerably smaller than it is on the 152 mm round, it is advisable to consider inclusion of a correction for this effect, particularly if the instrumentation is not complex.

3. Because of the length of the 105 mm gun tube, distortion due to temperature differentials that may exist in its structure is a significant source of error, These temperature differentials arise from firing the gun and from rapid changes in exposure of the tube to sunlight and wind, Experiments conducted at several test sites have revealed

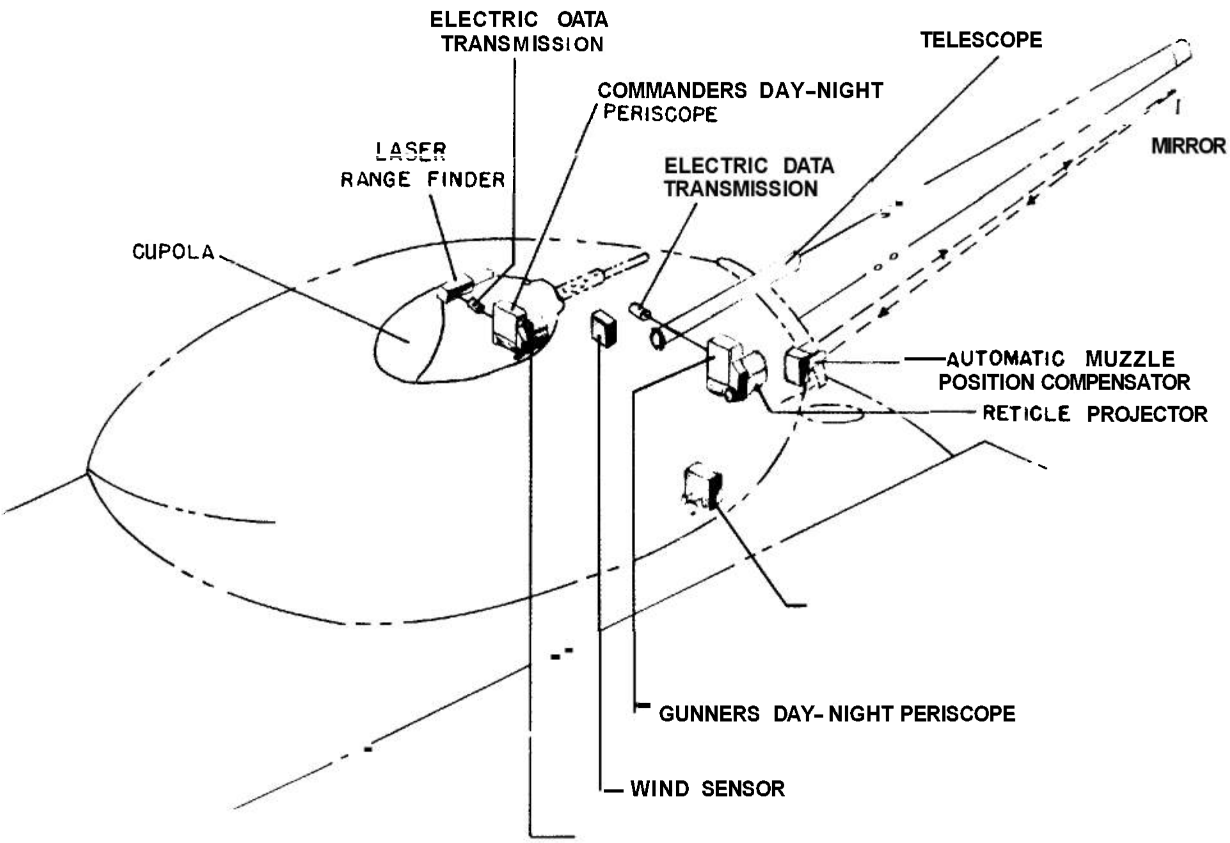


Figure 1-59. Schematic representation of an integrated fire control system for a high-velocity weapon system.

Example 1-2 (Continued)

;hat the deflections of a long tube gun may be 2 mils or larger. Because of these deflec- ;ions, the automatic muzzle position indi- cator is included in the system proposed for ;he high-velocity weapon. Deflections de- :ected by it are automatically fed into the ballistic computer, where corrections are generated.

4. Because there are currently ammunitions with three different muzzle velocities (4500 ft/sec, 3800 ft/sec, and 3000 ft/sec) fired from the 105 mm weapon, the computer must be capable of producing the appropriate range-elevation relationships for all three of these ammunition types. This is in contrast to the low-velocity sys- :em, whereinthere is only one range-eleva- :ion relationship required.

5. Based on the preceding considera- :ions, the computer to be provided for use .n the high-velocity system will generate corrections based on the following factors:

a. Vertical corrections will be based on:

Range*
Vertical jump*
Parallax*
Ammunition type*
Muzzle bend

b. Horizontal corrections will be based on:

Range*
Ammunition type*
Cant*
Drift*
Horizontal jump*
Parallax*
Cross wind
Muzzle bend

NOTE: Factors marked (*) are included in the correction computations of the XM16 computer.

6. Studies undertaken to determine the anticipated performance of the 105 mm gun firing the APDS projectile (M.V. = 4500 ft per sec), assuming the use of the fire control system described, have revealed the following interesting facts:

a. A fire control system including a computer with the characteristics of the XM16 coupled with the automatic muzzle position indicator will result in a significantly improved first-round hit probability as compared with the system currently in use in the M60 Tank.

b. For a high-velocity round such as the APDS projectile, the contribution of a 50-percent correction for crosswind is trivial. This value will increase, however, for the lower-velocity rounds that can be fired from the 105 mm gun.

c. In contrast to the low-velocity system, substitution of the laser range finder for the M17 results in a small but discernible improvement in the first-round hit probability.

The proposed fire control systems discussed in Examples 1-1 and 1-2 cannot be more precisely defined at present because two conditions are still to be met:

1. A detailed interchange of ideas must take place between the designers of the fire control system and the users of the weapon system into which the fire control system is to be incorporated.

2. The fire control system designers must know in detail what space will be available in the armored vehicles. A schematic presentation is used in Figs. 1-58 and 1-59, rather than the more realistic type of presentation used in preceding illustrations, because the appearance of the fire control systems depends greatly upon the actual configurations of the vehicles.

Despite the present detailed uncertainties, however, the general merits of the two proposed systems should be clearly evident. As compared with the latest equipment currently available in such vehicles as the M60A1, the proposed systems offer the following advantages:

1. While some components are larger than those currently used, the elimination of the xenon searchlight and the reduction in size of the laser range finder as compared with the M17 Range Finder will result in an overall system that is smaller, lighter, and easier to place in vehicles than existing equipment.

2. The proposed systems will be better suited for operation under a variety of battlefield conditions than any currently in existence. The flash-protection devices will offer some degree of eye protection against atomic flash. The passive night-sighting equipment will enhance the capability of fighting at night.

3. Crew security will be increased by elimination of the extremely vulnerable 18-inch-diameter xenon searchlight.

4. Accuracy will be increased.

5. While certain components are more costly than those currently in use, it is believed that as the prices of the image converter tubes and the laser range finder are reduced by quantity production the proposed systems will ultimately cost little if any more than comparable systems currently in use.

6. Because of the several automatic features included, plus the simplicity of using the laser range finder, the proposed systems should be easier to operate than those now in existence. This will result in faster reaction time and decreased training requirements.

Accordingly, it can be concluded that recently conceived components will make it possible to design integrated tank fire control systems that are significantly better than those currently in existence.

A final point should be noted with regard to the configuration of fire control systems for armored vehicles. The ultimate decision as to the fire control equipment to be provided for a given tank weapon system must take into account not only the aforenoted merits of the so-called "full-solution" systems but also the merits of ballistically matched spotting systems. An analysis of this highly complex problem is beyond the scope of this handbook. It can definitely be stated, however, that (1) development of "full-solution" systems will improve first-round hit capability, and (2) this will in turn diminish the chief virtue of spotting systems.

Work on tank stabilization systems has also continued with strong emphasis on the years subsequent to World War 11. For example, in 1950, Vickers, Inc. built a completely integrated fire control system that combined a coincidence range finder, an automatic lead computer and a stabilization system for the Light Tank T41. This system, like the Westinghouse system, employed hydraulic power drives. The British, on the other hand, developed a stabilization system for their Centurion III tank: that employed an electric power drive.

An extensive discussion of the various design considerations that apply to tank stabilization systems is reserved for Section 4 of the Fire Control Series (Weapon-Pointing Systems).

1-3.3 AA FIRE CONTROL SYSTEMS

Between World War II and the early 1960's the Army put a major effort into the improvement of antiaircraft fire to match the increasingly high speeds of modern aircraft. Special emphasis was placed in developing lightweight, mobile, semi- or fully-automatic radar-controlled tracking systems for use in forward areas.

Formidable difficulties were encountered, particularly in tracking low-flying aircraft. No radar was able to discriminate such aircraft from the "nap" of the earth consistently enough to lock on the target and track it automatically. Computers were unable to process the very rapidly changing inputs of target range and angular velocity, or they were too cumbersome and difficult to maintain in the field, or both.

Combination optical and radar systems were tried and other simplifications made to increase usefulness of front-line weapons.

Though some of the weapon systems proved effective against some kinds of targets, none achieved the specified minimum percentage of hits on high-speed maneuvering targets at low altitude. The last of these systems, the Vigilante, was phased out in 1962 in favor of the Mauler Forward Area Defense Missile System.

The very difficulty of the problems that prevented complete success in ground fire control against high-performance aircraft also lends interest to the research and development devoted to these problems. For this reason, some of the systems are described in the paragraphs that follow. Note that such items as doppler radar and tracking computers often proved the weakest links. As lighter, more effective and rugged radars and computers are developed, the concepts of such systems as Raduster and Vigilante may eventually be successfully applied.

1-3.3.1 Late World War II and Post-War Years[33, 34]

During World War 11, the increased activity of enemy aircraft in ground support and reconnaissance brought a demand for a light forward-area antiaircraft system to defend Army Field Forces. Since 40 mm cannon were being produced in large numbers, these were used as the basic weapon, and an on-carriage target indicating system and target designating system were developed. About the end of the war, a self-propelled twin 40 mm gun mount with a mechanical computing sight was introduced for use in forward areas.

Between 1947 and 1950, a program of improving 40 mm AA fire control resulted in Drive Controller T26, using a ball resolver type of tracking head as the gunner's control.

In the 1950's a number of different AA fire control systems were developed for weapons of various sizes. The most prolifically produced systems were for rather large AA weapons and were designed originally to be fully automatic. These were the M33 and M38 (Skysweeper).

1-3.3.2 T33/M33[35]

The T33 was developed in 1949 and 1950 as an electromechanical system designed to detect any aircraft and compute the necessary firing data to control 90 mm or 120 mm guns. It included two radars: a track radar and an acquisition radar. The track radar and its associated parts — tracking console, tracking antenna, tactical console, computer, data junction box, and early warning plotting board — were installed on a trailer; the acquisition radar, with antenna, antenna drive unit, antenna RF unit, and modulator unit, was set up separately from the tracking radar but controlled from it.

The acquisition radar operated on S-band frequencies and was designed to detect targets at a maximum range of 120,000 yd. The target tracking system was designed to track the target at ranges up to 99,000 yd; the tracking radar furnished present target information to the computer which then transmitted either predicted or present information to the gun battery.

The M33 system, introduced in 1952, was like the T33, except for an improved acqusition antenna enclosed in a Fiberglas dome, transported and emplaced from a flat bed trailer.

The M33 Search Radar was sensitive and powerful compared with other systems; return was 10 or more db stronger than, for example, the M38 and Porcupine. Detections of 80% were achieved in tests at Aberdeen Proving Ground and lock-ons of 72%, but lock-on required the use of both radars and three operators. Tracking of low-altitude aircraft was relatively poor but a capability for this type of tracking was not a design objective.

1-3.3.3 M38 (Skysweeper)[36, 37]

The AAFCS M38 Skysweeper was specif-

ically designed to provide fully automatic fire control for a 75 mm AA gun against low-flying, high-speed aircraft. This system was produced, tested, and used over a seven-year period from 1951 to 1958. As it turned out, many deficiencies were discovered and eliminated in the test program and in the field, and Aberdeen Proving Ground recommended that in similar circumstances in the future production be severely curtailed until the "bugs" are removed.[36]

Skysweeper included a medium-range 75 mm cannon with carriage-mounted fire control equipment, including electromechanical computer, radar tracker, periscope, power control, target selector, cable system, and wiring set. The gun itself had automatic loading and ramming and recoil mechanism. The whole thus formed a rapid-fire completely integrated AA weapon system.

The computer was mechanically connected to the radar tracker and to the azimuth power controls via ground reference shafting. The computer, of modular construction, converted angular data from the radar to rectangular (XYZ) coordinates in two converters. A prediction unit in the computer — with constant-speed motor, inverter, and ballistic unit with cam — determined target rate and multiplied it by time of projectile flight. Target lead distance and present position were then added in each coordinate and converted to future angular data. A ballistic unit added elevation and corrected time of flight.

The computer output was put in synchro form for transmission to the power controls. Data on wind, muzzle velocity, and air density could be inserted.

Skysweeper was effective against targets approaching or moving away at constant moderate speeds and altitudes but its effectiveness dropped rapidly at higher speeds or changes in speed and direction, and at low altitudes. For example, one series of tests produced 67% of hits (15 yd limit) with targets moving perpendicular to the line of site at 540 knots. When the speed increased to 810 knots, accuracy dropped to 17%. It was speculated that the trouble might lie in excessively low computer requirements.

At low altitudes, the lack of a doppler element in the radar made detection difficult; no target signal return was detected when ground-clutter signals exceeded target signals in intensity. In a series of low-altitude tests, detection of low targets was in the order of 50%. Lock-on also proved exceedingly difficult: 28% vs 70% for the M33, probably because the complex tasks of detection and lock-on were performed by a single operator. Once lock-on was achieved, however, tracking was more satisfactory.

1-3.3.4 Porcupine X-1[36, 38]

The Porcupine X-1 Antiaircraft Fire Control System was developed by Lincoln Laboratory, Massachusetts Institute of Technology, as a defense against high-performance, low-flying aircraft. It was primarily intended as a close-in defense weapon for cities and other civil installations of a permanent nature. It was housed in a large trailer, with a separately mounted optical sight (which could be stored in the trailer in transit). It was intended that a number of these systems, with associated rocket launchers or guns (the Porcupine could be adapted to various types of weapons), would be emplaced on the periphery of the area to be defended, with overlapping sectors of destruction.

In developing a simpler and more effective fire control system for low-flying targets, the Porcupine X-1 designers profited from the difficulties encountered with the immediately preceding systems. The chief means of achieving these goals were:

1. The use of a Doppler radar which distinguished moving targets from stationary targets and was able to detect many targets and track them in range when target-signal energy was lower than ground-clutter-signal energy.

2. The use of an optical sight for tracking in azimuth and elevation, and for providing the directional inputs to the computer. The designers felt that optical tracking was justified on the grounds that attacking low-flying aircraft could navigate effectively and identify ground targets only when visibility was reasonably good. At any rate, the sight eliminated the difficulties encountered in other systems in angular tracking by radar, and it simplified operation and maintenance.

The system detected targets, measured coordinates, and predicted future target po-

sition coordinates as follows:

1. An automatic-detection Doppler radar (whose antenna was mounted on the trailer roof during emplacement) detected and designated the approximate position of an approaching target; the radar established nine guard rings at radii of 3-1/2 to 6 miles from the site and any target entering that area tripped an alarm after the radar took two successive looks at the target. The optical sight operator, thus alerted and fed the target azimuth by the radar, aligned his sight with the target azimuth and moved it in elevation until he located the target; then he commenced tracking. Meanwhile, the Doppler radar searched automatically through a small range interval at 3 to 3-1/2 miles, locked on the target, and tracked it automatically in range unless the operator desired to shift targets.

2. An electromechanical analog computer received inputs from the optical sight, the Doppler radar, and a wind vane perched on the trailer roof; and converted the data to the correct azimuth lead angle and fire elevation angle for the weapon.

The operation of the Porcupine X-1 is shown in the block diagram of Fig. 1-60.

The system tracked low-flying targets far better than the M33 and M38 systems, and it could be operated efficiently by as few as two men. Its designers believed that a ring of such equipments could be nearly 100-percent effective against low-flying targets. The Porcupine's fate was sealed, however, when the concern of civil defense shifted to missiles, and it never got beyond the prototype stage. Nevertheless, its features are still of interest as a possible basis for new forward-area AA fire control designs.

1-3.3.5 Stinger

The Stinger Antiaircraft Fire Control System was a fully automatic system designed to position four 60 caliber machine guns in elevation and through 360° of azimuth. It was intended for low-flying high-performance aircraft and, in spite of the short range of the guns, the radar had a detection range of 20,000 yd and tracking capabilities of 16,000 to 150 yd. A gyroscopic computer automatically processed radar inputs and positioned the guns through electrohydraulic controls. The entire system was on-carriage.

Stinger was abandoned because of its complexity and the resulting unreliability for field use.

1-3.3.6 T50 Raduster[39, 40]

In the early 1950's, a fully automatic off-carriage radar and computer antiaircraft fire control system, the Rattrap, was developed for towed or self-propelled 40 mm AA weapons, especially the twin 40 mm self-propelled M42 Duster. This system proved to be too cumbersome and complex for use in forward areas. A similar system, but with relaxed requirements, the Mousetrap, was abandoned after a design study, and efforts were concentrated on the T50 AA Fire Control System, called the T50 Raduster (see Fig. 1-61).

The T50 Raduster was an on-carriage system designed for optical tracking and radar range input. It consisted basically of a range radar, computing sight, and range radar, computing sight, and range servo. The system depended on visual detection, acqusition, and directional tracking of a target; estimated range could also be introduced manually if the radar was not functioning properly. The computer generated angular leads on the basis of inputs from the optical equipment and radar, and the weapon was pointed to the target future position while the antenna and sight were pointed to present position.

The system was operated by a two-man crew — a tracker, who operated the optical sight, and a radar monitor.

The radar control unit was located in an armored turret. The radar antenna mount consisted of a cantilever and yoke assembly bolted to a mounting frame which, in turn, was welded to the gunner's shield. Thus the antenna and its associated RF unit moved with the turret and at the same time could move in respect to it in azimuth and elevation, to correspond with elevation and azimuth angles of the gunsight.

A mechanically interconnected drive controller and computer, and a sight mechanically connected to the controller also moved with the turret; the sight also could

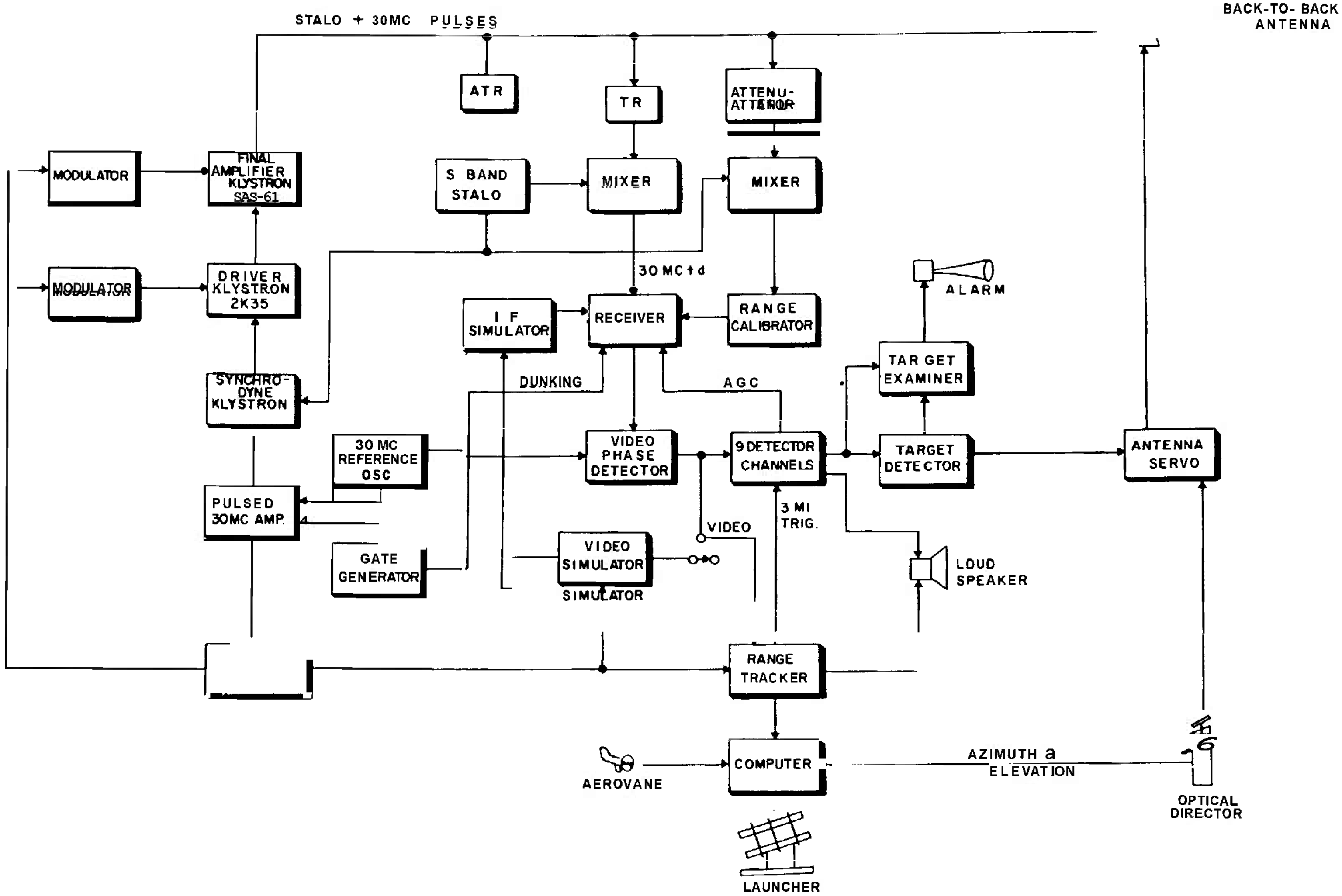


Figure 1-60. Functional block diagram of the Porcupine X- 1 System.

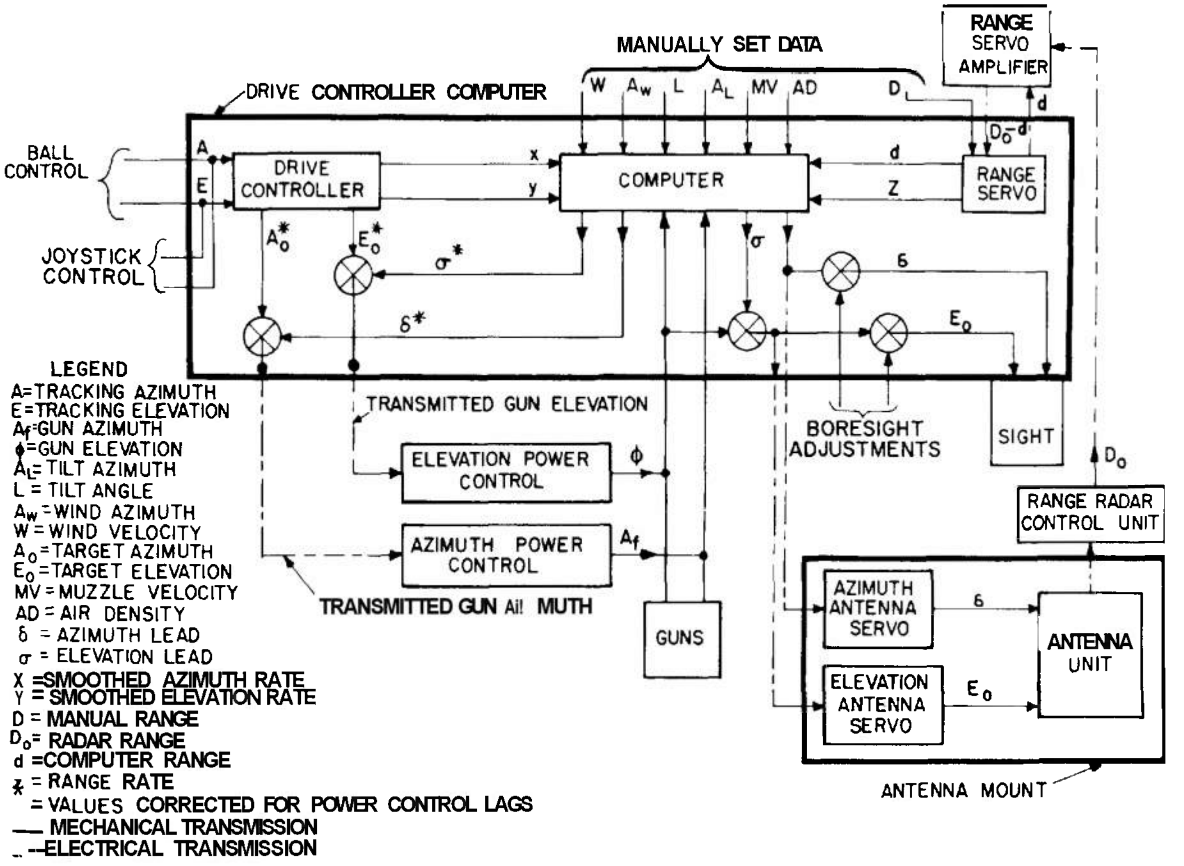


Figure 1-61. Functional block diagram of the Antiaircraft Fire Control System T50.

be moved in respect to the turret in azimuth and elevation to track the target.

The sight actually consisted of three optical units: a primary sight for accurate tracking in azimuth and elevation; a polaroid sight, mechanically linked to the primary sight, for quick target acquisition (the primary sight had too narrow a field of vision for this purpose); and a check sight for boresighting and training.

The primary sight tracked the target in elevation with a prism that could be rotated through 90°; the linkage used to rotate the prism was also geared to the computer controller and served to supply elevation information to the computer. The sight, as a whole, was similarly geared to the controller to supply azimuth information.

The drive controller computer, which included the range servomechanism, was coordinated with the range radar, sight, and guns so as to solve fire control problems in a single mechanical operation. The controller provided controls for acquisition and tracking, supplied angle and angular rate information to the gun power control system to keep the guns positioned, generated smoothed angular rate information in azimuth and elevation for computation of kinematic lead data by the computer, supplied sight-position information to the radar antenna servo to permit radar tracking, and provided a stable platform.

In normal operation – surface-to-air – a joystick type of control permitted rapidly slewing the guns onto fast targets as viewed through the acquisition sight. Then the target was tracked through the primary sight with a ball-type resolver designed for thumb operation. In surface-to-surface operation,

the joystick was inoperative and the ball control was used for tracking slow or stationary targets. In both the acquisition and tracking modes of surface-to-air operation, aided tracking was used; i.e., the guns moved at a rate that was proportional to the displacement of the joystick or the ball control.

The computer was primarily composed of mechanisms designed to accept target position, target rate, ballistic, and meteorological data and generate solutions for empirically developed equations representing solutions to fire control problems.

Meteorological data, ballistic data, and gun cant were handset with knobs. To compensate for tilt, bubbles were centered in an out-of-level indicator; this action entered a correction to position the guns in respect to a level coordinate system instead of the gun mount.

The range servomechanism consisted of (1) a range servo amplifier which received a scaled d-c voltage from the range radar and put out an a-c error voltage to drive a range motor and rate generator; and (2) a computing mechanism which generated range and range rate for the computer.

The range radar was a pulse type that searched and tracked in range only. Its output was a voltage proportional to range, transmitted to the range servo amplifier. The radar, as noted above, was slaved to the optical sight so that the tracker on the crew kept it on target by moving the sight. Normally, the radar locked on the first target encountered and tracked it in range. The monitor could also set a range gate marker so that the radar would automatically start tracking a distant target when it came within maximum range of 15,000 yd. The radar could also be used for manual tracking during periods of jamming or clutter. Range was presented on an "A" scope.

The Raduster presented an unusual challenge: developing a rugged, reliable fire control system that would be compatible with the carriage of an existing weapon. Components had to be developed both on an individual and system basis such that the overall performance of the fire control system, power control, gun system, and ammunition would permit maximum effectiveness against tactical aircraft. The mathematics were especially difficult and time consuming since it was desired to use a simple, small-sized, rugged computer; this restriction made it necessary to use exact prediction formulae based on linear, unaccelerated target motion. New techniques were required that involved mathematical analysis together with critical design computations. An explicit analytical expression was required for relating computer input to output; mathematicians used 57 target flight paths and approximately 2000 positions. Equations relating input and output variables in terms of arbitrary functions were set up using least-square methods of statistical analysis from the target flight path data.

Developing the optimum range finder radar was also a challenge; it involved selecting the most effective systems by "breadboarding" and empirical analysis.

Because of the foregoing difficulties, there were delays in the Raduster program and, since the M42 was only considered an interim weapon, the program was phased out at about the pilot model stage. Nevertheless, the Raduster holds more than usual interest to this day because of the several radically new approaches required and the lessons learned in trying to develop an effective and simple system by careful analysis and application of statistics.

Reference 33 is a complete report on the Raduster development program. Reference 40 is a report on radar test results.

1-3.3.7 Vigilante[41]

The Vigilante, developed in the years 1959 through 1962, was the final effort in the series that began with the use of mechanical computing sights for twin 40 mm AA guns at the end of World War II. Experience in the intervening years had shown that, in the existing state of the various arts involved:

1. Doppler radar was far superior to pulse radar in discriminating moving targets – low-flying aircraft – in ground clutter.
2. The optimum system for detecting and tracking high-performance tactical aircraft should use radar detection, range-only radar tracking, and optical position tracking.

The Vigilante was developed as an on-carriage system with a multibarrel Gatling

gun. Two systems were developed: the towed and the self-propelled. The problem of producing enough power for both self propulsion and turret operation, however, was never entirely overcome. Only one of each was actually produced, and the system was phased out in favor of the Mauler and Redeye.

The Vigilante was designed as a forward area system. It was located in a turret which could be mounted either on the self-propelled or the towed carriage. The turret was capable of 360° rotation. It contained the operator's compartment together with the controls and indicator, radar, computer, periscope, hydraulic power servos, and the main slip-ring assembly. The turret also contained the cannon and the ammunition feed assembly.

From the seat in the operator's compartment, all controls essential to the operation of the radar, computer, sight, and gun were accessible. A single eyepiece presented visual information from the radar and the periscope. Provision was also made for acquiring "targets of opportunity" by means of an open sight.

The radar was a pulse-Doppler system that detected only moving targets. The radar provided a search and range-tracking capability for the operator. A track-while-scan feature allowed automatic azimuth-tracking of a target while the antenna was scanning through 360°.

The fire-control computer and sight provided the required automatic range and angular tracking capability. The azimuth hydraulic power servo drove the turret and gun to the predicted azimuth of the moving target. The elevation power servo positioned the gun to the predicted target elevation. The computer provided primary and secondary ballistic inputs for positioning the power-control servos to the predicted target position.

Operation. The search mode of the Vigilante system was primarily a radar mode of operation. The operator viewed the radar indicator for moving targets.

The operator could be alerted to a detected target by three means: an indication on the radar indicator, an externally mounted horn, and the Doppler tone in his headset. Upon sighting the target, the operator rotated the turret to the target azimuth and proceeded to the acquisition mode.

During the acquisition mode, the turret automatically tracked the target in azimuth while the antenna wa's scanning through 360° for other targets. The operator visually located the target in elevation by means of the periscope and proceeded to the track mode by depressing a foot switch; the radar range gate then slewed and locked on the target range. Both the computer range rate and range settled to the radar range information. Simultaneously, the computer generated information to aid the operator in tracking the target and to generate the proper fire-control solution.

An operation spotting sight was included to aid the operator in rapidly acquiring close targets or "targets of opportunity". To track these targets, the operator switched directly to the track mode instead of utilizing the acquisition mode.

Controls. The system controls were designed for ease of operation and for a sequence of operation from left to right. The operator hand-control motions were designed to make target tracking an instinctive process utilizing the operator's previous training.

Field tests demonstrated that the controls of both Vigilante systems were easy to operate. It was demonstrated that, with relatively little practice, an inexperienced operator was able to track a moving target with acceptable accuracy. Each of five operators who operated the system during the firing tests was able to hit a drone target at least once.

Regenerative tracking. Early system studies revealed that gunsmoke and gunflash, together with possible transmission of gun shock to the operator's hand controls, would result in excessive tracking errors during the firing period. To minimize these errors, the computer was mechanized to provide regenerative tracking during the firing interval, eliminating the need for the operator to supply tracking inputs during this interval.

Regenerative tracking is defined as the capability of the computer, after settling, to continuously provide the changing inputs, normally provided by the operator, to maintain the sight on a nonmaneuvering, constant-velocity target. Although manual effort was required to settle the sight initially on the target, the operator then returned his hand controls to zero as the computer supplied the

tracking inputs from the regenerative circuitry. Small hand-control motions were required because of target maneuvering and because of imperfect computer regenerative solutions.

The computer was essentially composed of a hand control unit, a periscope optical system, six instrument servos, a gyroscopically controlled platform, and a dial-panel ballistics unit.

The hand control unit provided initial error-excitation to the platform, the angular-rate instrument servos, and the periscope optics. The operator kept the periscope optics on the target with the hand controls.

The two-degree-of-freedom gyro within the platform forced the platform to follow the gyro line, and the platform forced the periscope optics to follow the platform line. Thus, the sight line, the gyro line, and the platform line pointed to the target after the computer was settled. The platform iine became the present-position computing line upon which was built the fire-control prediction solution for the positioning of the gun. The gyro provided angular stability of the platform, independent of mount motion, and provided part of the required integration for the regenerative tracking solution.

As the platform line settled to the present-position target line, the range and range-rate instrument servos settled to the radar functions, and the two angular-rate instrument servos settled concurrently with the platform angular rates. These four servos served the dual purpose of producing the regenerative tracking functions and of providing the necessary present-position parameters for the prediction solution. The time of flight and quadrant elevation servos generated the future-position parameters for the remaining part of the prediction solution. Ballistic corrections to the prediction solution were manually made on the dial-panel ballistics unit.

For more information on Vigilante, see par 4-6, Chapter 4.

1-3.4 ARTILLERY FIRE CONTROL SYSTEMS

Since the days when artillery fire began to exceed the range of the gunner's eye, a more positive means of delivering an effective "first round" has been the artilleryman's greatest desire. In fulfilling this desire, the following five major elements have emerged as the basic requirements for accurately predicted fire:

1. Battery location
2. Target location
3. Meteorological data
4. Muzzle-velocity data
5. Computation of projectile flight incorporating nonstandard conditions.

The first four of these elements have been reduced to a fairly fine degree. However, because they are either relatively static or true static conditions and the techniques necessary to deal with them are used in many other applications, these elements have had the benefit of much thought and effort. Battery and target locations represent true static conditions at the time of firing. Meteorological data, though not static, has a sufficiently slow rate of change that modern analysis methods reduce such changes to a relatively static condition. Changes in muzzle velocity occur slowly enough to be considered a nearly static condition. The remaining element, the computation of projectile flight, involves elements such as muzzle velocity, propellant temperature, projectile weight, ballistic coefficient, drift, and rotation of the earth.

A modern digital computer, when appropriately programmed, can provide the information necessary to give an accurate prediction of projectile flight in a matter of seconds. In the past, a computer with this capability required highly skilled operators and the equipment was not suited to the demands of the military environment.

The computer require; to meet the demands of the military must incorporate the following features:

1. Reliability
2. Simplicity of operation
3. Portability
4. Rugged construction
5. Ease of maintenance

Since all these features are not to be found in readily available digital machines employed by business and industry, the United States Army initiated an intensified research and development program to produce a digital computer compatible with these requirements. This program ultimately produced

the M18 Gun Direction Computer, commonly referred to as FADAC (Field Artillery Digital Automatic Computer). FADAC, along with ancillary equipment which greatly expands its capabilities, has been thoroughly tested and type classified standard A. A summary of FADAC's background and its capabilities follows.

Computers were used by Artillery to advantage during World War II. However, only restricted calculations could be completed by such types because the program had to be manufactured into the analog computer and, consequently, did not have the versatility of the later digital type.

The desire for computer-generated firing data increased substantially when the earliest practicable digital computer came into being. Initially, digital computers were large, bulky, delicate in many respects, and difficult to maintain. The development of the transistor made a smaller and more-rugged computer possible, with capabilities increased beyond the earlier vacuum-tube types.

What the U. S. Army desired was a computer that could withstand the rigors of field use and readily accept a wide range of data for all types of commands; yet be small, light, and, above all, simple to operate. In successfully combining all of these attributes into FADAC, the Army has produced a computer which has far wider applications than was even remotely anticipated.

FADAC is a portable all-transistorized general-purpose computer specifically designed to withstand the rigors of rough transportation and varying climatic conditions. The efficiency of the computer and ancillary equipment remains unimpaired when operated in severe rain, salt-laden air or dust storms. FADAC is of modular construction.

FADAC presents the ultimate in simplicity of computer operation. Switches, controls, keyboards, and all displays are directly in front of the operator, thereby requiring a minimum of movement on his part. The basic functions, which represent the lowest level of artillery fire control automation, are to perform computations that had formerly been done manually and to store other information formerly in manual form. Inputs to FADAC are received by manual methods, voice, or written message and are entered into the computer by the operator.

Much of the effectiveness of cannon artillery fire is negated by the fact that adjustment of fire or registration to obtain predicted fire data normally precedes fire for effect. This, of course, informs the enemy as to the target area and permits him to prepare countermeasures. Given the proper information, FADAC fires theoretical registration rounds within itself until the target is "hit", at which time it will display the required information so that the real round will obtain first-round hits on the target, providing target location is known. This process requires only a few seconds and is extremely accurate. It is readily apparent that two things are achieved: (1) registration firing is unnecessary, saving the cost of ammunition; and (2) fire for effect is initiated with the first round fired, thereby denying the enemy the time and opportunity for countermeasures.

FADAC has many uses beyond its artillery-firing computational ability. Some of those outlined below have been tested and found highly satisfactory. Others that are suggested represent areas that should not present any particular difficulty of utilization in that many digital-computer applications have been made with units closely approximating the capability of FADAC.

In the military area, some possibilities are:

1. Weapon Effects Analysis. This program can compute the effects of different projectile-fuze combinations from the same weapon. Or it can be used to compare the effects of two different calibers when fired on the same target. Or it can analyze and determine the best combination of weapons or the best combinations of ammunition to neutralize a target.

2. Counter Battery Computation. With appropriate input, FADAC can determine the location of hostile artillery units.

3. Sound and Flash Ranging. Plotting hostile battery positions from sound or flash can be done more rapidly and more accurately by FADAC than by manual methods. The application of computer meteorological data plus rapid calculating ability make FADAC a most worthwhile tool for this purpose.

4. Mapping: Long-Range Survey. By integrating FADAC with radio-frequency devices, accurate surveys can be made in

ranges far beyond visual capabilities.

5. Mapping: Analytical Triangulation in Photogrammetry. The exacting requirements of determining real positions or aerial photographs can be rapidly determined by FADAC when properly programmed.

6. Meteorological Data Reduction. The integration of FADAC with weather sounding devices, such as the Meteorological Data Sounding System, can produce significant weather data almost as fast as it is received.

7. Satellite Tracking. The integration of FADAC with satellite tracking devices can produce position, speed, and path data and can convert messages to or from satellites into proper codes.

8. Field Automatic Checkout Systems. The Multipurpose Automatic Inspection and Diagnostic System (MAIDS) is an integrated concept for a family of automatic diagnostic equipments to be used for malfunction isolation in various types of Army materiel. The concept is based upon the utilization of a standard militarized digital computer (FADAC) as the central control element of the various subsets of the MAIDS. The computer-controller will be programrned to locate and diagnose faults down to the replaceable module or component.

Other potential military uses could be:

(a) Processing target imagery
(b) Tactical data systems
(c) Inventory control
(d) Surveillance and reconnaissance
(e) Monitoring operations.

For information on the design principles of FADAC, see Chapter 13 in Section 3 of the Fire Control Series (Fire Control Computing Systems).

1-3.5 ENVIRONMENTAL PROTECTION[14, 15]

1-3.5.1 Antiglare Filters and Protective Lens Coatings

Considerable fire-control research was conducted in antiglare filters and protected-lens coatings during World War 11. Tests made by the Desert Warfare Board in 1942 indicated some advantages to the use of red, amber, and neutral filters for sighting equipment, but reported that none justified adoption. Better results were obtained with a nonreflecting coating on glass surfaces and by the substitution of solid-glass prisms in telescopes for mirrors. Further work expanded into the development of antirain and antifog coatings, hoods for protection against sun and rain, as well as mechanical modifications to sighting equipments in order to simplify and facilitate operation.

1-3.5.2 Unusual Environments

The effects of extreme cold on the performance of all types of fire control equipment was investigated by the Army at Fort Churchill in Canada during the winter of 1943-44 and yielded valuable design and maintenance-engineering information. The use of fire control instruments in tropical theatres of warfare, on the other hand, soon revealed the ravaging effects of fungus growth and other types of destructible deterioration. In June 1944, a committee was formed at Frankford Arsenal to study the protection of fire control instruments with its efforts directed toward the use of protective coatings, the development of moisture-proof sealing, the incorporation of silica-gel desiccants, and the employment of a volatile fungicide with the instruments.

The new environmental problem for fire control equipment is the survival of operational capability in the vicinity of a low-yield nuclear blast. Part of the solution to this problem may be accomplished by (1) viewing targets through unusually thick turret walls in order to protect the operators and most of the fire control system components and (2) minimizing the effects of radiation on those elements of the viewing system that must be exposed. Sighting devices based on the use of fiber optics may be applicable here. The advantage of fiber optics over the usual type of optical system is their relative simplicity, Figure 1-62 indicates the simplicity of a telescopic system based on fiber optics as compared with some of the more conventional telescopes that have been designed in the United States. Further information on optical systems of this type will be given in Section 2 of the Fire Control Series (Acquisition and Tracking Systems).

Apart from the comparatively long-range problem of radiation, there is the almost in-

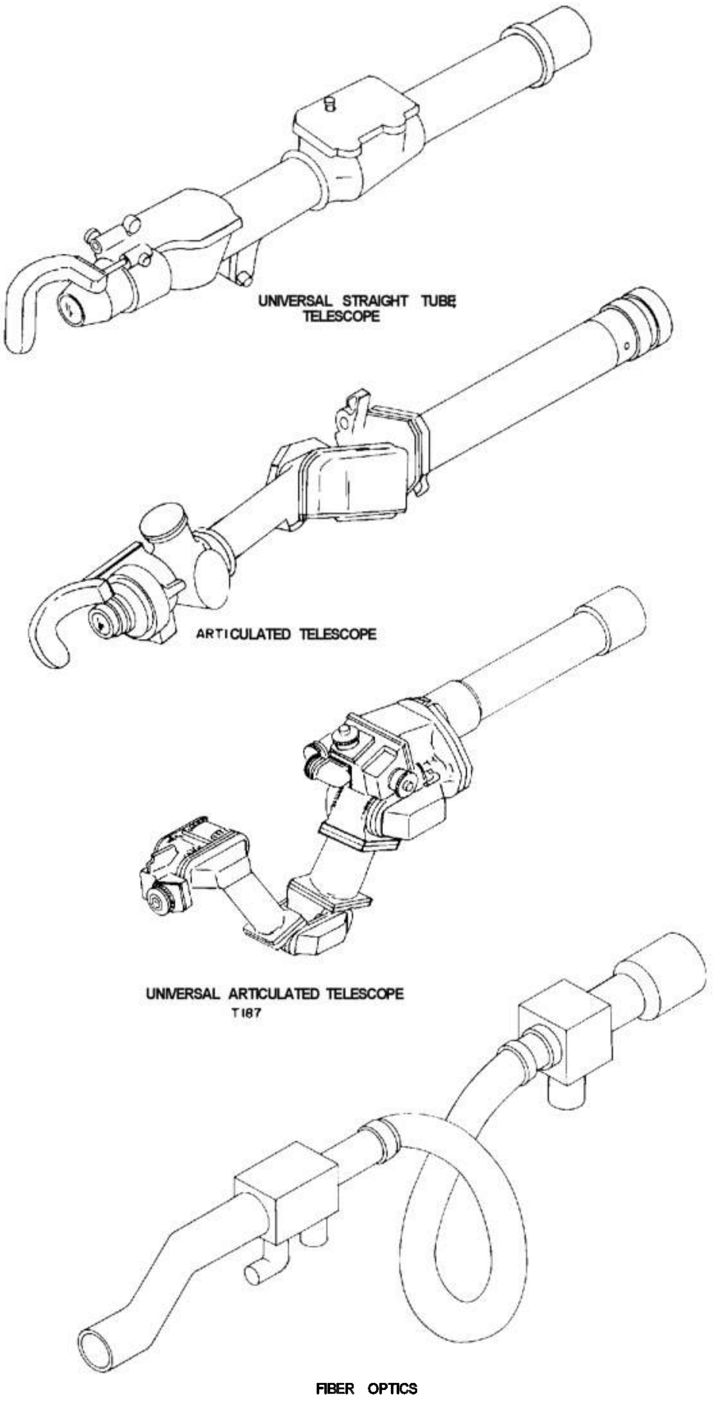


Figure 1-62. The comparative simplicity of a telescope using fiber optics.

stantaneous problem of affording flash protection against atomic blasts to operators of sighting devices. As indicated in Example 1-1 in par 1-3.2, in connection with fire control equipment for armored vehicles, a variety of quick-acting shutters are under development that would be activated by the flash of an atomic blast. The object is to obtain such a rapid reaction that the shutter will be closed before the operator's eyes are burned.

1-3.6 CONCLUSIONS

The following conclusions can be drawn from the history of fire control development, particularly from the crash program of World War II and the sustained program of the following years:

1. Fire control development, like most technical development for weapons programs, occurs most rapidly during times of greatest need. Even *so*, it is important to note that the impact of a research program upon society is always delayed. One authority (Fry of OSRD; see p. 81 of Reference 25) estimated in 1942 that in normal peacetime this delay is "pretty long, not often less than five years and sometimes, as in the case of the automobile, a good quarter of a century. In war the process is speeded up, but certainly cannot average much under two years." Much of the research undertaken during World War II did not actually see application until long after the war was over. (This was particularly true of tank fire control systems.) The average lead time required for the Army to develop new weapon systems and bring them to operational status has been placed at about ten years, though it is hoped that the recent reorganizations will result in a significant reduction in this lead time.

2. Research and development during the stress of wartime must be done under the principle of getting something workable within the time available. Getting the best result is usually impracticable. Normal development and test procedures often have to be dispensed with. Such procedures were sometimes very effective, as in the development of the proximity fuze, but this pragmatic approach obviously represents a gamble; not all developments can be expected to turn out as fortunately as the proximity fuze did. With the type of weapon threat present in the world today, it is doubtful that even the limited time that was available for research and development during World War II will be available during any future conflict. Hence, it becomes imperative to make the most of the time and funds (however, limited) that are available during times of peace or cold war.

3. Fire control analysis, to be effective, must extend beyond the design of weapons to include studies of their optimum use.

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CHAPTER 2

THEORETICAL ASPECTS OF THE FIRE CONTROL PROBLEM AND ITS SOLUTION

2-1 INTRODUCTION

The problem of control over weapon fire and its solution as discussed in the Fire Control Series does not apply to those projectiles known as "guided missiles". Instead, the term projectile is used here in a more limited sense, which includes bullets, shells, and rockets. For information on fire control as it applies to guided missiles, the reader is referred to the Ballistic Missile Series of the Engineering Design Handbooks, AMCP 706-281, -282, -283, -284(C) and -286.

The present chapter comprises two main parts. The first part, pars 2-2 through 2-2.7, is concerned with the fire control problem. It states the problem and then follows this statement with a summary of generalized fire control theory.

The second part, pars 2-3 through 2-3.4, discusses the solution of the fire control problem in general terms in a manner that parallels the treatment given in the first section. The solution of the fire control problem is broken down into three distinct phases and each phase is then treated separately in turn.

It should be noted that a broad discussion of the functional elements employed in the solution of the fire control problem and examples of how such elements are utilized in actual systems is reversed for Chapter 3. Detailed discussions of the functional elements employed in the solution of the fire control problem are reserved for subsequent sections of the Fire Control Series as follows:

(a) Section 2 – Acquisition and Tracking Systems

(b) Section 3 – Fire Control Computing Systems

(c) Section 4 – Weapon-Pointing Systems

2-2 THE FIRE CONTROL PROBLEM

2-2.1 STATEMENT OF THE FIRE CONTROL PROBLEM

The general fire control problem may be stated as follows: "How can a projectile be fired from a weapon (that may be in motion) at a target (that may also be in motion) in such a way as to score a hit on the target?" Implicit in this problem statement is the fact that the effects of certain phenomena, unless they are compensated, will produce errors in weapon fire. These phenomena, which are common to all types of weapon fire, are reducible to corrective terms when defined by a suitable analytical or geometrical approach.

It is obvious that the path of the projectile must be made to intersect the path of the target *so* that a hit is obtained. (If the target is stationary, the target path reduces to a point, of course, and the fire control problem is considerably simplified.) Inasmuch as there is usually a finite period of time during which the required intersection of paths can be obtained, there is no single solution, but rather a different solution for each moment in real time. Thus, an implicit part of many fire control problems is the determination of when fire can profitably be opened and when it should be stopped, and how to make the most of the opportunity for fire.

2-2.2 GENERALIZED FIRE CONTROL THEORY[1-12]

2-2.2.1 Basic Concepts

Analysis of the over-all problem of controlled weapon-fire brings out an important concept: <u>There is basically only one fire control problem</u>. All fire control. problems re-

solve into variations of a single fundamental situation – the launching of a projectile from a weapon at a target in such a manner as to score a hit on the target.

In order to solve the fire control problem, the element of probability must be taken into account. For example, based upon observations of present target motion, the future target position at the time of hit must be predicted if the effects associated with the phenomenon of relative target motion are to be adequately compensated. In addition, the concept of prediction is associated with the in-flight characteristics of a projectile during its time of flight. During the projectile time of flight, the projectile is entirely under the influence of natural phenomena – e.g., gravity drop, drift, and precession – that lie beyond the control of operating personnel; from the time the projectile is fired, its trajectory is irrevocably dependent on gravity, wind and the ballistics* of the projectile. Thus, because the exact nature of these quantities and their interplay can not be exactly predicted, the element of probability must be taken into account in this connection also,

In order to compensate for the effects of the various phenomena that enter into the fire control problem, the use of certain corrective measures is necessary. The determination of the required corrective measures by fire control equipment is made possible by the application of suitable analytical approaches. These approaches, which are primarily algebraic in nature, may be expressed in terms of various types of models and are dealt with in Chapter 4 (Design Philosophy).

2-2.2.2 The Geometrical Approach

For understanding the true nature of the fire control problem, however, it has been found more effective to treat generalized fire control theory in geometrical terms rather than in algebraic terms. This is because the basic fire control problem is a kinematic and dynamic problem, i.e., one involving the relative motion between points in space (weapon, projectile, and target) and the forces acting on the projectile. It, therefore, lends itself to expression in terms of the pertinent kinematics (velocities) and dynamics (forces), rather than to a purely numerical treatment. The algebraic approach must come into application, of course, in the actual solution of any particular fire control problem.

The geometry involved in the geometrical approach is not a matter of triangulation, but rather one of vectors that are related by the laws of physics. Vector diagrams and vector operations may be used extensively, therefore, to relate the physical parameters of the fire control problem. For a complete unified treatment of the basic physics and geometry applicable to any fire control problem, see Reference 1.

2-2.2.3 Common Geometrical Factors

Three quantities that remain constant regardless of the reference coordinate frame† selected for expression of the fire control problem are the following (see Fig. 2-1):

1. The <u>line of site</u> which is the straight line between the weapon and the target. It should be noted that the line of site does not necessarily represent a line of visibility between weapon and target. When such visibility is present, direct fire control applies; otherwise, a requirement for indirect fire control exists (see pars 1-1.3.1 and 1-1.3.2).

2. The <u>weapon line</u> which is the prolongation of the weapon axis. It is a straight line along the direction in which the projectile should be fired in order to score hits on the target.

3. The <u>prediction angle</u> which is the total offset angle between the line of site and the weapon line. As used in the Fire Control Series, the term "prediction angle" is a general designation that for moving targets corresponds to "lead angle" plus any supplementary corrections for gravity drop, drift and the like. As mechanized in the solution of the fire control problem, the prediction angle is equal to the combination of the angle of elevation and the angle of deflection (see par 1-1 of Chapter 1). For stationary targets, where

*<u>Ballistics</u> is the science that is concerned with the motion of projectiles. That part of ballistic theory that is concerned with the motion of the projectile after it leaves the muzzle of the gun is termed <u>exterior ballistics.</u> That part concerned with the motion of the projectile while it is still in the bore of the gun is called <u>interior ballistics</u>. The theory of fire control is primarily concerned with exterior ballistics.

† For a discussion of reference coordinate frames, their applications, and significance, see par 2-2.6.

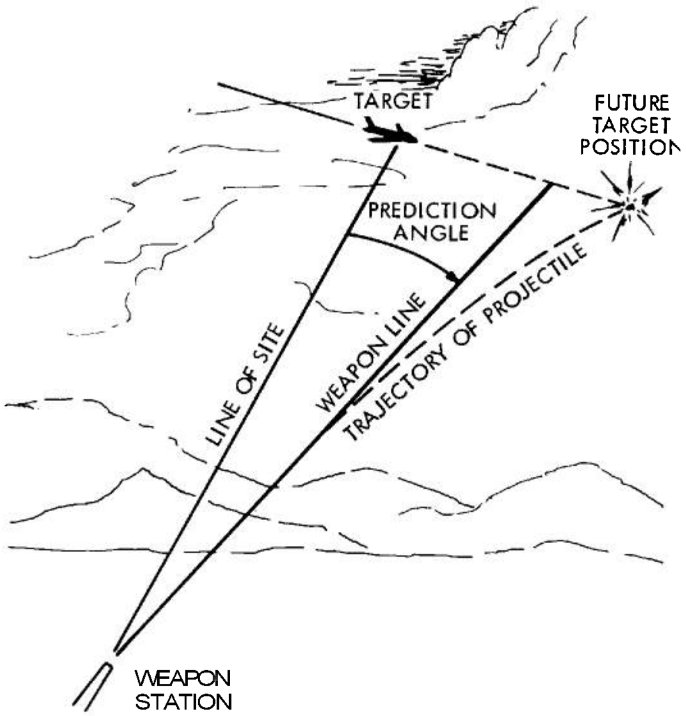


Figure 2-1. The fundamental geometry of a typical fire control problem.

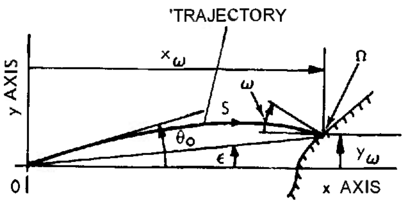
no lead angle is involved and where reference is usually to geographical coordinates rather than the line of site, the prediction angle is mechanized by the combination of the angle of elevation and the angle of azimuth.

It should be noted that the terminology used for these and other aspects of fire control differs in the various branches of Army fire control and in the various publications on fire control. For example, the line of site is sometimes referred to as a "line of position" in coast artillery literature. In the Air Force and Navy, on the other hand, the line of site is referred to as the "line of sight".

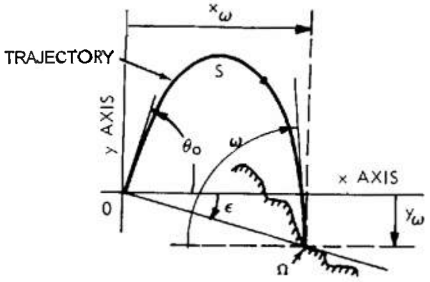
2-2.3 FACTORS AFFECTING THE PROJECTILE PATH

A weapon-launched projectile cannot be fired directly along a line of site to score a hit on a fixed target because of certain phenomena that are applicable to all problems of fire control. These phenomena consist of (1) the curvature of trajectory of the projectile, (2) the effects of jump, and (3) variations from standard conditions. The influence of these factors on the projectile would cause it to miss the target if it were fired along the line of site from the weapon to the fixed target. It is, therefore, necessary to apply corrective measures (to compensate for these factors) to obtain the direction along which the weapon should be fired, i.e., the correct orientation of the weapon line. Accordingly, except for certain small arms fire for which the effective ranges are *so* short that the phenomena noted are of no consequence, the line of site and the weapon line do not coincide at the time of firing. Ideally, the application of dimensional corrections will result in a weapon-line orientation that compensates for the factors that affect the projectile path and the projectile will strike the target.

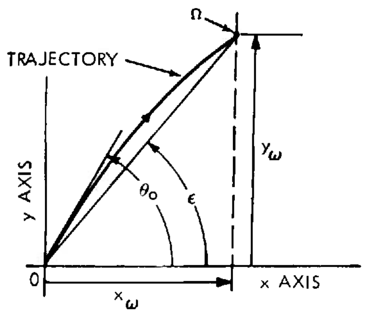
Figure 2-2 shows the projectile trajectories that are required for various typical weapon-fire situations in order for the projectiles concerned to hit their respective targets. In each case, the trajectory is determined by (1) the position of the origin of fire, i.e., the location of the weapon, (2) the conditions under which the projectile is projected from the weapon, i.e., the quadrant angle of elevation θ_0 and the muzzle velocity, (3) the ballistic characteristics of the air through



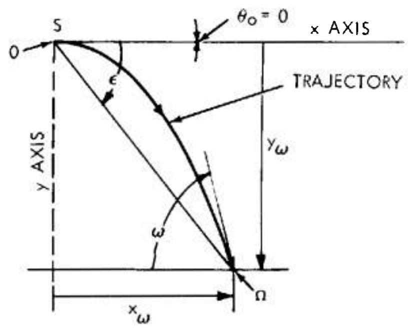
(A) Trajectory associated with a direct-fire field-artillery situation



(B) Trajectory associated with an indirect-fire field-artillery situation



(C) Trajectory associated with antiaircraft fire



(D) Trajectory associated with air-to-groun'd weapon fire by bombs or rockets

0 = ORIGIN OF TRAJECTORY

Ω = POINT OF IMPACT, OR POINT OF BURST IN ANTIAIRCRAFT FIRE

S = SUMMIT OF TRAJECTORY

$0S$ = ASCENDING BRANCH OF TRAJECTORY

$S\Omega$ = DESCENDING BRANCH OF TRAJECTORY

θ_o = QUADRANT ANGLE OF DEPARTURE

ω = ANGLE OF FALL

ϵ = ANGLE OF SITE

β = RANGE ANGLE

x_ω = HORIZONTAL RANGE

0Ω = SLANT RANGE

Y_ω = ALTITUDE OF BURST OR IMPACT WITH RESPECT TO WEAPON

(E) Definitions

Figure 2-2. Typical trajectories projected onto the plane of departure.

which the projectile must pass in order to reach the target.

Figure 2-2(A) illustrates the trajectory of a projectile fired from a field-artillery weapon having a high initial velocity and a small quadrant angle of departure. Figure 2-2(B), on the other hand, represents the trajectory of a projectile fired from a field-artillery weapon having a much lower initial velocity and a large quadrant angle of departure. These two examples, of course, represent direct-fire and indirect-fire situations, respectively.

Figure 2-2(C) shows the type of trajectory associated with antiaircraft fire. In antiaircraft fire, the whole trajectory is generally considered to comprise the ascending branch, inasmuch as the descending branch has no significance. This is in contrast to the trajectories associated with bombs and rock-

ets released in air-to-ground weapon fire, which trajectories are represented by Fig. 2-2(D). For such trajectories, only the descending branch is used; i.e., there is no ascending branch,

All four representative trajectories are shown projected on the plane of departure of the trajectories. This plane is the x, y-plane of the coordinate system customarily used in the computation of trajectories. In this system, the x-axis is horizontal, and the y-axis is vertical. The z-axis lies in a horizontal plane and is perpendicular to the plane of departure. Forming a part of Fig. 2-2 is a set of definitions (see Fig. 2-2(E)) that applies to the various situations depicted.

Inasmuch as the usual trajectory is three-dimensional in nature, it does not lie entirely in the x, y-plane but also has a projection onto the x,z-plane. This projection is represented in Fig. 2-3; it should be noted, however, that the projectile deflection shown in the x,z-plane is exaggerated for the purpose of illustration. As indicated, the z coordinate of the point of impact or burst is designated by the symbol z_ω and is called the deflection. The component of deflection that is not due to the effect of wind on the trajectory is called drift.

The paragraphs below treat, in turn, the influence of trajectory curvature, jump effects, and variations from standard conditions on the overall trajectory of a projectile.

2-2.3.1 Curvature of the Trajectory

The curvature of the trajectory of a projectile in motion is caused by many forces that act on the projectile during its time of flight. The principal effects that influence the shape of the trajectory are the gravitational field of the earth and the characteristics of the air through which the projectile passes (see pars 2-2.3.1.1 and 2-2.3.2.2). Other effects contributing to the form of the projectile path include drift, wind, and meteorological conditions. These effects will be considered in connection with paragraph 2-2.3.3, Variations from Standard Conditions.

The force of gravity and air resistance are both generally considered in respect to an air structure referred to as the standard atmosphere. This standard structure provides a mathematical point of departure from which essential ballistic data can be obtained by applying corrections to such variations as may exist in the actual air structure at a particular time.

2-2.3.1.1 Gravity

Gravity is a primary factor that influences the path of a projectile in motion. If a projectile were fired in a vacuum and in the absence of a gravity field, it would maintain a constant direction and continue to rise indefinitely at a constant vertical velocity dependent only on the muzzle velocity and the angle of departure from the weapon (see Fig. 2-4). The kinetic energy imparting this motion would produce both vertical and horizontal components of velocity, the combined effect of which would form a resultant velocity along the straight-line path of projectile motion.

With gravity effects only considered, the flight path changes as follows (see Fig. 2-5). Because no air resistance would be encountered in the vacuum, the horizontal component of velocity would remain constant. On the other hand, because the projectile would be acted upon by the force of gravity during the time of flight, the vertical component of velocity would diminish at the rate of about 32 feet per second each second, This component, then, would first reduce to zero, at which

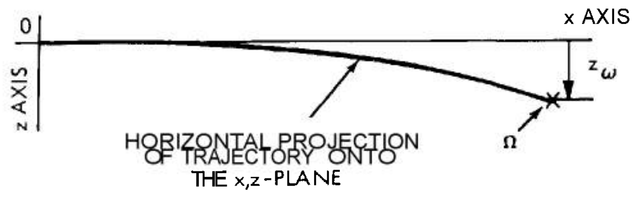
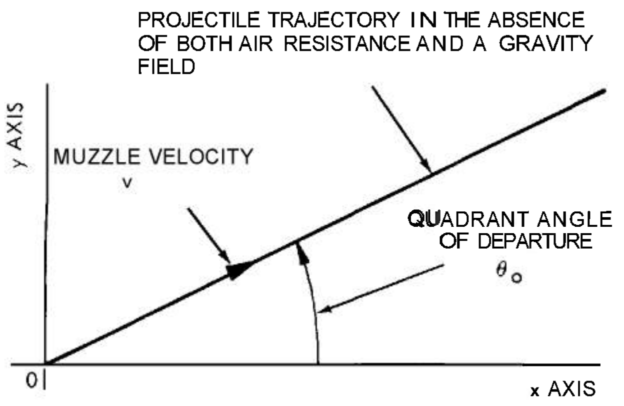


Figure 2-3. The horizontal projection of a typical trajectory.



THE EQUATIONS OF MOTION FOR A PROJECTILE AS IT MOVES ALONG ITS STRAIGHT-LINE TRAJECTORY ARE AS FOLLOWS:

$$x = (v_o \cos \theta_o)\, t \qquad\qquad y = (v_o \sin \theta_o)\, t$$

WHERE

x = HORIZONTAL DISTANCE FROM THE ORIGIN
y = VERTICAL DISTANCE FROM THE ORIGIN
t = TIME FROM THE INSTANT THE PROJECTILE LEAVES THE GUN MUZZLE
v_o = MUZZLE VELOCITY
θ_o = QUADRANT ANGLE OF DEPARTURE

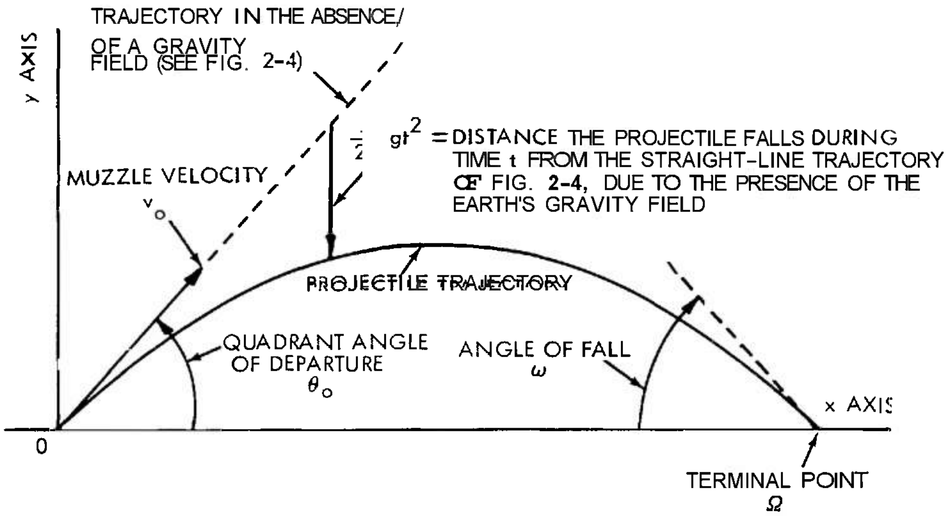
Figure 2-4. The trajectory of a projectile fired in a vacuum and in the absence of a gravity field.

time the projectile could no longer rise with respect to the earth's surface; thereafter, gravity would further cause the projectile to fall back toward the earth*. The form of the path generated by the projectile under such theoretical conditions would be a perfect parabola with the angle of fall equal to the angle of elevation and with the summit midway between the origin and terminal points. Figure 2-5 summarizes the pertinent characteristics and mathematical relationships that pertain to a parabolic trajectory.

2-2.3.1.2 Air Resistance

The standard trajectory described by a projectile under atmospheric conditions assumed to be standard becomes a more complex curve than it would be in a vacuum. The air resistance acting along the axis of the projectile produces a downward component that adds to the effect of gravity on the vertical component of projectile velocity during the ascending portion of the trajectory and subtracts from the effect of gravity during the descending portion. The air resistance also acts to decrease the horizontal component of projectile velocity over the entire trajectory. The net result is that the angle of fall becomes greater than the angle of elevation, the summit is displaced closer to the point of impact than to the origin, and the range of the projectile is greatly reduced. This is shown by Figure 2-6(A), which is a projection of a typical standard trajectory on the plane of departure.

* With respect to the initial line of departure of the projectile along the weapon line, "falling" actually starts, of course, just as soon as the projectile is free of the weapon; *i.e.*, there is a constant acceleration acting towards the earth's center throughout the flight of the projectile.



THE EQUATIONS OF MOTION FOR A PROJECTILE AS IT MOVES ALONG ITS TRAJECTORY IN THE ABSENCE OF AIR RESISTANCE BUT IN THE PRESENCE OF A GRAVITY FIELD ARE AS FOLLOWS:

$$x = (v_o \cos \theta_o)\, t \qquad y = (v_o \sin \theta_o)\, t - (1/2)\, g t^2$$

WHERE g IS THE ACCELERATION DUE TO THE EARTH'S GRAVITY FIELD AND THE OTHER QUANTITIES ARE AS DEFINED IN THE FIGURE. THE ELIMINATION OF TIME FROM THESE EQUATIONS OF MOTION GIVES THE FOLLOWING PARABOLIC EQUATION FOR THE TRAJECTORY IN TERMS OF THE y COORDINATE AS A FUNCTION OF THE x COORDINATE:

$$y = (\tan \theta_o)\, x - \frac{g}{2 v_o^2 \cos^2 \theta_o}\, x^2$$

FOR SUCH A TRAJECTORY, THE FOLLOWING CHARACTERISTICS PERTAIN:

(1) THE SHAPE IS THAT OF A PARABOLA WITH A VERTICAL AXIS.

(2) THE MAXIMUM ORDINATE IS LOCATED HALFWAY BETWEEN THE WEAPON AND THE TERMINAL POINT Ω

(3) THE ANGLE OF FALL ω IS EQUAL TO THE QUADRANT ANGLE OF DEPARTURE θ_o

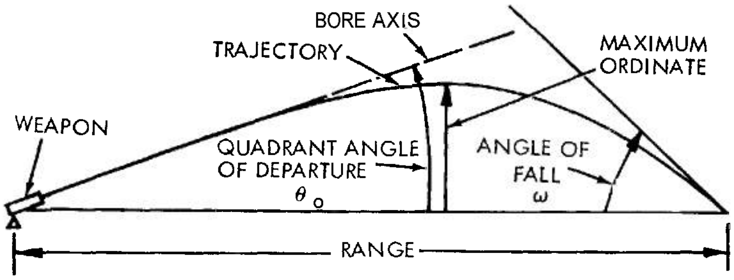
(4) THE STRIKING VELOCITY AT THE TERMINAL POINT IS EQUAL TO THE MUZZLE VELOCITY.

(5) THE MAXIMUM RANGE IS OBTAINED WITH A QUADRANT ANGLE OF DEPARTURE OF 45°.

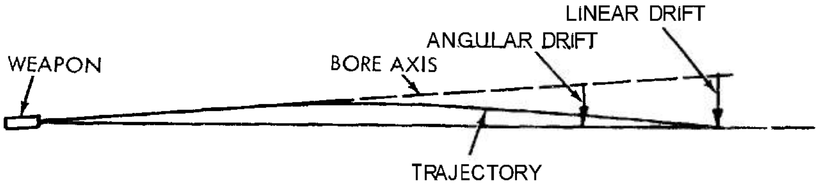
Figure 2-5. The trajectory of a projectile fired in a vacuum but with gravity effects considered.

Resistance of the air to the forward motion (range motion) of a projectile greatly influences not only the shape of the trajectory in elevation but also adversely affects the azimuth direction of the projectile. This is shown by Figure 2-6(B), which is a projection of a typical standard trajectory on the horizontal plane. For reference purposes, the characteristics of a standard trajectory are summarized in Figure 2-6(C).

The following factors must be taken into account in ascertaining the difference in the



(A) The projection of a typical standard trajectory onto the plane of departure



(B) The projection of a typical standard trajectory onto the horizontal plane

WHEN A PROJECTILE IS FIRED IN AIR AND IS UNDER THE INFLUENCE OF GRAVITY, THE THEORETICAL TRAJECTORY IN A VACUUM (SEE FIG. 2-5) IS MODIFIED BY AIR RESISTANCE, WITH THE FOLLOWING RESULTS:

(1) THE TRAJECTORY IS NOT A TRUE PARABOLA IN THAT THE RANGE TO THE SUMMIT OF THE TRAJECTORY IS MORE THAN HALF THE TOTAL RANGE.

(2) THE ANGLE OF FALL IS GREATER THAN THE ANGLE OF ELEVATION.

(3) THE STRIKING VELOCITY IS LESS THAN THE INITIAL VELOCITY.

(4) THE MAXIMUM RANGE OF THE PROJECTILE IS OBTAINED CLOSE TO, BUT NOT PRECISELY AT, A QUADRANT ANGLE OF DEPARTURE OF 45 DEGREES. FOR SMALL BALLISTIC COEFFICIENTS, THE ANGLE IS LESS THAN 45°; FOR LARGE BALLISTIC COEFFICIENTS, THE ANGLE IS GREATER THAN 45°.

(C) Characteristics of a standard trajectory

Figure 2-6. The trajectory of a projectile fired under standard atmospheric conditions (both gravity and air resistance present).

trajectory characteristics of a projectile fired in air from the characteristics of one fired in a vacuum:

1. The density of the atmosphere. The air offers resistance to the projectile that substantially alters the characteristics of the trajectory. Since the density of the atmosphere differs from time to time in accordance with changes in temperature and barometric pressure, and also in accordance with altitude, air resistance as well varies not only with time but also with altitude as the projectile travels the course of its trajectory.

2. The characteristics of the projectile. The specific characteristics of a projectile that influence its retardation in passing through air of a given density are (1) its weight, (2) its cross sectional area which is, of course, proportional to the square of the projectile's diameter, and (3) its shape. A

projectile that has a streamlined front end encounters less resistance than one having a short blunt nose. The shape of the base also affects the air resistance encountered by the projectile.

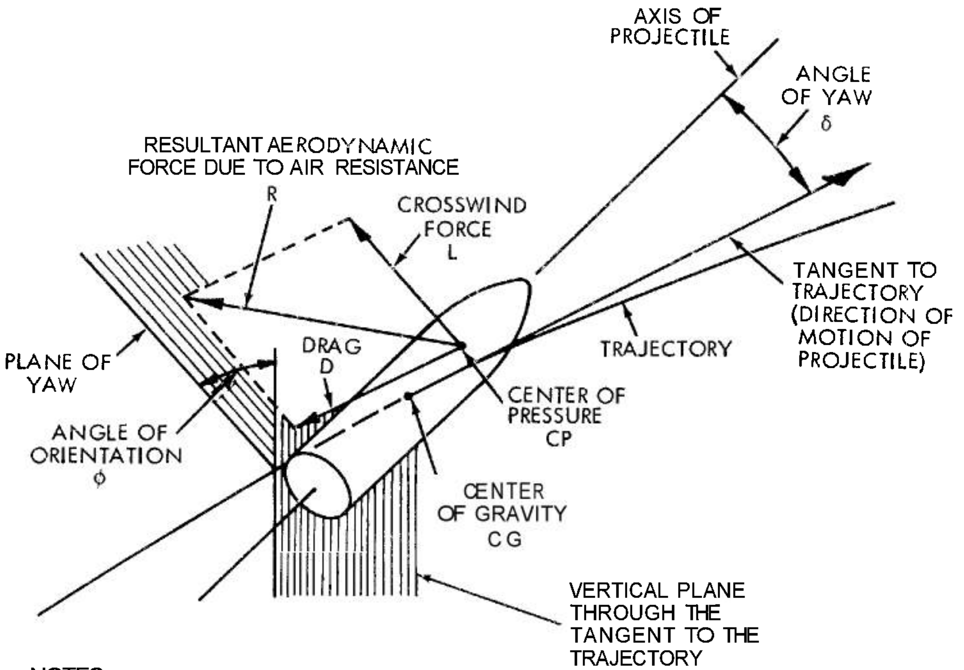
3. The initial velocity. With the air density and the design of the projectile considered to remain constant, the initial velocity of the projectile affects the characteristics of the trajectory because the amount of resistance offered by the air varies with the projectile velocity.

While, in general, air resistance is consideredto be a resultant vector force, many significant force and moment factors make up the total effect that causes retardation and mis-direction of the projectile in flight. (See Reference 3 for an excellent summary of these factors.) The resultant aerodynamic force R that acts on a moving projectile as a result of air resistance can be treated as two component forces (see Fig. 2-7):

1. Crosswind force L which at any instant is a force lying in the plane formed by the tangent to the trajectory and the axis of the projectile (the plane of yaw) having a direction perpendicular to the direction of projectile motion.

2. Drag D which at any instant is a force acting in the same plane as the crosswind force, and having a direction parallel and opposite to the direction of projectile motion.

As the attitude of the projectile varies with respect to the instantaneous direction of motion of the projectile over the course of the trajectory, the direction of the crosswind force also varies. In addition, the magnitude of the crosswind force increases as the angle



NOTES:

1. THE PLANE OF YAW IS AN INSTANTANEOUS PLANE FORMED BY THE TANGENT TO THE TRAJECTORY AND THE AXIS OF THE PROJECTILE.
2. THE DIHEDRAL ANGLE BETWEEN THE PLANE OF YAW AND THE VERTICAL PLANE THROUGH THE TANGENT TO THE TRAJECTORY IS KNOWN AS THE ANGLE OF ORIENTATION ϕ. THE ANGULAR MOTION OF A PROJECTILE ABOUT ITS CENTER OF GRAVITY IN THREE DIMENSIONS IS DESCRIBED IN TERMS OF THE ANGLE OF YAW δ, AND THE ANGLE OF ORIENTATION ϕ.

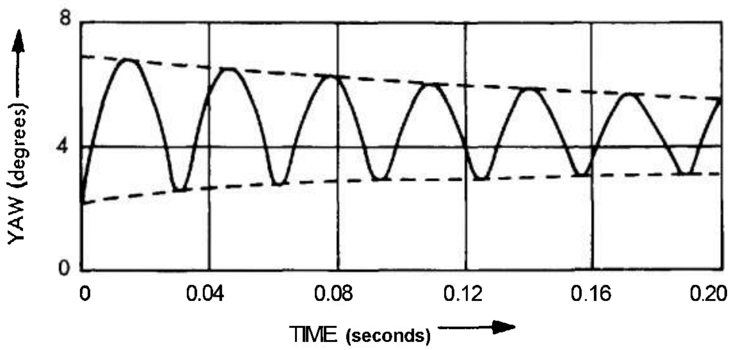
Figure 2-7. The forces on a projectile moving in still air.

of yaw increases. As indicated by Figure 2-7, the attitude of a projectile with respect to the direction of motion of the projectile is completely specified at any particular instant by the angle of yaw δ, and the angle of orientation ϕ. Figures 2-8 and 2-9 show typical variations of these quantities with time and with one another. The train of events pictured stems from a combination of projectile precession (see par 2-2.3.3.6) and the resulting variations of air pressure on the projectile nose. In order to meet projectile-stability criteria, the oscillations in yaw (called nutations) must be damped out, as shown in Figure 2-8. (For a summary of how projectiles can be designed to achieve appropriate control of their flight characteristics, the reader should consult References 3 and 7.)

Drag, the force component of the total air resistance that acts in the direction opposite to the direction of motion of the projectile, is generated by the resistance of the projectile nose, the skin friction caused by translation and rotation, and the formation of eddy currents and a partial vacuum at the base of the moving projectile. The behavior of airflow over the surface of the projectile in its passage through the air is affected by the form of the projectile. A blunt-nosed projectile encounters greater air resistance than a projectile with a pointed nose. A square-base projectile similarly offers more impedance to air flow than a tapered-base projectile. Projectile size also influences drag: the larger the diameter of the projectile, the greater the surface area exposed to the air; consequently, the more the drag effect tends to retard the projectile (for a given mass).* Again, the larger the projectile, the greater is the volume of air that must be displaced from the path of the projectile: a portion of the kinetic energy imparted to the projectile at the instant of firing must be used to perform the work of displacing this air.

Skin friction, too, plays an effective role as a component of drag. A rough surface on the projectile increases air resistance and, accordingly, decreases the range. As the projectile penetrates the air at high speed, the viscosity of the air affects projectile motion as described below. Layers of air adjacent to the surface of the projectile are dragged along with it; other layers of air above and contiguous with these are not. The air, therefore, submits to this layer-sliding action with a reluctance that is manifested by shearing stresses on the projectile surface; here, again, drag results in retardation of the forward motion of the projectile.

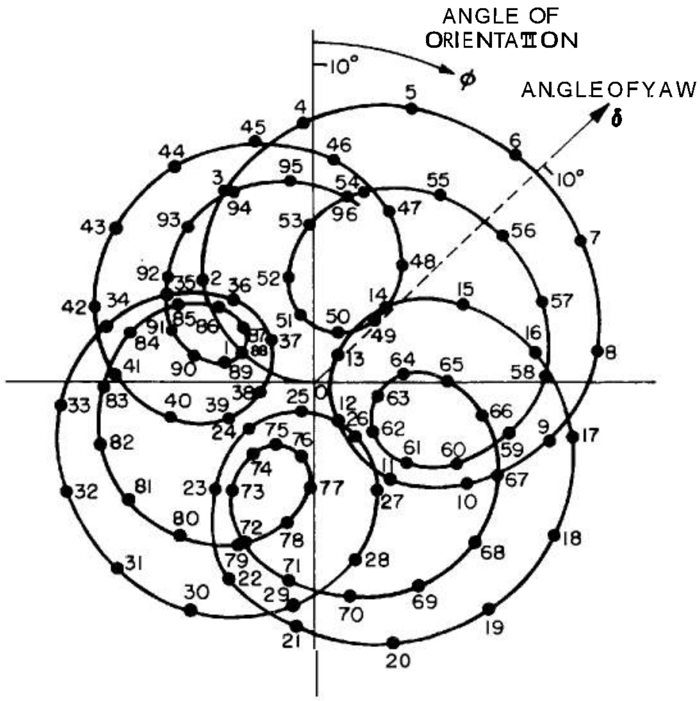
The velocity of the projectile along its curved trajectory also influences drag, as shown by Figure 2-10 for various projectile



NOTE: THE DAMPED-OSCILLATION CONDITION REPRESENTED HERE IS EXPERIENCED BY THE PROJECTILE AS IT LEAVES THE GUN AND ALSO WHEN THE DIRECTION OF PROJECTILE MOTION IS CHANGED AT THE TOP OF THE TRAJECTORY.

Figure 2-8. Plot showing a typical variation of the angle of yaw with time.

* In general, however, the larger the diameter of a projectile, the greater its mass, and hence the greater its momentum. This increase in momentum with size usually overcomes the increased drag due to air resistance.



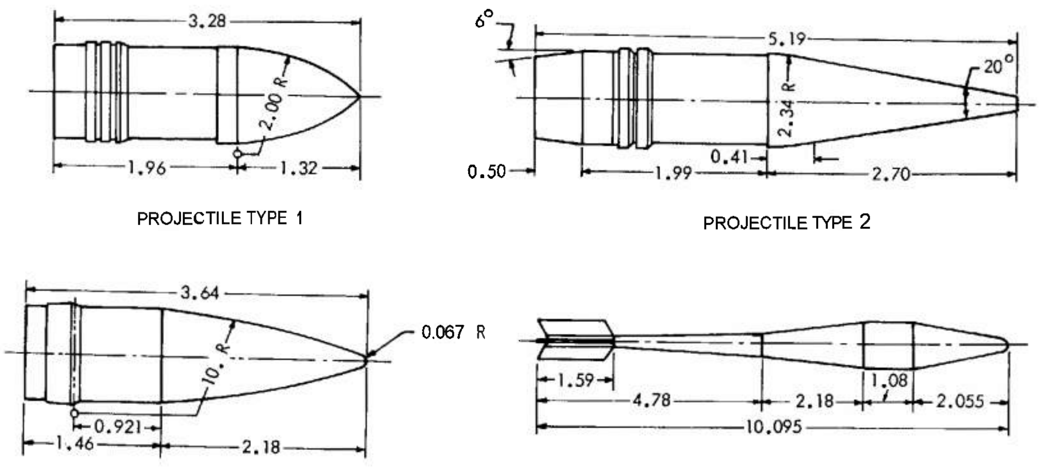
NOTES:

1. YAW IS PROPORTIONAL TO THE RADIAL DISTANCE FROM ORIGIN.
2. NUMBERS DENOTE TIME IN UNITS OF 0.0025 SECOND.

Figure 2-9. Polar plot showing a typical variation of the angle of orientation and the angle of yaw with time.

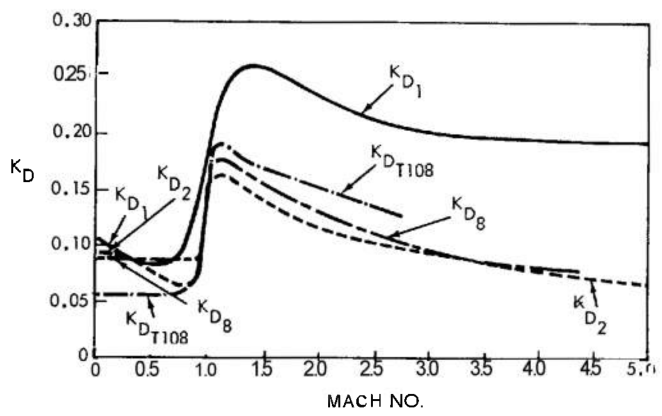
shapes. Below the speed of sound, skin friction constitutes the primary retardation effect and drag is approximately proportional to the square of the velocity of the projectile. For increasing projectile velocities in the subsonic range, the retarding effect also increases but at a faster rate. As the projectile velocity approaches the speed of sound, a sudden increase in drag occurs as a result of local velocities on the surface of the projectile exceeding the speed of sound and a shock wave being set into motion. Since energy is required not only to establish but to maintain any wave motion set up in the air by the projectile, the energy that is contained in the shock wave is derived from the kinetic energy imparted to the projectile at the instant of firing. Thus, the shock wave represents an energy loss that is continuously being dissipated through compression and irreversible heating of the air passing through the shock wave. The continuous drain of energy at this level of velocity obviously contributes to the retardation of the projectile. Furthermore, as the velocity of the projectile increases beyond the speed of sound, the airstream passing over the surface is unable to effect closure behind the base of the projectile; this inability creates turbulence, or wake, behind the projectile. At supersonic velocities, more shock waves are generated that add further drag or retardation effect. The total effect of friction, wake and shock waves, therefore, alters the range of the projectile.

A final factor that influences drag is yawing which, because of aerodynamic effects, results in a motion that fails to present the projectile to the air point first and thus requires the projectile to move through the air with a projected area greater than its diameter. If the angle of yaw exceeds 2° or 3°, the air resistance increases sufficiently to induce a retarding effect.



NOTE: ALL DIMENSIONS ARE IN CALIBERS

(A) Typical projectile shapes



(B) Drag coefficient vs Mach number for the projectile shapes shown in part A.

Figure 2-10. The influence of projectile velocity on drag for various projectile shapes.

In addition to the dominant aerodynamic forces of drag and crosswind force that act on a projectile as a result of air resistance, a dominant moment, called the overturning moment OM, must also be considered. (Other moments, such as the Magnus moment due to yawing and the yawing moment due to yawing, can normally be neglected.) This moment (see Fig. 2-7) is the moment of the resultant aerodynamic force R (which acts through the center of pressure of the projectile) about the center of gravity of the projectile. It varies with the sine of the angle of yaw δ.

2-2.3.1.3 Mathematical Relationships Associated With the Trajectory

The exact calculation of a projectile's trajectory under standard conditions would pose no serious problem if accurate data

were available on the inertia, gravitational, and aerodynamic forces exerted on the projectile as it moves through the air. This is particularly true in the present era of high-speed digital computers which were originally developed in order to solve the trajectories of projectiles and bombs. Unfortunately, the prediction of the aerodynamic forces just discussed is a matter of considerable difficulty and thus represents the primary problem associated with the exterior ballistics of a projectile.

The mathematics that pertain to the exterior ballistics of a projectile (see summary in Appendix 2-1) are discussed in References 3 and 7. These references provide an excellent description of the present-day means employed to analyze trajectories and to obtain firing tables, via high-speed digital computers, that very closely reproduce data obtained from test firings. In addition, Chapter 4 of Reference 7 provides an example of a desk-computer method of trajectory calculation. For additional information on the various aspects of exterior ballistics, the reader should consult References 4 through 6 and 8 through 12.

As shown by Reference 4, the equations of motion of a projectile under standard conditions can be put in the form

$$\ddot{x} = -\frac{GH}{C}\dot{x}$$

$$\ddot{y} = -\frac{GH}{C}\dot{y} - g$$

where

- x = instantaneous horizontal range of the projectile
- y = instantaneous projectile altitude
- G = drag function, proportional to the product of the drag coefficient, K_D (see Appendix 2-1) and the velocity u of the projectile relative to the air
- H = air density ratio
- g = gravitational acceleration
- C = ballistic coefficient, a measure of the relative air resistance of the projectile (see Appendix 2-1).

and the dots denote derivations with respect to time. While these equations cannot be solved analytically, they can be solved by a method of numerical integration. This method, which superseded the so-called short-arc method, was the principal method used in the United States subsequent to World War I until the advent of high-speed digital computers during World War II. The method is explained in References 5, 8, 9 and 10. Figure 2-11 is a plot of trajectories thus obtained for an initial projectile velocity of 2,800 ft per sec, a quadrant angle of departure of 45°, and various values of the ballistic coefficient. Reference 5 gives several sets of curves, including time-of-flight data, for various projectile types, elevation angles and muzzle velocities. For a discussion of the application of high-speed digital computers to a different numerical-integration approach, see Appendix 2-1.

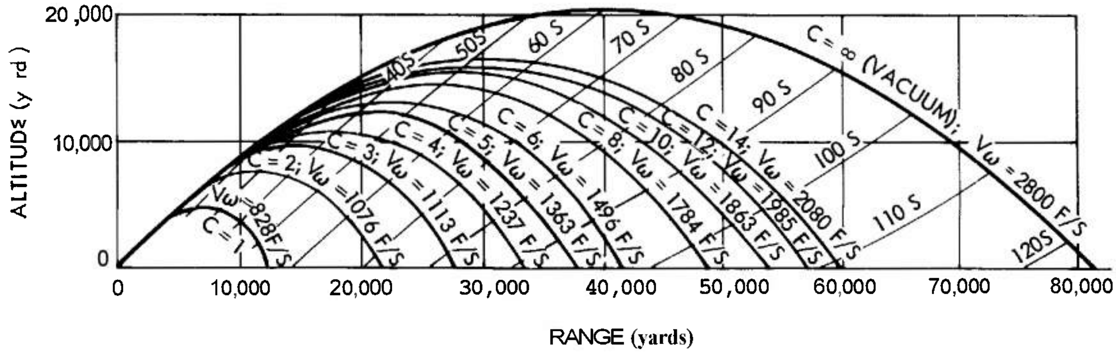


Figure 2-11. Plots of trajectories for v_o = 2,800 feet per second, θ_o = 45°, and C variable.

2-2.3.2 Effects of Jump

The combination of factors that determine the total velocity of the projectile are responsible for creating the phenomenon of jump that causes the initial projectile-velocity direction to differ from the direction in which the weapon is aimed. When a projectile is launched from a gun, the phenomenon of a jump usually occurs as a combination of a vertical jump effect and a horizontal jump effect.

2-2.3.2.1 Vertical Jump

Four factors, as follows, contribute to vertical jump:

1. The bore axis of a gun at rest does not exist as a straight line because of the cantilever construction of the gun tube. The axis of the bore has the characteristics of a curve that becomes more pronounced, the longer the bore. This characteristic is generally referred to as gun-tube droop. The projectile in passing through the bore at high velocity tends to straighten out the droop with an upward whipping effect. Because of the elasticity of the gun-tube metal and the forces involved, the gun tube is curved slightly concave upwards at the instant of release of the projectile.

2. The reaction of the gun tube to the rotation of the spinning projectile as it moves along the bore also influences vertical jump. As the projectile rotates clockwise (when viewed from the breech of the gun) the twisting moments that are induced tend to twist the gun tube in a counterclockwise direction.

3. As the projectile moves down the bore, the center of gravity of the projectile-gun tube system shifts and tends to displace the muzzle toward the ground.

4. The lack of complete rigidity of various parts of the gun and its carriage, combined with lack of complete stability (due to terrainal effects), also influences vertical jump.

It should be observed that these four factors affecting vertical jump do not necessarily exist in the same degree nor act in the same direction. Because the magnitude and direction of these factors cannot be determined by practical means, vertical jump is determined experimentally for each weapon.

2-2.3.2.2 Lateral Jump

Lateral jump is an effect similar to vertical jump; however, it represents the difference in azimuth — i.e., in the horizontal plane — between the line of bore sight and the line of departure. When the phenomenon exists, it has a magnitude much less than that of vertical jump.

Lateral jump generally occurs as a result of an unbalanced carriage condition, although some of the vertical-jump factors may contribute to lateral jump also. For a given unbalanced carriage condition, lateral jump increases slightly as gun traverse increases. On the other hand, lateral jump is usually considered negligible in stable carriages.

Frequently, curvature of the gun tube is a factor that produces lateral jump; it may be considered to be the counterpart of gun droop but derives from improper manufacturing techniques. When held within specified limits, gun-tube curvature produces negligible lateral jump effects.

2-2.3.3 Variations from Standard Conditions

The factors that influence the motion of a projectile are related to certain presupposed conditions involving the weather, the weapon, the projectile, and a motionless earth. Such conditions are referred to as standard conditions. Because these conditions do not exist at a particular time of weapon firing, variations from the assumed and accepted standard conditions introduce differences that influence the behavior of the projectile. These variations are referred to as nonstandard conditions. It should be noted that while some of the factors that make up nonstandard conditions are not natural phenomena, they are generally treated as variations from the norm.

Nonstandard conditions include the following quantities:

1. Propellant characteristics
2. Projectile weight
3. Air density
4. Air temperature
5. Differences in muzzle velocity
6. Drift
7. Wind
8. Effects of the rotation of the earth
9. Nonrigidity of the trajectory.

2-2.3.3.1 Propellant Characteristics

The characteristics of propelling charges used to fire projectiles vary from standard conditions because of differences in propellant temperature and moisture content. These differences cause variations in ignition, rates of burning, gun-tube temperature, and seating of the projectile that produce variations in muzzle velocity; therefore, variations in range result.

2-2.3.3.2 Projectile Weight

Projectiles of the same caliber are specified as having a standard weight. However, variations from standard — i.e., heavier-than-standard or lighter-than-standard — often occur among projectiles of the same caliber. A description of the effects of projectile weight follows. A projectile that is heavier than standard will acquire from the propellant about the same amount of energy as a lighter projectile; consequently, the projectile leaves the muzzle with a muzzle velocity less than that possessed by a projectile of standard weight. But the heavier projectile, because of the greater sectional density, has an improved ballistic coefficient, and the effect is toward an increase in range. For heavier-than-standard projectiles, the net effect of the two factors is to decrease the range over short times of flight and increase the range over the longer times of flight. For lighter-than-standard projectiles, the reverse is true.

2-2.3.3.3 Air Density

The density of the air is an important factor related to drag because any increase in air density causes greater resistance to the forward motion of the projectile, which results in a decrease in projectile velocity and range. However, air density also influences the path of the projectile and its time of flight because it is a measure of the mass that must be displaced by the projectile along its flight path. The greater the density of the air, the more kinetic energy must be consumed to overcome the compactness of the air and, as a consequence, the greater is the retardation of the projectile. Over long times of flight, the projectile may pass through several layers of air having different densities which may have significant effect on range. For short times of flight, the range effects due to density variations in the atmosphere are negligible.

2-2.3.3.4 Air Temperature

Nonstandard temperature affects the path of a moving projectile in an oblique manner. Because air temperature and air density are interdependent, variations in the former influence the latter. It was established in the discussion of factors relating to drag that air density affects the retardation of the projectile. Therefore, since a variation in temperature brings about a variation in density, the latter in turn causes a variation in range. As the temperature of the air increases, the range of the projectile may increase or decrease, depending on the velocity of the projectile. The relationship of drag to the Mach number of the projectile (projectile velocity/velocity of sound) changes abruptly when the projectile velocity is in the vicinity of Mach 1. As the velocity approaches the speed of sound, the effect of drag increases. But as the air temperature increases, the velocity of sound also increases. In this way, the differential effects of air temperature influence the location of the point on the trajectory at which the change in retardation due to the initial speed of sound occurs. Over a considerable amount of a projectile's trajectory, the projectile velocity may be below the speed of sound. The supersonic range of the Caliber .30 projectile, for example, is quite short.

2-2.3.3.5 Differences in Muzzle Velocity

Among the deviations from the standard conditions that cause a projectile to impact or burst at some point other than the target, are variations in muzzle velocity. The muzzle velocity is the maximum speed attained by a projectile while under the influence of the propellant gases and occurs shortly after it leaves the muzzle of the weapon. The greater the muzzle velocity of a particular projectile, the greater is the range it can attain. Accordingly, variations in the actual muzzle velocity from the standard value upon which a particular set of firing tables is based will result in range inaccuracies.

Variations in muzzle velocity result from a number of causes which can be summarized as follows:

1. As already noted in pars 2-2.3.3.1 and 2-2.3.3.2, respectively, variations in propellant characteristics (temperature and moisture content) and variations in projectile weight contribute to changes in muzzle velocity.

2. Erosion of the weapon tube enlarges the bore. This allows the propellant gases to escape, thereby reducing gas pressure and hence the muzzle velocity.

3. Lack of hard, uniform ramming of separate-loading ammunition from round to round causes variation in the seating of the projectile, which results in nonuniform velocities at the muzzle.

4. Rough surfaces on the rotating band of a projectile prevent its proper seating. As a result, the propelling gases escape and the muzzle velocity decreases.

5. Even such minor factors as manufacturing tolerances and oily weapon tubes result in minor and abnormal variations in the muzzle velocity of the projectile.

2-2.3.3.6 Drift

As indicated in Figure 2-6(B), the trajectory of an elongated, rotating projectile deviates laterally from its plane of departure in such a manner that the horizontal trace of the trajectory is a curved, rather than a straight, line. This lateral deviation is called drift and is measured as the perpendicular distance from the end of the trajectory to the plane of departure. It is sometimes referred to as linear drift in order to differentiate it from angular drift, which is the angle subtended by the linear drift between the plane of departure and the vertical plane containing the line of site.

Drift may be considered to result from the following three causes:

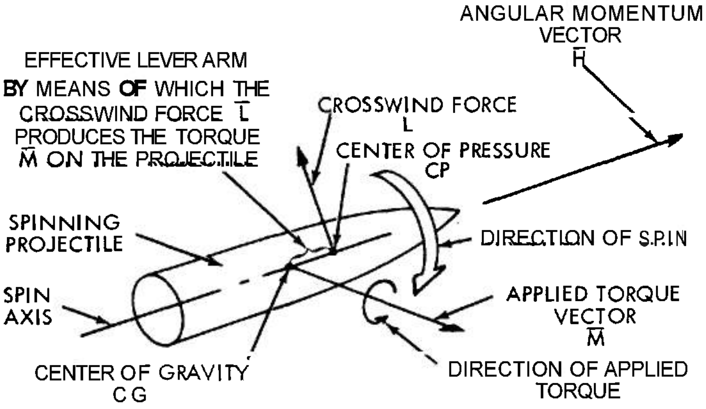
1. Gyroscopic action
2. Magnus effect
3. Cushioning effect.

It is reasonably certain, however, that the combined effect of the last two causes named is minor compared with the effect of the first.

The part played by gyroscopic action will be considered first. A projectile when fired from a weapon is given a rotating motion or spin about its longitudinal axis by means of the rifling, i.e., the lands and grooves of the tube. This spinning action prevents tumbling of the projectile during its flight. In U. S. Army weapons, rifling is always right-hand twist, so the projectiles spin clockwise when viewed from the base of the projectile. The spinning action is accomplished at a rotational speed sufficient to make the projectile behave as a gyroscope during its time of flight. Although this gyroscopic behavior serves to stabilize the projectile in flight, it does at the same time subject the spinning projectile to gyroscopic precession. Gyroscopic precession is a change in the orientation of the spin axis of a rotating body that takes place as the result of an applied torque. The direction or axis about which the rotating body will turn, or precess, is such as to bring the spin axis into alignment with the direction or axis about which the torque is applied (see Fig. 2-12). The particular precession of concern here results from the interaction of the torque produced by the crosswind force L* (see par 2-2.3.1.2), which acts at the center of pressure of the projectile, with the angular momentum of the spinning projectile.

The gyroscopic precession of a projectile occurs as a result of the curvature of the flight path due to gravity (see par 2-2.3.1.1) in the following manner. Because of the stability of the projectile arising from its spin, the projectile tends to maintain its original flight orientation in space even though the trajectory itself does curve. This means that as the trajectory drops away from the initial flight direction due to the action of gravity, the nose of the projectile points slightly above the trajectory. The air pressure acting on the underside of the nose of the spinning projectile causes the projectile to precess clockwise (as viewed from above the trajectory). This shift of the longitudinal axis of the projectile now exposes the left side of the nose (as viewed from above the trajectory) to the

* The principal force involved is the crosswind force since, with the small angles of yaw encountered in practice, ***drag*** produces negligible torque on the projectile.

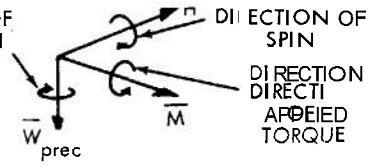


NOTE: THE CROSSWIND FORCE REPRESENTED IS THAT CAUSED BY AIR PRESSURE ACTING ON THE UNDERSIDE OF THE PROJECTILE NOSE, WHICH IS THE RESULT OF CURVATURE OF THE TRAJECTORY.

THE BASIC LAW OF MOTION OF A SPINNING BODY, SUCH AS A PROJECTILE IN FLIGHT, IS THAT WHEN A TORQUE $\overline{M}$ IS APPLIED (SEE ABOVE) THE SPIN AXIS OF THE BODY ROTATES (PRECESSES) ABOUT AN AXIS THAT IS PERPENDICULAR TO BOTH THE SPIN AXIS AND THE APPLIED TORQUE VECTOR. THE RATE OF PRECESSION IS DIRECTLY PROPORTIONAL TO THE APPLIED TORQUE. THE DIRECTION OF PRECESSION IS SUCH AS TO ALIGN THE ANGULAR MOMENTUM VECTOR $\overline{H}$ (WHICH IS COINCIDENT WITH THE SPIN AXIS) WITH THE APPLIED TORQUE VECTOR. THIS ACTION IS REPRESENTED BY THE FOLLOWING VECTOR RELATIONSHIP BETWEEN THE ANGULAR MOMENTUM VECTOR, $\overline{H}$, THE APPLIED TORQUE VECTOR $\overline{M}$, AND THE ANGULAR VELOCITY OF PRECESSION, $\overline{W}_{prec}$:

$$\overline{M} = \overline{W}_{prec} \times \overline{H}$$

THE ASSOCIATED VECTOR DIAGRAM IS SHOWN AT THE RIGHT. (FOR A RIGID DERIVATION OF THIS BASIC RELATIONSHIP, SEE DERIVATION SUMMARY 4-1 IN CHAPTER 4 OF REFERENCE 1.



THIS VECTOR RELATIONSHIP AND VECTOR DIAGRAM REPRESENT THE PHYSICAL FACT THAT WHEN A TORQUE IS APPLIED TO A SPINNING PROJECTILE, AS SHOWN BELOW, THE PROJECTILE ACTUALLY ROTATES (PRECESSES) IN A DIRECTION THAT IS AT RIGHT ANGLES TO THE ROTATION THAT WOULD RESULT FOR A NON-SPINNING BODY.

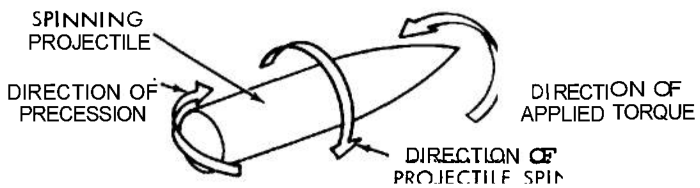


Figure 2-12. The gyroscopic precession of a spinning projectile.

air pressure. Continuing gyroscopic behavior then precesses the spinning projectile nose-downward. This kind of precession action continues until the projectile is once again positioned with the underside of the nose exposed to the pressure of the air. This train of events continues, causing the axis of the projectile to oscillate about the instantaneous tangent to the trajectory. The oscillating cycle repeats itself with diminishing effect (see Figs. 2-8 and 2-9). The predominant orientation of the projectile nose, how-

ever, is upward. Therefore, since the maximum air pressure is always on the underside of the projectile, the net precession is always toward the right.

The phenomenon of precession has negligible effect on the trajectory of spin-stabilized projectiles launched at high angles of elevation. However, for projectile flight paths at low angles of elevation, the projectile continues its precessional orientation toward the right. As a result of this crabwise movement, the lateral component of air resistance continues to push the projectile further towards the right, thereby causing the projectile to drift to the right from the initial vertically-oriented plane of the fire. The magnitude of the drift — expressed as a lateral distance on the ground — is dependent on the rotational speed of the projectile, the curvature of the flight path due to gravity, and the time duration of flight. As shown by the vector relationship of Figure 2-12, the amount of precession — and hence the drift — varies inversely with rotational speed of the projectile. Drift increases with an increase of the other two factors, however.

A description of the part played by the Magnus effect in producing trajectory drift follows. As has already been noted, the initial tendency of a projectile to maintain the original direction of its axis as it falls away from the axis of the weapon tube causes the air stream to strike the lower side of the projectile. The air stream then splits, with part going past the projectile on the left-hand side (as viewed from the rear of the projectile) and part going past the right-hand side. Because of this and the projectile's right-hand spin, the air adhering to the right-hand side of the projectile meets and opposes that part of the air stream passing on the right-hand side of the projectile, with a resulting increase in pressure on that side. At the same time, there is a corresponding rarefaction on the left-hand side of the projectile. This results from the fact that the air adhering to the left-hand side of the projectile is moving in consonance with that part of the air stream passing on the left-hand side of the projectile. Accordingly, the projectile tends to move to the left, the side of lesser pressure. This effect — known as the Magnus effect — is the same phenomenon that causes a golf ball to hook or to slice. The Magnus effect can be important in the descending end of a trajectory for a projectile fired at high elevations of the weapon tube. This is because the steepness of the descent causes the air stream to hit the projectile nearly perpendicular to its axis, and therefore with maximum Magnus effect. As can be seen, the Magnus effect opposes the gyroscopic effect.

The cushioning effect stems from the fact that the air tends to pile up on the underside of the projectile and, therefore, forms a cushion. The projectile tends to roll on this cushion because of its spin and the friction existing between the projectile and the cushion. This rolling movement is to the right in a projectile with right-hand spin. Thus, the cushioning effect opposes the Magnus effect but adds to the gyroscopic effect.

2-2.3.3.7 Wind

The lateral deviation of a projectile from its standard trajectory results from two principal factors. The first factor, drift, has been discussed previously. The second factor is wind. For purposes of practicality, a theoretical wind that is assumed to be constant is sometimes employed for correction purposes. This constant wind, termed ballistic wind, is expected to have the same effect on a projectile during its flight as the varying winds actually encountered.*

The ballistic wind is considered to be horizontal. In general, it therefore has components that are parallel and perpendicular to the line of fire. Accordingly, the ballistic wind generally influences both the range and direction of a projectile's trajectory. The component of ballistic wind that blows at right angles to the line of fire is called the cross wind or lateral wind and causes the projectile to be displaced laterally with respect to the line of fire. The component of ballistic wind that blows in the plane of fire, on the other hand, is referred to as range wind.

With respect to the relationship between the projectile velocity and the velocity of the

* It should be noted that the concept of a ballistic wind is necessary only for hand solutions; machine solutions use a "Meteorological Message" in which winds are taken altitude by altitude.

air adjacent to the projectile, range wind may produce positive or negative effects. If the air moves with the projectile — i.e., if a tailwind is present — the velocity relative to the air is reduced, the projectile encounters less air resistance and therefore less drag, and a longer range results. On the other hand, if the air moves in direct opposition to the forward motion of the projectile — i.e., a headwind exists — the velocity relative to the air is increased, drag is increased and the range decreases.

The cross or lateral wind component does not, of course, affect the trajectory range, being responsible only for such deflection of the trajectory that is not attributable to drift. The direction and magnitude of this deflection is dependent on the azimuth and velocity of the wind.

The effect of the ballistic wind has negligible influence on flight paths having a short time duration. Conversely, considerable effect on the accuracy of weapon fire results for flight paths of long time duration.

2-2.3.3.8 Effects of Rotation of the Earth

The rotation of the earth is a factor that affects both the range and azimuth of the terminal point of a projectile's trajectory. Because the earth rotates from east to west at an angular velocity of 15 degrees per hour — producing a tangential velocity of 1024 miles per hour at the equator — the effect of the earth's rotation on the movement of a projectile fired to a high altitude on a very-long-range trajectory is highly significant from the standpoint of accuracy. Unless the earth's angular velocity is accounted for in the differential equations of motion for the projectile — e. g., in Eqs. 9, 10, and 11 of Appendix 2-1 — errors in trajectory calculations will result.

For very long-range trajectories, two other factors come into play: the variation of gravity with altitude and the curvature of the earth. An explanation of their influence on projectile motion follows. Rotation of the earth is considered as a nonstandard condition involving the factors of direction of fire, angle of departure and velocity of the projectile, and aspects of longitude and latitude, i.e., the relative positions of weapon and target with respect to geographic location. For long ranges, these aspects represent a departure from the standard structure; here, projected motion can no longer be considered from the standard conditions of air resistance, a flat earth, and a homogeneous field of gravitation. The variation of gravity with altitude and the curvature of the earth's surface influence projectile motion in the following ways:

1. The influence of the variation of gravity with altitude has a minor deviational effect on long-range projectiles. Only when the maximum ordinate of the trajectory reaches 100 miles or more is this factor significant. At this altitude, at the equator, the acceleration due to gravity decreases approximately 5%, the effect of which would increase the range beyond predicted values obtained under standard conditions.

2. The curvature of the earth affects computation of the trajectory in two ways. In the first place, the direction of the downward force of gravity is established at the origin of the trajectory. At long range, the direction of the force of gravity at the point of impact is not parallel to the gravity force at the origin. In the second place, under short-range, standard conditions, the coordinates of the particular target are determined on the basis of a horizontal plane; at long ranges, the curvature of the earth must be accounted for in computing the point of fall. With the curvature of the 'earth changing at a rate of approximately one degree for each 70 miles, the range of a projectile over a long trajectory increases over that range computed by simplified solutions. It should be noted also that the influence of the curvature of the earth and variations in the gravitational field always act on a given projectile path irrespective of the direction of fire.

2-2.3.3.9 Nonrigidity of the Trajectory

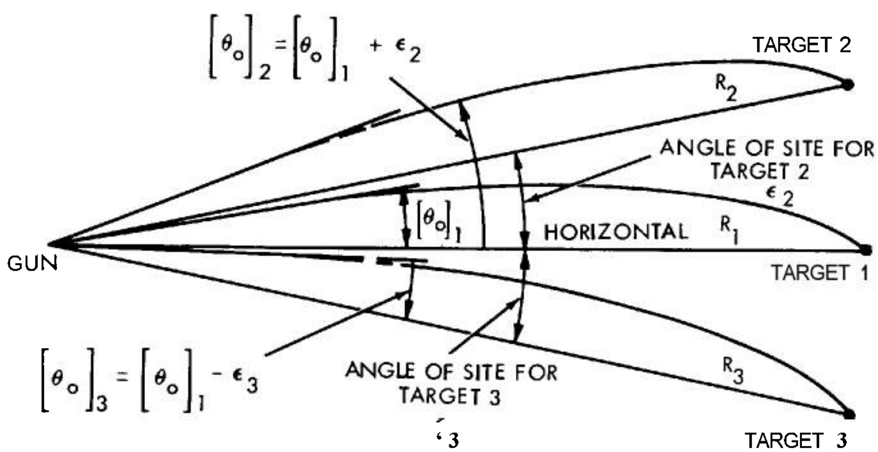
In the standard structure assumed and accepted as the basis of weapon fire, the standard trajectory of a projectile is referred to the horizontal plane that passes through the weapon and the fixed target. In actual weapon fire, however, targets may be located at various heights above or below the horizontal plane at different values of range, resulting in various angles of site. For small angles of site, it is satisfactory to rotate the

trajectory about the origin through these small vertical angles in the plane of fire. In theory, this may be accomplished without materially influencing the curvature of the trajectory (see Fig. **2-13**). This assumption, known as the theory of rigidity of the trajectory, is generally applicable to field-artillery and small arms fire, and introduces significant range error only when the ratio of target height to target range is large so that the angle of site is large. In particular, the assumption is not applicable to antiaircraft-fire trajectories when the angle of site is normally quite large.

2-2.4 EFFECT OF TARGET MOTION

The physical phenomena that, unless compensated, will produce errors in weapon fire do not relate entirely to the projectile or the weapon; even if such phenomena did not exist, any target motion during the time of flight of the projectile would cause it to miss the target if it were fired along the line of site.

The motion of the target during the time of flight of the projectile between the time of launching and the moment of impact has kinematic characteristics that derive from the integrated effects of target velocity and acceleration during the interval of flight. These effects, which involve the rotation of the line of site to the target, vary with target velocity, the angles of the space geometry, and target range. When the range to the target is long, the angular velocity of the line of site (the apparent motion of the target to the tracking system) is relatively low; hence target motion imposes relatively little influence on the angular velocity of the line of site in space. Conversely, when the range to the target is short, a small amount of target motion results in a relatively large angular velocity of the line of site. Because target motion that occurs during the time of flight would cause the projectile to miss the target if it were directed along the line of site to the target, it is necessary to provide compensation for target motion. This compensation is directional in nature and is applied to the weapon-aiming line before launching or firing so that an angle exists between that line and the line of site. It is generally applied in the form of component corrections in elevation and azimuth. The total angular correction provides a weapon orientation that nullifies



NOTE: THE ANGLES OF SITE ARE GREATLY EXAGGERATED HERE FOR ILLUSTRATIVE PURPOSES. WHEN THEY ARE SMALL, R_1, R_2 AND R_3 CAN BE CONSIDERED TO BE EQUAL FOR THE QUADRANT ANGLES θ_o THAT ARE NORMALLY EMPLOYED.

Figure **2-13**. The rigidity of the trajectory for small angles of site.

the miss-producing effect of target motion and allows projectiles to score hits on the target.

2-2.5 THE PREDICTION ANGLE

After a projectile has been launched, it is acted upon by various forces peculiar to the weapon, to the environment through which it passes, and to the motion along its path of flight. The associated corrections that must be applied as compensation are directional in nature, i.e., each correction takes the form of an angle. Similarly, compensation for the effects of target motion during the time of flight of the projectile also takes the form of an angular correction. The total correction angle, made up of the sum of these individual correction angles, forms the required angle between the weapon line and the line of site for scoring hits on a target. This angle is referred to for generality as the prediction angle since it is the overall angle that must be predicted in advance of firing in order to aim the weapon so as to obtain hits on the target. It is this angle that must be generated by fire control equipment by one means or another in order to effect the required offset angle of the weapon line from the line of site that is required to obtain hits on the target. The prediction angle for the case of a stationary target is depicted in Figure 2-14, while Figure 2-15 represents the prediction angle for the case of a moving target.

The prediction angle is composed of three major components: kinetic lead, ballistic lead, and compensating corrections. Each is discussed in turn in the following paragraphs. The geometry associated with these prediction-angle components is portrayed in Figure 2-16, which is based on the fire-control situation shown in Figure 2-15.

Kinetic lead is the angular correction required to compensate for target motion

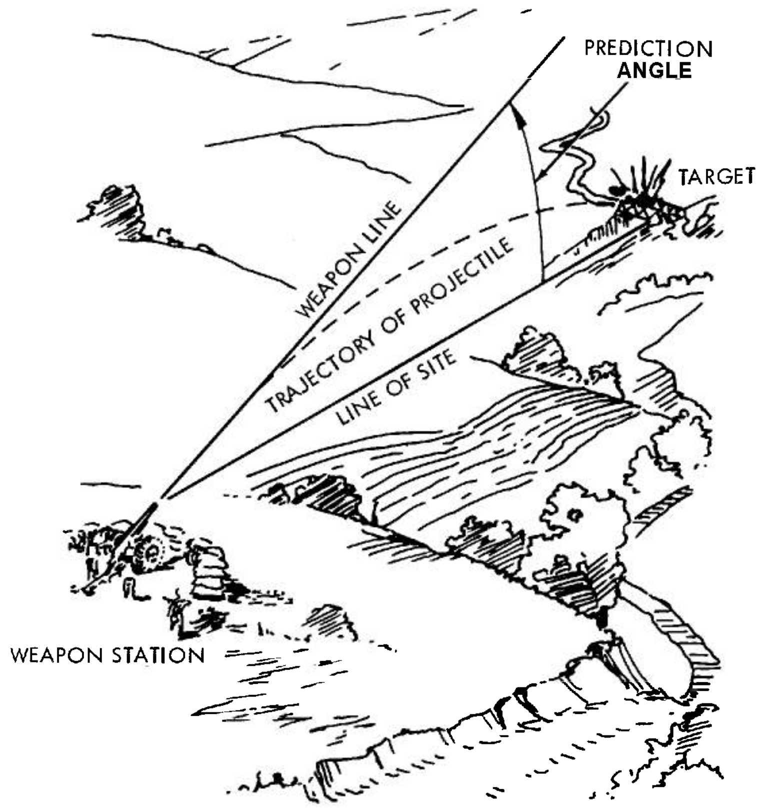


Figure 2-14. The prediction angle for a stationary target. (Adapted from FIRE CONTROL PRINCIPLES by W. Wrigley and J. Hovorka. Copyright © 1959 by McGraw-Hill, Inc. Used by permission of McGraw-Hill Book Company.)

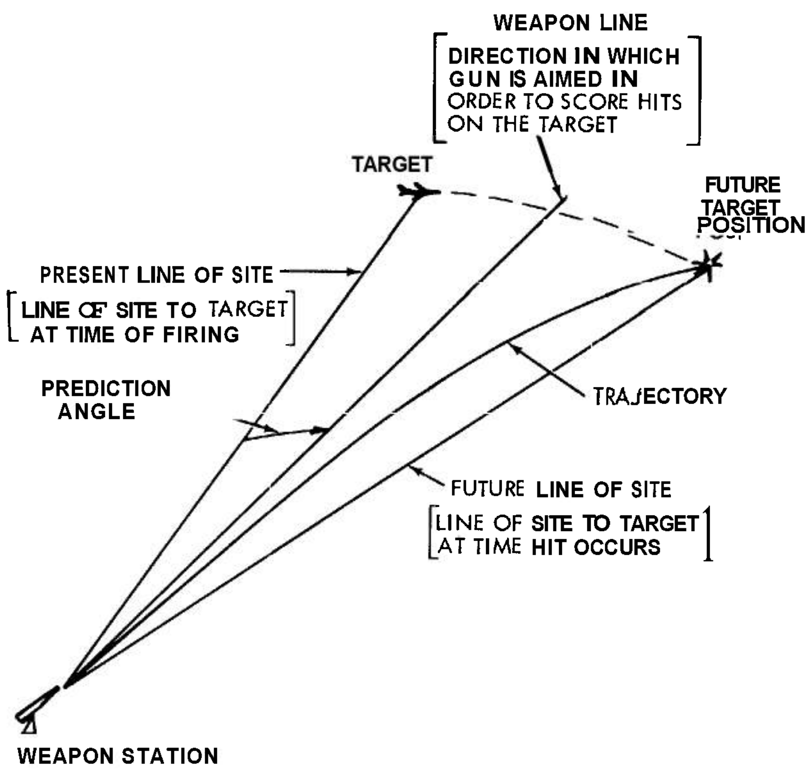
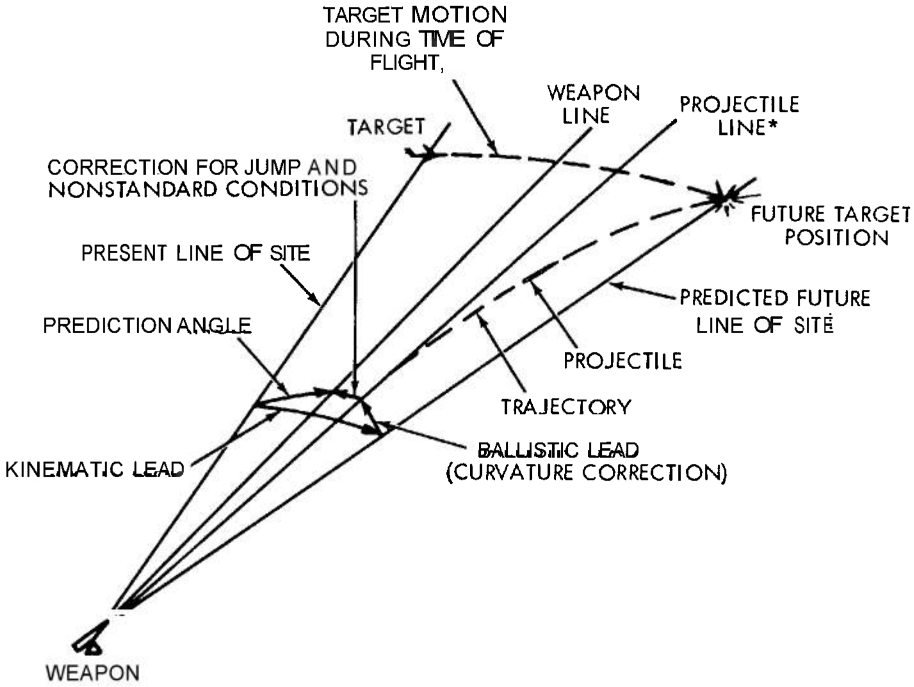


Figure 2-15. The prediction angle for the case of a moving target.

during the time of flight of the projectile and is a function of that time of flight. It is the angle between the line of site to the target at the time of firing and the predicted future line of site to the target at the time a hit occurs. It should be noted that in the case of a stationary weapon and a stationary target, the need for the kinetic lead component of the prediction angle does not exist. An example of this situation is given by Figure 2-17, which represents those aspects of field-artillery fire control problems that lie in the elevation plane. Here, the elevation component of the total prediction angle is the angle of elevation. It is comprised of the quadrant angle of departure, the angle of site, and a correction for vertical jump. No kinetic lead correction is required.

Ballistic lead, or curvature correction, is an angular correction required to compensate for the effect of the various in-flight forces, such as air resistance and gravity, that act on a projectile during its time of flight and result in a curved trajectory. As in the case of the kinematic-lead correction, it is also a function of the time of flight of the projectile. Geometrically, it is the angle between the predicted future line of site to the target and the projectile line.

The compensating corrections correct for jump and variations from standard conditions. Jump correction compensates for initial velocity effects. It may be defined as that correction required to compensate for the nonparallelism of the weapon line and the initial projectile velocity vector in the particular coordinate reference system chosen. Unlike lead correction and curvature correction, jump correction is not a function of time of flight of the projectile. It can be visualized geometrically as being the angle between the projectile line and the weapon line, the former being the direction of the initial velocity of the projectile. Corrections for variations from initial conditions are made on the basis of available information concerning propellant temperature, projectile weight, air density, etc.



* THE PROJECTILE LINE IS THE DIRECTION OF THE INITIAL VELOCITY OF THE PROJECTILE.

Figure 2-16. The prediction angle and its major components. (Adapted from FIRE CONTROL PRINCIPLES by W. Wrigley and J. Hovorka. Copyright © 1959 by McGraw-Hill, Inc. Used by permission of McGraw-Hill Book Company.)

2-2.6 COORDINATE FRAMES FOR FIRE CONTROL

As has been noted, the fire control problem is inherently kinematic and dynamic by nature. Its solution is, therefore, readily expressible in geometric terms by means of vectors related by the laws of physics.

Certain vectors (e.g., velocity) require a coordinate-frame reference in order that they may be properly specified. For instance, while air speed and ground speed both may be considered to be vector velocities, they differ vectorially simply because their frame of reference in each case is different; air speed must be associated with an air-mass reference frame and ground speed must similarly be associated with a ground reference frame. It should be observed that, unless some reference frame is specified, the concept of velocity can have no meaning. Since, among the general cases of fire control, the weapon as well as the target may be in motion – e.g., a moving tank firing at another tank in motion – the specification of vectors may also be made with respect to moving coordinates.

In general, there are two broad classes of coordinate reference frame:; that find application in connection with fire control problems:

1. One class is used in stating the fire control problem without reference to actual fire control equipment.

2. The other class is used in solving the fire control problem and, accordingly, pertains to reference frames that are fixed in relation to the fire control equipment itself.

A great variety of reference frames have been used in connection with fire control. The paragraphs below describe the most important of these frames and classify them in accordance with the aforenoted scheme.

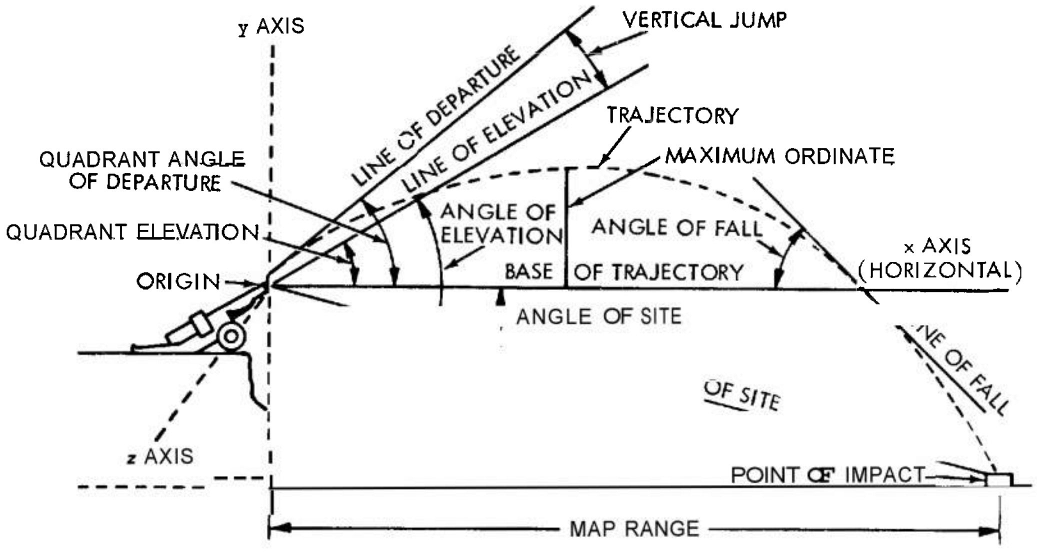


Figure 2-17. Aspects of the field-artillery fire control problem associated with the elevation plane.

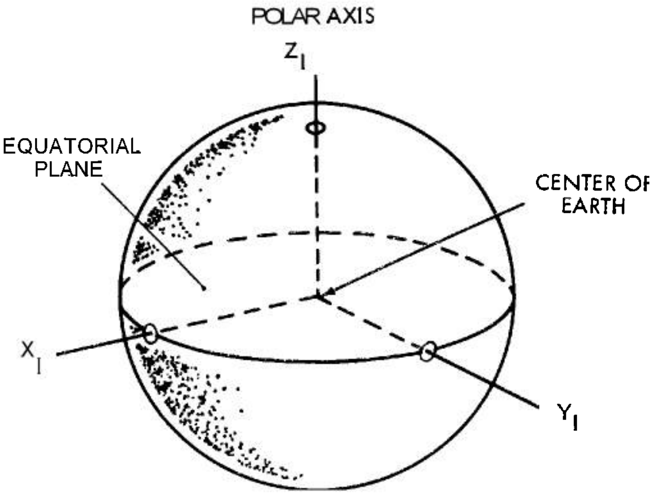
2-2.6.1 Primary Coordinate Frames of Use for Stating the Fire Control Problem

There are four primary coordinate reference frames in which the fire control problem can be defined. The first is an inertial reference frame and is usually referred to simply as "inertial space". This is the framework in which the laws of physics are expressible in their simplest form. An inertial reference frame is generally considered to be unaccelerated — i.e., of constant velocity and nonrotating — with respect to the so-called "fixed stars". For convenience, it is generally taken with its center at the center of the earth. This inertial frame, referred to as a geocentric inertial reference frame (see Fig. 2-18), is taken as a reference only for those fire control problems in which the time of flight of the projectile is so long that the effects of the diurnal rotation of the earth cannot be ignored, for example, long-range weapon fire.

A second useful reference system is an earth coordinate system, which may be considered fixed with respect to the earth, but not necessarily centered at the earth's center (see Fig. 2-19). If the frame of reference has its origin at the center of the earth and rotates with the earth (see Fig. 2-19(A)), it is referred to as a geocentric earth reference frame. If, on the other hand, the frame of reference is centered at some convenient point on or near the surface of the earth (see Fig. 2-19(B)), it is referred to as a vehicle-centered earth reference frame. In general, the earth frame of reference is extremely useful for those fire control problems in which the weapon is either stationary or is moving at a very slow ground speed. This reference frame would, therefore, be applicable to most Army fire control problems.

A third useful reference frame may be described as an air-mass coordinate system in which the frame is considered fixed in the air mass. The air-mass reference frame (see Fig. 2-20) may be visualized as a frame fixed in a free balloon. This frame is particularly useful for problems associated with airborne fire control, e.g., a helicopter fire control system. It should be noted that in this type of fire control problem the times of flight are generally of short duration; hence the air mass is considered to be inertial. From the standpoint of the air-mass reference frame, the ballistics of a moving projectile reduce to their simplest analytic form.

The fourth useful frame of reference is the stabilized weapon-station coordinate system (see Fig. 2-21). This frame of reference has its origin centered in the weapon station and translates with the vehicle that carries



THE MOST CONVENIENT ORIENTATION OF AXES IS WITH ONE AXIS, E.G., Z_I, PARALLEL TO THE EARTH'S POLAR AXIS AND X_I AND Y_I THEN ARBITRARILY LOCATED IN THE EQUATORIAL PLANE. THE GEOCENTRIC INERTIAL REFERENCE FRAME $X_I Y_I Z_I$ IS, BY DEFINITION, NONROTATING RELATIVE TO THE "FIXED STARS."

Figure 2-18. The geocentric inertial reference frame. (Adapted from FIRE CONTROL PRINCIPLES by W. Wrigley and J. Hovorka. Copyright © 1959 by McGraw-Hill, Inc. Used by permission of McGraw-Hill Book Conipany.)

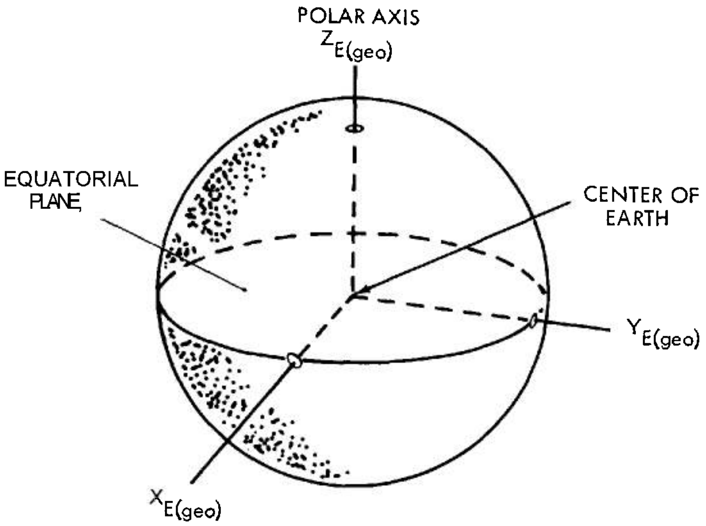
the weapon. The frame is considered, however, to be free from any of the rotational motion of the weapon-carrying vehicle, i.e., motion about the reference coordinate axes in the roll, pitch, and yaw modes. This inertial reference frame is generally useful when the linear motion of the vehicle is readily distinguishable from the roll, pitch, and yaw of the vehicle. Thus, the stabilized weapon-station coordinate system would be useful for a tank weapon system designed for firing while the tank is in motion.

2-2.6.2 Coordinate Frames of Use in Data Handling and Computing*

As indicated in paragraphs 2-2.2.3 and 2-2.5, the basic parameters of the fire control problem are the line of site, the weapon line, and the prediction angle. In the mechanization of the solution to the fire control problem, actual indications of the line of site are provided by means of some physical tracking mechanism. This indicated line of site is usually referred to as the tracking line. The weapon line, of course, is coincident with the axis of the weapon tube. Accordingly, both the tracking line and the weapon line represent driven lines that are firmly fixed with respect to physical equipment in any particular weapon system. Each of these lines must intrinsically have reference coordinate frames associated with them, i.e., there must be a reference frame for the data-gathering function of the acquisition and tracking portions of the fire control system and there must be a reference frame for the data-utilization function of the weapon-pointing system.† These two frames may or may not be identical. The computed prediction angle itself must also be generated in a reference coordinate frame. For sake of distinction, this frame is sometimes referred

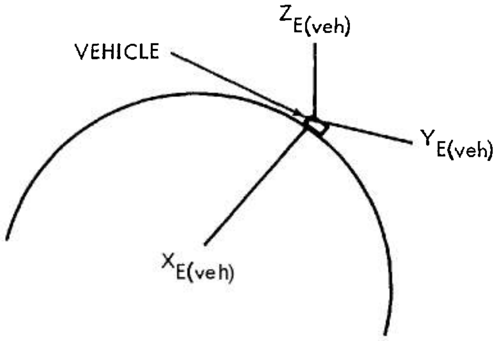
* While this topic ties in with the solution of the fire control problem which is discussed in paragraphs 2-3 through 2-3.4 following, it is presented at this point in order to complete the discussion of coordinate frames for fire control.

† Acquisition and tracking systems are covered in Section 2 of the Fire Control Series; fire-control computing systems are covered in Section 3; and weapon-pointing systems are covered in Section 4. The present discussion of reference frames is for background information only.



THE MOST CONVENIENT ORIENTATION OF AXES IS WITH $Z_{E(geo)}$ PARALLEL TO THE POLAR AXIS AND WITH $X_{E(geo)}$ AND $Y_{E(geo)}$ ARBITRARILY LOCATED IN THE EQUATORIAL PLANE.

(A) Geocentric Earth reference frame



THE ORIENTATION OF THE AXES IS ARBITRARY.

(B) Vehicle-centered Earth reference frame

NOTE: THESE FRAMES MAKE ONE ROTATION ABOUT THE POLAR AXIS WITH RESPECT TO AN INERTIAL REFERENCE FRAME IN ONE SIDEREAL DAY.

Figure 2-19. Geocentric and vehicle-centered Earth reference frames. (Adapted from FIRE CONTROL PRINCIPLES by W. Wrigley and J. Hovorka. Copyright © 1959 by McGraw-Hill, Inc. Used by permission of McGraw-Kill Book Company.)

to as the computation reference frame. It is this computing frame that largely dictates the choice of the other coordinate frames used in carrying out the solution of a given fire control problem.

Because the computation reference frame is the frame in which the fire control problem is actually solved, it is necessary in the design of a fire control system that this frame be selected in advance even though – because of the

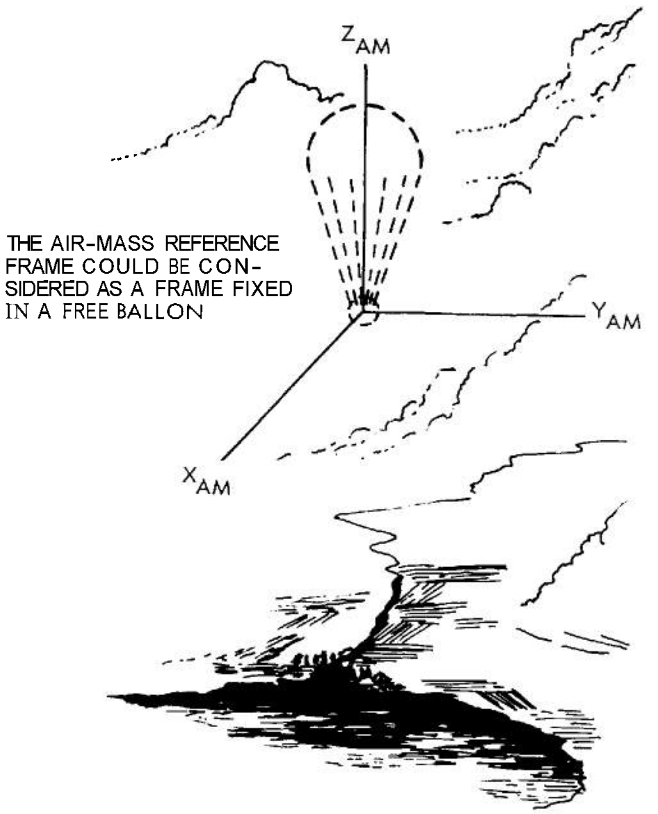


Figure 2-20. The air-mass reference frame. (Adapted from FIRE CONTROL PRINCIPLES by W. Wrigley and J. Hovorka. Copyright © 1959 by McGraw-Hill, Inc. Used by permission of McGraw-Hill Book Company.)

selection sometimes being obvious – it is not always explicitly stated by the system design agency. It is obvious that the computation reference frame selected should be one that is naturally suited to the fire control problem at hand rather than one into which the fire control solution is forced. For example, two suitable types of earth reference frames and their applications to practical antiaircraft fire control problems are discussed in paragraph 2-3.3. The applications of these same reference frames, plus another alternate reference frame, to the data-gathering function associated with sighting and ranging are discussed in paragraph 2-3.2.1.

Computation reference frames can be classified into either of the two following basic types:

1. A reference frame in which both the tracking line and the weapon line may be rotating with respect to the coordinate axes of the frame. This type of frame is generally fully stabilized geometrically with respect to the earth. The instantaneous orientations of the tracking line and the weapon line are then specified by numerical angular measurements relative to the coordinate axes of the frame.

2. A reference frame in which one of the three coordinate axes is chosen to be coincident with either the tracking line or weapon line. The line that is not aligned with one of the coordinate axes is then measured relative to the other line.

More detailed information relating to reference coordinate frames – as applicable to acquisition and tracking systems, computing systems, and weapon-pointing systems, respectively – will be given in Sections 2, 3, and 4 of the Fire Control Series.

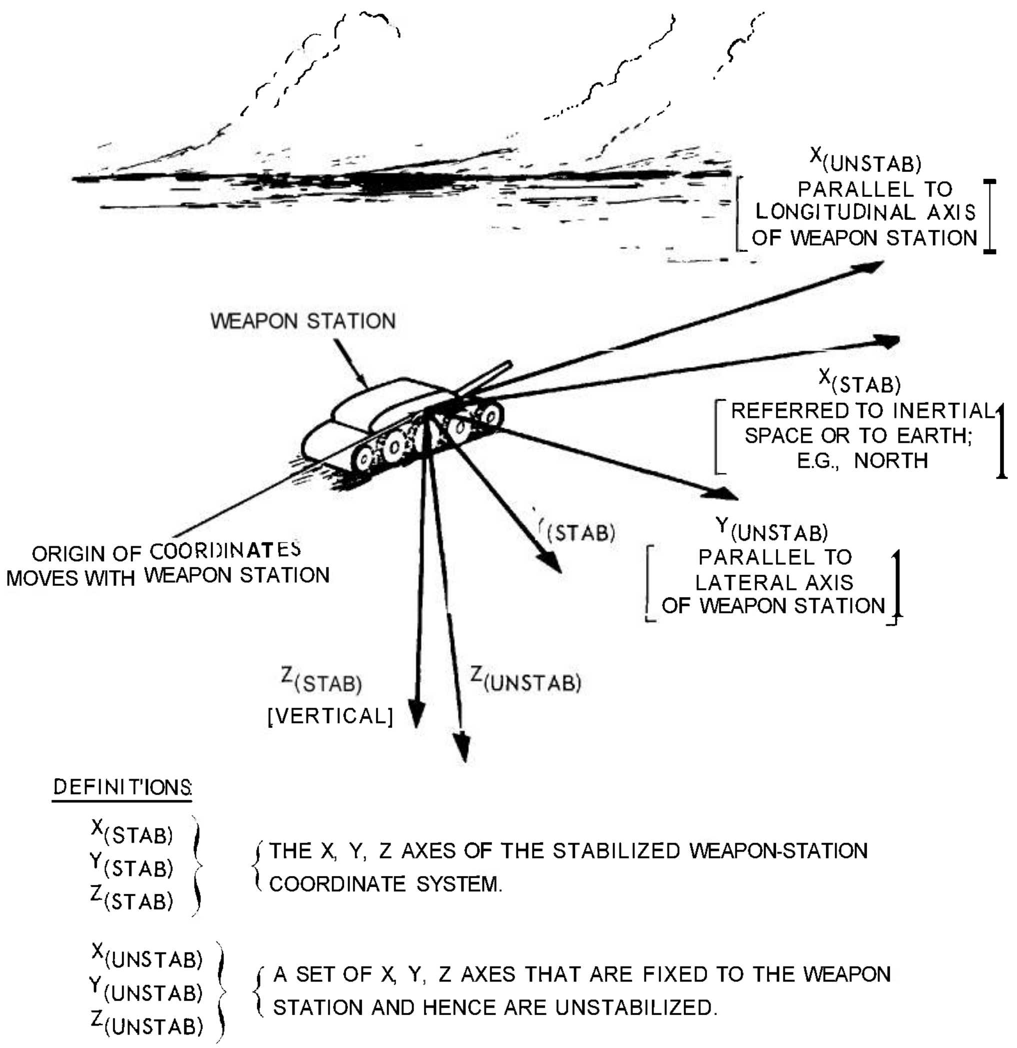


Figure 2-21. The stabilized weapon-station coordinate system. (Adapted from FIRE CONTROL PRINCIPLES by W. Wrigley and J. Hovorka. Copyright © 1959 by McGraw-Hill, Inc. Used by permission of McGraw-Hill Book Company.)

2-2.6.3 Effect of the Reference Coordinate Frame on the Prediction Angle and Its Components

In the previous discussion of the coordinate reference systems used in the solution of fire control problems, it has been noted that certain vectors required, from a geometrical approach, a frame of reference in order to be properly specified. Because error-producing effects and their assignable corrective measures – e.g., target motion and the associated kinetic lead correction – are reducible to vectors, they must be considered in relation to a specified, albeit arbitrary, reference coordinate frame. As shown by Figure 2-16, kinetic lead is the angle between the present and future lines of site. The present line of site, of course, is the direction from the weapon station to the target at the instant of firing; accordingly, it is invariant with the reference coordinate frame selected. The future line of site, on the other hand, varies with the reference coordinate frame that is chosen. (For example, the future line of site from a moving tank to a stationary target is different for a reference frame fixed to the tank than it is for a reference frame fixed to the earth.) Figure 2-22 represents the future-range vector, in relation to the

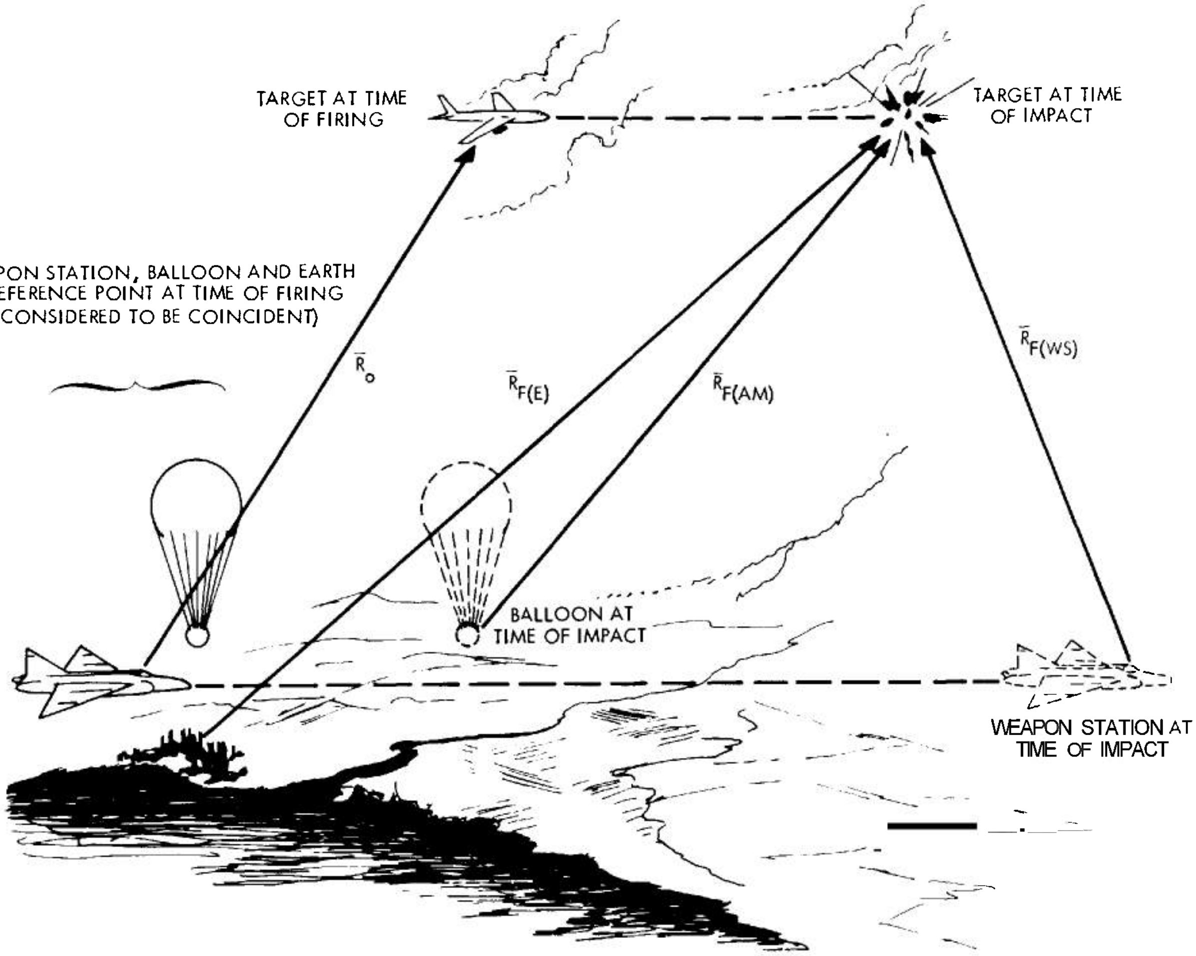


Figure 2-22. Relationships between the future-range vector and the present-range vector from the standpoint of Earth, air-mass, and weapon-station coordinates, respectively. (Adapted from FIRE CONTROL PRINCIPLES by W. Wrigley and J. Hovorka. Copyright © 1959 by McGraw-Hill, Inc. Used by permission of McGraw-Hill Book Company.)

present-range vector, as it might appear from the standpoint of earth, air-mass, and weapon-station coordinates, respectively. For the purposes of illustrating the effect of weapon-station velocity, the weapon station is depicted as a high-speed jet aircraft. Therefore, lead is also dependent upon the reference frame chosen. Similar considerations apply to the other components of the prediction angle, e.g., ballistic lead and the correction for jump.

On the other hand, the prediction angle need not be considered in relation to a specified reference coordinate frame because its definitive limits (the line of site and the weapon line) are determined by quantities that are not influenced by the selection of the reference space. For example, the weapon line is coincident with the gun bore in the case of guns; in rocket launchers, it bears a similar significance. Since the weapon line represents a physical, extensible line on the weapon, its specification is independent of the reference coordinate-frame. The line of site, of course, as already noted, is similarly invariant with the reference coordinate frame selected. Therefore, the prediction angle also remains invariant with the reference

frame selected; i.e., irrespective of the coordinate system chosen, the prediction angle is seen to be the same to the observer in any selected reference frame.

For a more detailed discussion of the effect of the reference coordinate frame and illustrative examples, see Reference 1.

2-2.7 DIFFERENCES BETWEEN FIRE CONTROL FOR GUNS AND ROCKETS

It was noted at the start of the discussion of the fire control problem that the term "projectile" was used in its general sense to include bullets, projectiles and rockets. However, from the standpoint of the geometrical approach, it becomes necessary to delineate the differences that may exist between firing bullets and projectiles on the one hand, and firing rockets on the other.

Gun fire and rocket fire are similar; the essential difference between them lies in the method of propulsion. In gun fire, the propellant and its gases are confined in the gun tube and the projectile is ejected by the pressure produced by these gases. In rocket fire, the propellant and its gases travel with the rocket during the burning of the propellant. A pseudo or fictitious initial velocity thus must be used to account for its continued propulsion after launching.

In general, bullets and projectiles are fired with a relatively high initial (muzzle) velocity; the military rocket, by contrast, has a low initial velocity when fired from a static launcher. For a given target range, this low initial velocity increases the time of flight and lessens the chances of scoring a hit on a moving target. If the rocket is fin-stabilized (in contrast to spin-stabilized bullets or projectiles), the low initial velocity results also in reduced stability during flight and, therefore, in greater dispersion.

Rocket fire tends to be less accurate than gun fire. A gun-fired projectile is usually guided very accurately along the bore during the burning time of the propellant; the turbulent action of the expanding gases behind the projectile have little effect on the path of flight. In contrast, similar turbulences developed in the rocket exhaust gases are unrestrained and free to produce variations in the direction of flight. For this reason, rockets fired from a static launcher are less accurate than gun-fired projectiles. When rockets are fired forward from high-speed aircraft, rocket-fire accuracy is greatly increased because of the high initial velocity and the aerodynamic effectiveness of the large protruding fins. This method of aircraft rocket fire effectuates long-range artillery equivalence for low-velocity, short-range rockets when used in air-to-surface weapon fire.

It should be noted in this connection that correction for jump effects apply both to gun fire and rocket fire. However, in the case of gun fire, jump phenomena result from the elastance of the weapon whereas, in the case of rocket fire, the phenomena known as weather-cocking result from the influence of the folded rocket fins on the rocket path as the rocket is fired from the aircraft launching tube into the airstream.

The rocket-assisted projectile (or, equivalently, gun-boosted rocket) is a new development in which a rocket motor is added to a projectile and the combination is fired from a gun, The result will be either an increase in range, an increase in the payload that can be carried to the same range as that obtained by the projectile alone with its normal payload, or an increase in the projectile velocity at target impact – in each case with no decrease in the mobility of the gun. The advantages and disadvantages of using the rocket-assisted projectile – particularly from the standpoint of accuracy – are discussed in Reference 7.

2-3 SOLUTION OF THE FIRE CONTROL PROBLEM

2-3.1 GENERAL

The solution of the fire control problem can be considered to comprise three distinct phases:

1. Sighting and ranging (or tracking*)
2. Computation of firing data

* As explained in Chapter 3 which describes the functional elements of fire control equipment employed in the solution of the fire control problem, the term "tracking" denotes the action of keeping target-locating equipment continuously pointed at a moving target. Sighting and ranging, on the other hand, denotes the action of determining the position and range of a stationary target with respect to the weapon.

3. Application of firing data.

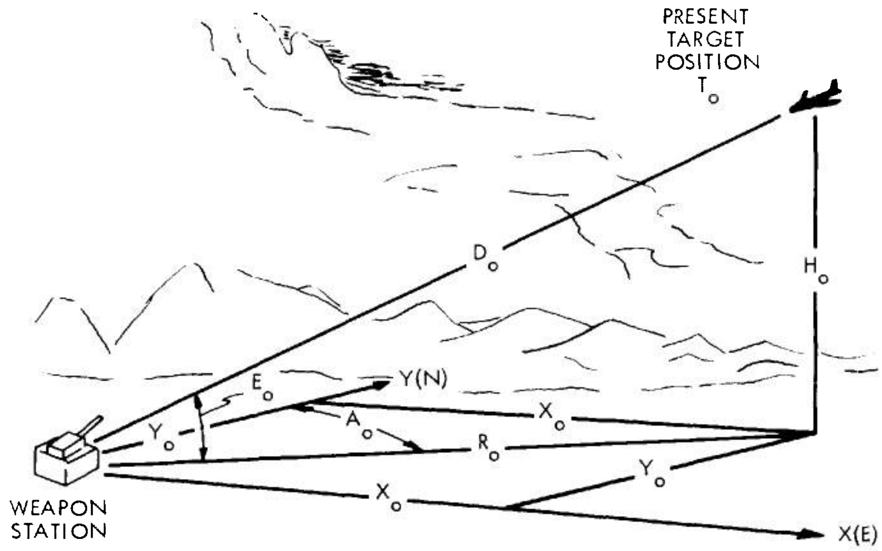
Each of these three phases is treated, in turn, in the remainder of this chapter.

2-3.2 SIGHTING AND RANGING

2-3.2.1 General

The first requirement in solving any fire control problem is to continually locate the target with respect to the weapon. This requirement is satisfied by the use of sighting and ranging procedures as described in paragraphs 2-3.2.2 and 2-3.2.3.

Target location is usually established in spherical polar coordinates in an earth reference frame. Figure 2-23 shows how the target is located with respect to the weapon by this method in a typical antiaircraft fire control problem. The line of site between weapon and target is established when the target's azimuth angle A_O and elevation angle E_O are determined. The third required element of data is the target's slant range D_O. When



DEFINITIONS:

X(E)	X AXIS OF THE XYZ REFERENCE COORDINATE FRAME (DIRECTED TOWARD GEOGRAPHIC EAST)
Y(N)	Y AXIS OF THE XYZ REFERENCE COORDINATE FRAME (DIRECTED TOWARD GEOGRAPHIC NORTH)

(NOTE THAT THE Z AXIS THAT COMPLETES THIS REFERENCE COORDINATE FRAME, ALTHOUGH NOT SHOWN IN THE ILLUSTRATION, IS DIRECTED UPWARD FROM THE ORIGIN, AT THE WEAPON STATION, IN A VERTICAL ORIENTATION)

X_o	TARGET DISTANCE COMPONENT ALONG THE X AXIS (FOR TARGET PRESENT POSITION T_o)
Y_o	TARGET DISTANCE COMPONENT ALONG THE Y AXIS (FOR TARGET PRESENT POSITION T_o)
H_o	TARGET HEIGHT ABOVE THE HORIZONTAL XY PLANE (FOR TARGET PRESENT POSITION T_o)
A_o	TARGET AZIMUTH ANGLE; MEASURED CLOCKWISE FROM TRUE NORTH, THAT IS, FROM THE Y(N) AXIS
E_o	TARGET ELEVATION ANGLE WITH RESPECT TO THE HORIZONTAL
D_o	TARGET SLANT RANGE
R_o	PROJECTION OF THE SLANT RANGE D_o, TO THE PRESENT TARGET POSITION T_o, INTO THE HORIZONTAL XY PLANE

Figure 2-23. Reference coordinate frames for locating the target with respect to the weapon station.

there is relative motion between the weapon and the target, the target must be "tracked" to determine the rates of change of these three basic elements of data — azimuth, elevation, and range — in order that proper leads may be computed.

Figure 2-23 also indicates two alternate coordinate systems (also in an earth reference frame) that are used for establishing the location of a target with respect to a weapon:

1. In the rectangular coordinate system, the mutual orthogonal distance vectors X_O, Y_O, and H_O fix the target. (Note that the positive directions for the X- and Y-axes are East and North, respectively.) The rectangular coordinate system is used in many automatic data computers, the raw data being obtained in polar coordinates and the data computer converting it either electrically or mechanically to rectangular coordinates and then solving for firing data.

2. A second alternate system uses the quantities A_O, R_O, and H_O. This system is used when the target-position data is measured from maps.

Additional methods for locating the target for particular fire control applications would be in terms of the other coordinate reference frames discussed in paragraph 2-2.6.1.

2-3.2.2 Sighting

There are two general ways of sighting on a target:*

1. The direct laying method which is associated with direct fire control (see par 1-1.3.1).

2. The indirect laying method which is associated with indirect fire control (see par 1-1.3.2).

The direct laying method is used when the target can be sensed — via optics, radar, etc. — directly from the weapon. The simplest means is to mount a front and a rear sight on the weapon, adjust their alignment so that the sight line is parallel to the axis of the bore (the weapon line), and then move the weapon in elevation and azimuth until the sights are aligned with the target. For rifle fire, the range would, of course, be estimated and set on the sights before actually sighting on the target. For larger caliber guns, for which various forms of optical sights might be used (see pars 1-2.4.3 through 1-2.4.3.5 of Chapter 1), the sighting would be maintained during the period that the sight was being adjusted for the actual conditions of the fire-control situation, i.e., the target range and the angle of site.

The indirect laying method is used either when the target cannot be sensed directly from the weapon or when remote control is employed. This method requires that the azimuth and angular elevations of the line of site be determined by some independent means such as map data or a remote observation post. If the weapon is equipped with calibrated and oriented angle-measuring devices similar to those on a surveyor's transit, it can be laid on the target's azimuth and angular elevation, and the weapon line extended would intersect the target.

From the foregoing, it can be seen that direct sighting is the simplest since it is only necessary that the sights be capable of being aligned with the weapon line. In the indirect sighting method, the sights must not only be capable of being aligned with the weapon line but they must also be capable of being leveled and oriented on the same reference — grid north, magnetic north, an aiming stake, the longitudinal axis of an aircraft, etc. — as that on which the target-angle data were based. The latter sighting system is obviously more complex, and subject to error and time lag in functioning. However, it is more flexible and capable of engaging unseen as well as visible targets.

As discussed in Chapter 1, the means for sighting on a target include simple mechanical sights, various types of optical sights, radar, active infrared viewing devices, and various types of passive night sighting equipment. Radar is particularly well suited for tracking moving targets — where ground clutter is not a problem — because automatic tracking capability can be readily designed into the tracking equipment.

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* It should be noted that, in general, sighting on a target (locating the target with respect to the weapon) differs from aiming the weapon, which (see par 2-3.3.4) can take place only after the target has been located (and tracked, if moving) and firing data has been computed.

2-3.2.3 Ranging

Target ranges are sometimes obtained by visual estimation of range as, for example, in the case of rifle fire or machine gun fire. Although rangefinding for tank fire once also depended on visual estimation, it now utilizes more sophisticated range-finding techniques. As discussed in paragraph 1-3.2, these techniques include the use of optical range finders of both the coincidence type and the stereoscopic type. In addition, promising effort is now being directed toward the development of laser range finders.

For field-artillery fire, optical range finders remain the primary means of ranging, with conventional spotting techniques used to correct for inaccuracies of fire. Map data are also used when applicable to the fire control situation at hand. Laser range finders, however, are also being developed for the ranging required with field-artillery fire control.

For antiaircraft-artillery fire, radar ranging is the accepted method of ranging, particularly because of the automatic-tracking capability of radar.

2.3,3 COMPUTATION OF FIRING DATA

With the target's position data known (or with tracking data available in the case of a moving target), the next step is to solve the complete fire control problem involved, utilizing the weapon's known ballistic performance data and correcting the standard ballistic data to allow for non-standard meteorological, ammunition, or weapon conditions. The objectives are firing azimuth, firing elevation (or their equivalents in whatever coordinate system is used, see par 2-3.2.1), and, when applicable, time of flight. Four general cases exist:::'

1. Weapon and target both stationary
2. Weapon stationary and target moving
3. Weapon moving and target stationary
4. Weapon and target both moving.

2-3.3.1 Weapon and Target Both Stationary

In 'this case, only the present-position data of the target need be known and the solution of the ballistic problem is relatively uncomplicated. A firing table or – in the case of certain short-range weapons such as a rifle, a mortar or a tank gun – calibrated sights (see Fig. 2-24) are all that are necessary to provide the necessary firing data.

2-3.3.2 Weapon Stationary and Target Moving

Here, as in duck shooting, it is necessary to "lead" the target and the present-position data is used to determine the future position of the target based upon the rates of change of present-position data. In short-range, direct-fire weapons, kinetic lead is frequently estimated as a function of range (and hence time of flight) and target speed. Figure 2-24 shows the sight reticle pattern used with a tank gun where one lead line gives a 5-mil lead.

For more-accurate weapon fire, there are two general types of prediction processes that can be used by computers for determining kinetic lead:

1. The angular-rate-of-travel method
2. The linear-speed method.

The angular-rate-of-travel method gives the fastest solution. In Figure 2-25, it can be seen that if the weapon is fired when the target is at point T_o, by the time the projectile arrives there, the target will be at T_p, the predicted future target position. Considering azimuth only, if the time of flight to the predicted future target position T_p is known (this time of flight is designated t_p), then the product $t_p \times \frac{dA_o}{dt}$ will approximate the necessary kinetic lead correction. The computer obtains t_p as a function of present-position data (i.e., A_o, D_o, and E_o), measures the rate of change of A_o by measuring the angular rate of tracking in azimuth, and multiplies them to obtain the azimuth component of kinetic lead. Using stored ballistic data, the computer then adds the necessary drift and windage corrections, and arrives at A_f, the firing azimuth. A similar process that uses elevation-tracking-rate data and adds a correction for the effect of gravity obtains

* For specific examples, see Part III of Section 3 of the Fire Control Series.

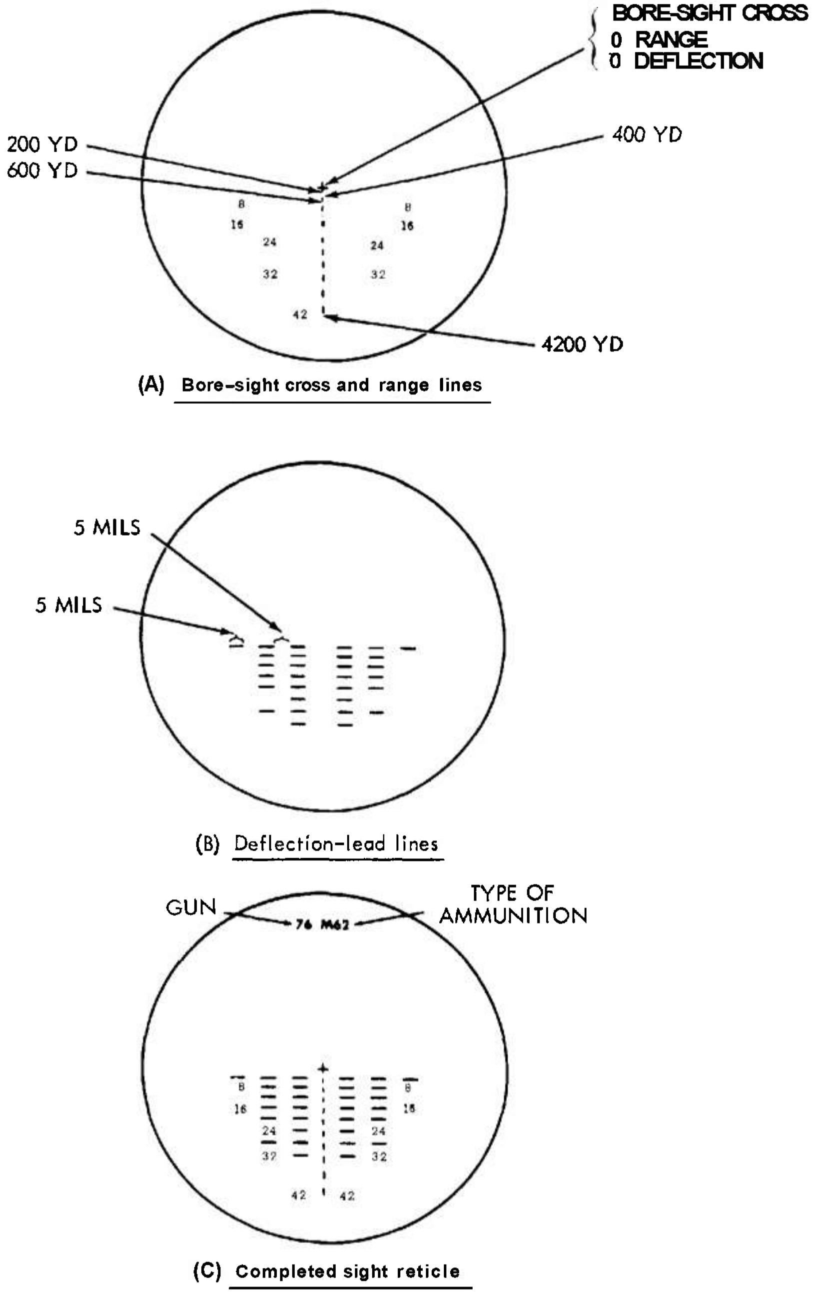
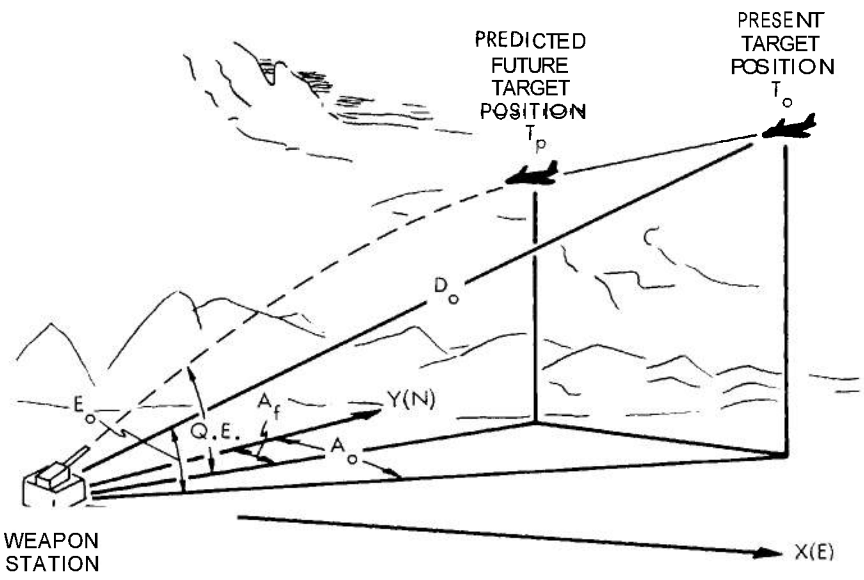


Figure 2-24. A calibrated sight such as used on a rifle, a mortar, or a tank gun.

Q. E., the quadrant elevation. Inasmuch as $\frac{dA_o}{dt}$ and $\frac{dE_o}{dt}$ are seldom constant and t_p is not equal to t_o, the time of flight to the present target position, it can be seen that this solution is only an approximation. It is suitable for short-range fire with automatic weapons against high-speed targets, being rapid and relatively simple in the mechanisms required. The volume and dispersion pattern of automatic weapons fire compensate for the errors resulting from approxi-



DEFINITIONS:

X(E)	X AXIS OF THE XYZ REFERENCE COORDINATE FRAME (DIRECTED TOWARD GEOGRAPHIC EAST)
Y(N)	Y AXIS OF THE XYZ REFERENCE COORDINATE FRAME (DIRECTED TOWARD GEOGRAPHIC NORTH)

(NOTE THAT THE Z AXIS THAT COMPLETES THIS REFERENCE COORDINATE FRAME, ALTHOUGH NOT SHOWN IN THE ILLUSTRATION, IS DIRECTED UPWARD FROM THE ORIGIN, AT THE WEAPON STATION, IN A VERTICAL ORIENTATION)

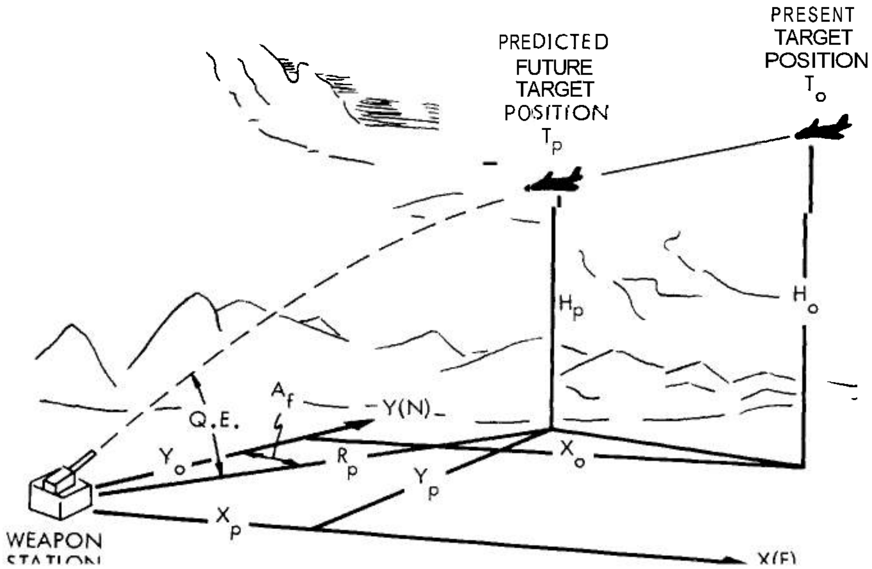
A_o	TARGET AZIMUTH ANGLE; MEASURED CLOCKWISE FROM TRUE NORTH, THAT IS, FROM THE Y(N) AXIS
E_o	TARGET ELEVATION ANGLE WITH RESPECT TO THE HORIZONTAL
D_o	TARGET SLANT RANGE
A_f	FIRING AZIMUTH; MEASURED CLOCKWISE FROM TRUE NORTH
Q.E.	QUADRANT ELEVATION

Figure 2-25. A stationary weapon firing at a moving target, using the angular-rate-of-travel method of prediction.

mation of angular rates.

The linear-speed method is more exact in its solution but is slower and requires more intricate, cumbersome equipment. In the linear-speed method, the computer converts A_o, E_o, and D_o, which are supplied as input data from the tracking system, to X_o, Y_o, and H_o (see Fig. 2-26).* The computer then takes the first derivative of these values with respect to time $\frac{dX_o}{dt}$, $\frac{dY_o}{dt}$ and $\frac{dH_o}{dt}$ and, by multiplying them by the time of flight the projectile, obtains future position data, X_p, Y_p, and H_p. The time of flight used is the actual time of flight to the future target position t_p, which the computer obtains by a successive-approximation method. Using stored ballistic data, the computer then corrects for gravity,

* See, for example, the similar computation described in paragraph 1-2.4.8.3 in connection with early mechanical directors.



DEFINITIONS:

X(E)	X AXIS OF THE XYZ REFERENCE COORDINATE FRAME (DIRECTED TOWARD GEOGRAPHIC EAST)
Y(N)	Y AXIS OF THE XYZ REFERENCE COORDINATE FRAME (DIRECTED TOWARD GEOGRAPHIC NORTH)

(NOTE THAT THE Z AXIS THAT COMPLETES THIS REFERENCE COORDINATE FRAME, ALTHOUGH NOT SHOWN IN THE ILLUSTRATION, IS DIRECTED UPWARD FROM THE ORIGIN, AT THE WEAPON STATION, IN A VERTICAL ORIENTATION)

X_o	TARGET DISTANCE COMPONENT ALONG THE X AXIS	FOR TARGET PRESENT POSITION
Y_o	TARGET DISTANCE COMPONENT ALONG THE Y AXIS	
H_o	TARGET HEIGHT ABOVE THE HORIZONTAL XY PLANE	
X_p	TARGET DISTANCE COMPONENT ALONG THE X AXIS	FOR TARGET FUTURE POSITION TP
Y_p	TARGET DISTANCE COMPONENT ALONG THE Y AXIS	
H_o	TARGET HEIGHT ABOVE THE HORIZONTAL XY PLANE	
R_p	PROJECTION OF THE SLANT RANGE TO THE FUTURE TARGET POSITION T_p ONTO THE HORIZONTAL XY PLANE	
A_f	FIRING AZIMUTH, MEASURED CLOCKWISE FROM TRUE NORTH	
Q.E.	QUADRANT ELEVATION	

Figure 2-26. A stationary weapon firing at a moving target, using the linear-speed method of prediction.

drift, wind and other meteorological and ballistic factors; delivers A_f and Q. E.; and, where necessary, fuze setting. The accuracy of the linear-speed method is dependent upon the target maintaining a constant course and speed. It finds application with antiaircraft guns and guided missiles.

2-3.3.3 Weapon Moving and Target Stationary

This is the complement of the preceding case. The problem is similar to that of precision bombing or aircraft gunnery against stationary or near-stationary ground targets,

where negative kinetic lead angles are required. The angular-rate-of-travel and linear-speed prediction methods are applicable for this case as well but must be used in modified form.

2-3.3.4 Weapon and Target Both Moving

This is the most complex of the four general cases. However, it is very similar to the preceding case of "weapon stationary, target moving" in that it involves relative motion between target and weapon. This fourth case would apply to weapon fire at moving targets from such Army vehicles as helicopters and moving tanks. For this case, the fire control problem is generally solved by angular-rate-of-travel prediction for high speeds. For lower speeds, linear-speed prediction can usually be utilized.

2-3.4 APPLICATION OF FIRING DATA

Having located the target and computed the firing data, it is next necessary to aim the weapons accordingly. For some weapons, this function is performed by the weapon's sighting system. Sights are essentially angle-measuring devices, calibrated for the ballistics of the weapon and ammunition with which they are used. Sights are classified as either optical (glass sights) or mechanical (iron sights). Figure 2-27 shows a simple elevation sighting arrangement and its application to laying a weapon in elevation. The parallax can be ignored in most weapons, since it is usually merely a matter of a few inches, but it can be reduced by having the sight axis depressed to converge with the gun axis at some convenient range. (For a complete discussion of the parallax problem and its solu-

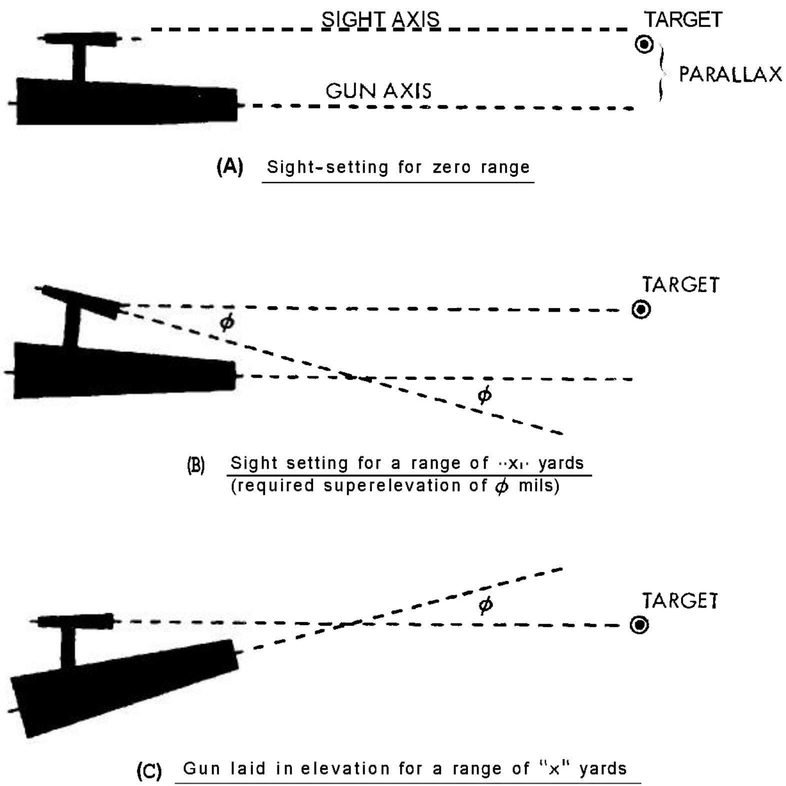


Figure 2-27. The use of a simple sighting arrangement for laying a weapon in elevation.

tion, see Reference 13, which forms part of the Fire Control Series.)

Assumethat it is desired to hit a target at a range of "x" yards and that the firing table shows that a superelevation of ϕ mils is required to compensate for the effect of gravity on the projectile during its time of flight. If the sight axis is depressed ϕ mils below the gun axis, then elevating the gun until the sight is back on the target will place the gun axis on the proper superelevation angle ϕ with respect to the horizontal plane. By properly calibrating the elevation sighting controls interms of yards of range rather than mils, it is possible to eliminate the firirig-table steps, as is done in the cases of the M1 Rifle, tank guns, etc. (see Fig. 2-24). This is the procedure usually employed with direct-fire weapons. Sights forindirect fire weapons, on the other hand, such as mortars and howitzers, are usually calibrated in angularunits and, instead of usingthe line of site to the target as a reference, refer to some arbitrary aiming point, See paragraphs 1-2.4.2 through 1-2.4.3.5 of Chapter 1 for an overall summary of sighting equipment that has been developed during the 20th century.

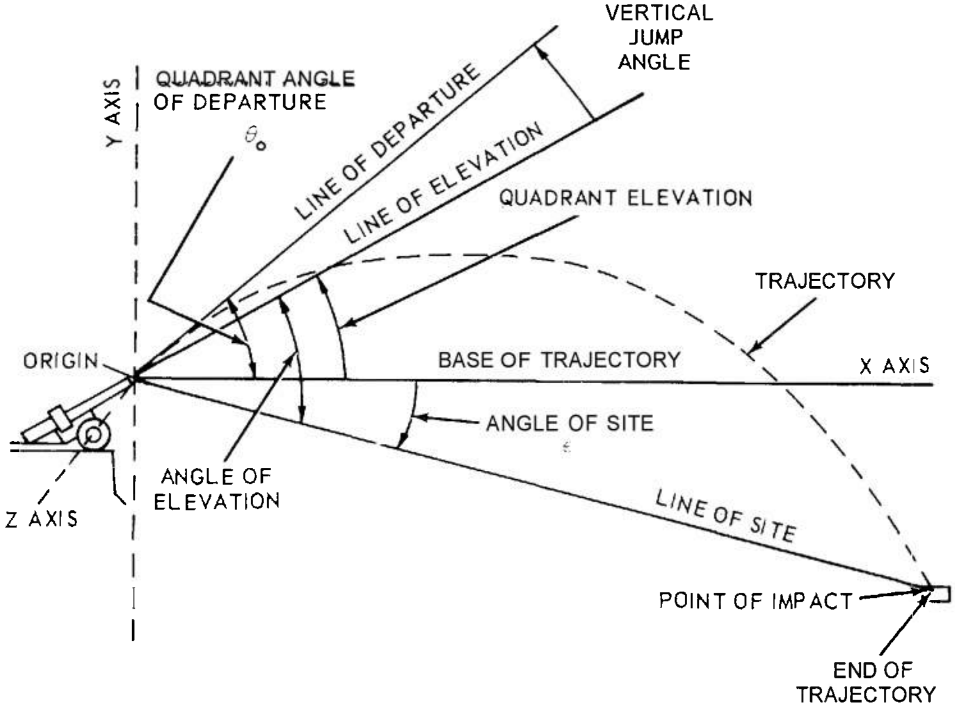
Figure 2-24 shows the type of reticle patternthat would be found in a tank-gun telescopic sight. The range reticles are marked in hundreds of yards; therefore, by placing the 800-yard reticle on the target image, the gun becomes elevated to the proper superelevation necessary to carry that distance. Similarly, drift, windage, and kinetic lead corrections for moving targets are applied by aligning the target on the proper horizontal lead line. The same sight may thus be used for both azimuth and elevation, although on larger caliber weapons separate sights may be used for functional ease. Again, instead of using the target for an aiming point, an arbitrary aiming point such as north, a terrain feature, or the like may be used in indirect firing. In all cases, the basic requirement is that the sighting system be oriented on the same references used to compute firing data, and that it be capable of positioning the weapon on the proper horizontal and vertical angles, namely, the firing azimuth A_f and the quadrant elevation Q. E.

Many weapons, particularly aircraft turrets and antiaircraft guns, are positioned by remote control. Their sightingand computing equipment is located remotely from the weapon and the firing data are transmitted, usually electrically, to the guns. Synchro electrical systems are most commonly used for this purpose. At the gun, servomechanisms employ this electrically-transmitted firing data to position the gun. Remote-control systems offer the advantages of smoother, more-accurate tracking rates against high-speed targets. They also permit mounting weapons in locations where optimum fields of fire may be obtained but which could not be utilized, because of either space or vulnerability considerations, if the gunner had to be located there. Theirdisadvantages are mainly in their complex operating machinery and the need for a power supply.

Note: As indicated in the introductionto this chapter (see par 2-1), the broad scope of the preceding discussion concerning the solution of the fire control problem is intended to supplement the more-detailed information presented in subsequent chapters of Section 1 and in Sections 2, 3, and 4 of the Fire Control Series.

Appendix 2-1. Summary of the mathematics associated with the exterior ballistics of a projectile.

Elements of a Trajectory



Definitions Associated with a Trajectory

Trajectory -- the curve in space traced by the center of gravity of a projectile in its flight through the air.

Origin of Trajectory -- the position of the center of gravity of the projectile at the instant it is released by the projecting mechanism.

End of Trajectory -- the position of the center of gravity of the projectile when it bursts or encounters some medium other than air.

Line of Departure -- the tangent to the trajectory at its origin.

Quadrant Angle of Departure -- the angle that the line of departure makes with the horizontal.

Plane of Departure -- the vertical plane that includes the line of departure. This plane is also known as the plane of fire. In this plane lie the X (horizontal) and Y (vertical) axes of the coordinate system used in the computation of trajectories; the Z axis lies in the horizontal plane and is perpendicular to the plane of departure.

Line of Elevation -- the extension of the bore axis of the gun.

Vertical Jump Angle -- the angle between the line of elevation and the line of departure of the trajectory. (This angle is included in the diagram for reference purposes only, to remind the reader that the projectile departs from the weapon along a line that differs from the bore axis of the weapon as a result of jump. As indicated by the mathematical development contained in this table, jump is not accounted for in the differential equations of projectile motion.)

Angle of Elevation -- the angle between the line of site and the line of elevation.

Quadrant Elevation -- the angle of the line of elevation with respect to the horizontal.*

Angle of Site -- the angle of the line of site with respect to the horizontal.*

Mathematical Description of a Trajectory

A trajectory can be completely described by specifying the instantaneous x, y, and z coordinates of the projectile's center of gravity in the X, Y, Z coordinate system at any time t after release of the projectile by the projecting mechanism. The trajectory starts at the muzzle of the gun, which is the origin of the X, Y, Z coordinate system. At t = 0, as the projectile leaves the muzzle along the line of departure, the x, y and z coordinates of the projectile are zero; i.e., $x_o = y_o = z_o = 0$. From that point on, the x, y and z coordinates are influenced by the earth's gravitational field and the aerodynamic forces acting on the projectile as it passes through the air in accordance with the following mathematical relationships (which are all based on the relationships that the net force acting on a body is equal to the product of the mass of that body and its acceleration):

$$F_x = m \frac{d^2 x}{dt^2} = m \ddot{x} = -D_x + L_x \tag{1}$$

$$F_y = m \frac{d^2 y}{dt^2} = m \ddot{y} = -D_y + L_y - mg \tag{2}$$

$$F_z = m \frac{d^2 z}{dt^2} = m \ddot{z} = -D_z + L_z \tag{3}$$

where, as shown in the accompanying sketch,

$\ddot{x}, \ddot{y}, \ddot{z}$ = components of projectile acceleration along the X, Y, and Z directions

F_x, F_y, F_z = components of the total force acting on the projectile along the X, Y, and Z directions

D_x, D_y, D_z = components of drag acting on the projectile along the X, Y, and Z directions

L_x, L_y, L_z = components of the crosswind force acting on the projectile along the X, Y, and Z directions

m = mass of the projectile

$= \frac{W}{g}$

g = acceleration due to gravity

W = weight of the projectile

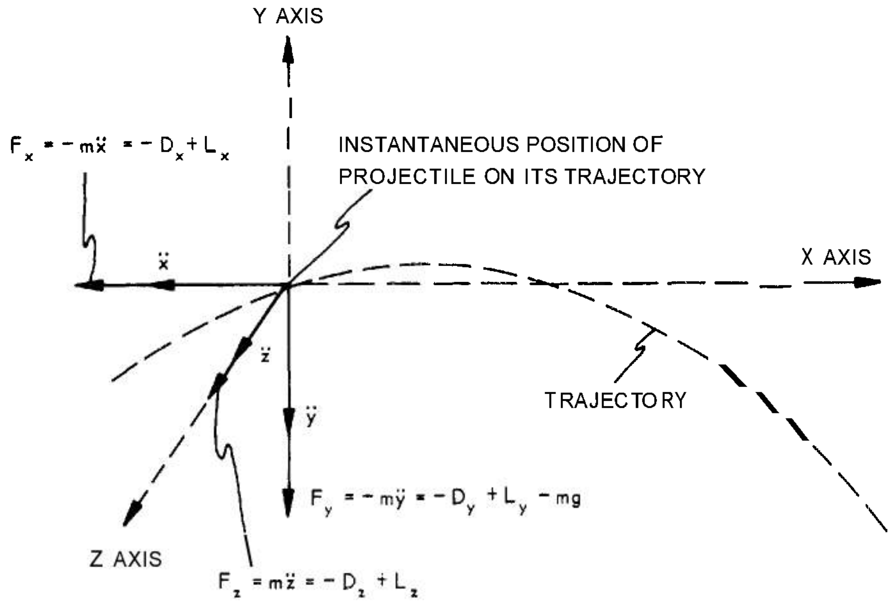
x, y, z = coordinates along the X, Y, and Z directions at any time t.

For a given projectile shape, empirical relationships for the drag D and the crosswind force L acting on the projectile can be expressed as follows:

$$D = K_D \rho d^2 u^2 \tag{4}$$

$$[illegible] = [illegible]\ pd^2 u^2 \sin \delta \tag{5}$$

* The horizontal plane provides a convenient reference plane from which to measure vertical angles and is used as the basis of application of the various types of quadrant-laying devices discussed in Chapter 1.



vhere, in a consistent set of units,

d = diameter of projectile, meter

p = density of air, g/meter^3

δ = angle of yaw, degrees or rad

u = projectile velocity relative to air, meter/sec

K_D = drag coefficient, dimensionless*

K_L = crosswind force coefficient, dimensionless:::

Both K_D and K_L are functions of $\rho ud/\mu$ (the Reynolds number), u/a (the Mach number), and), where

a = speed of sound in air, meter/sec

μ = viscosity of air, g/meter-sec

Inasmuch as K_D and K_L also vary with projectile shape and position of the center of gravity, extensive experiments are conducted at proving grounds to determine the effect of these variables on K_D and K_L.

The complete set of differential equations describing projectile motion, given by solvings Eqs. 1, 2 and 3 for $\ddot{x}$, $\ddot{y}$, and $\ddot{z}$, is as follows:

$$x = \frac{-D_x + L_x}{m} \tag{6}$$

$$\ddot{y} = \frac{-D_y + L_y}{m} - g \tag{7}$$

$$z = \frac{-D_z + L_z}{m} \tag{8}$$

These coefficients in the ballistic system (which are usually denoted by the letter K with an appropriate subscript) can be converted into the corresponding C-notation aerodynamic coefficient slopes of National Advisory Committee on Aeronautics (N. A. C. A.) terminology (or directly into those coefficients that are not functions of yaw) by multiplying the ballistic-system coefficient by $8/\pi$. For a more-detailed discussion of this matter, together with specific examples of usage and variations in usage, see Reference 7 which employs the N. A. C. A. notation, i. e., the letter C with appropriate subscripts.

where the x, y, and z components of D and L can be determined by the application of appropriate direction cosines to the drag and lift expressions given by Eqs. 4 and 5.

During past centuries, hand-computational methods of solving differential Equations 6 through 8 for explicit functions of x, y, and z have resulted in extremely lengthy calculations. Accordingly, the solution has been traditionally handled by such approximate methods as the Siacci method, a discussion of which can be found in any standard text book on ballistics and by such more exact but laborious methods as the short-arc method and the numerical integration method[5]. The development of high-speed digital computers*, however, has now progressed to a state where a different numerical-integration approach to the trajectory problem can be economically employed.

The differential equations of motion that are most commonly employed by high-speed computers are based on the Point-Mass theory and are exemplified by the following set:

$$x = -E(x - w_x) + \lambda_1 \dot{y} \tag{9}$$

$$\ddot{y} = -E\dot{y} - g - \lambda_1 \dot{x} \tag{10}$$

$$\ddot{z} = -E(\dot{z} - w_z) + \lambda_3 \dot{y} + \lambda_2 \dot{x} \tag{11}$$

where, in a consistent set of units:†

w_x, w_z = range-wind and cross-wind components of wind velocity in meters per second after conversion from knots, the units in which wind velocity is usually measured (the wind velocity is assumed to be horizontal, so that the y component is negligible).

λ_1, λ_2, λ_3 = components of the earth's angular velocity in radians per second (these components vary in accordance with the geographical latitude of the gun position and the azimuth of the gun; the λ product terms are included to account for the coriolis force due to the earth's rotation).

E = resistive function of the form $\dfrac{\rho u K_D(M)}{C}$

g = $g_o(1-2y/r)$, meter/sec^2

go = constant = 9.80665 meter/sec^2

Y = altitude above the earth's surface, meter

r = earth's radius, meter

C = $\dfrac{W}{id^2} = \dfrac{mg}{id^2}$, g/meter^2

= a ballistic coefficient that indicates the relative air resistance of the projectile (the larger the value of C, the less the retardation due to air resistance); it is expedient to use slightly different values for C for different sections of the trajectory

W = weight of the projectile, g

d = diameter of the projectile, meter

i = a dimensionless empirical factor, called the "form factor", that compares the drag coefficient of the particular projectile under consideration, at a given velocity, with that of an arbitrary standard at the same velocity.

$K_D(M)$ = dimensionless drag coefficient, that varies as a function of the Mach number M of the projectile

* Begun during World War II in connection with the trajectory problem; see Par 1-2.4.9.

† Any system of units can be used, of course, provided they are consistent. Since the metric system is becoming the common system, it has been used in the present example. Because of the magnitude involved, the metric unit of distance employed is the meter, rather than the centimeter of the conventional cgs system.

M = Mach number of projectile = $\frac{u}{a}$

ρ = absolute density, g/meter^3; varies with altitude in accordance with the atmospheric standard chosen. For the Ordnance Standard Atmosphere $\rho = \rho_0 e^{hy}$, where ρ_0 and h are constants, y is altitude and e is the base of natural logarithms = 2.7183. The I. C. A. O. Standard Atmosphere is the current standard for U. S. and NATO. use. (See U. S. Extension to I. C. A. O. Standard Atmosphere, published by the U. S. Dept. of Commerce Weather Bureau, Washington, D. C., 1958.)

u = projectile velocity relative to air, meter/sec

a = speed of sound in air, meter/sec

It is apparent that the equations of motion given by Eqs. 9 through 11 can account for the effects of both range wind and crosswind and the Coriolis force due to the earth's rotation, in addition to the effects of gravity and aerodynamic drag forces.

For a discussion of how Eqs. 9 through 11 are employed in the production of firing tables for fire-control purposes, see Reference 3. Figure 3-11 therein provides a flow chart for the computation of firing tables.

Firing tables supply the necessary data for the correct aiming of weapons. Since firing tables apply to particular projectiles, it is necessary that test firings be obtained for each round in order to obtain applicable values of the ballistic coefficient C. For example, the ballistic reductions of the range firing data on the projectile, HE, M106 (used for the 8-inch Howitzer M2, M2A1 and M47) conducted at Aberdeen Proving Ground, Maryland produced the following ballistic coefficients which were used in the computation of the firing tables:[14]

Elevation (mils)	Charge 1-4, 6 and 7	Charge 5
0-800	3.255	3,323
900	3.229	3.297
1000	3.153	3.22 1
1100	3.025	3.093
1200	2.846	2.914

Firing tables are predicated on standard conditions (arbitrarily chosen conditions of weather, location, and material) that, individually, are physically possible. Wherever practicable, corrections to be applied for variations from these standard conditions are tabulated. For example, Firing Tables FT8-J-2 previously referenced provide the following corrections:

1. Corrections to azimuth to compensate for drift. (Although a standard trajectory has drift, it is assumed, for simplicity, that drift is a deflection effect; any condition causing the shell to depart from the plane of fire is considered as a deflection effect.)
2. Corrections to azimuth to correct for crosswind.
3. Corrections to elevation to correct for change in projectile weight from the standard weight.
4. Corrections to elevation to compensate for an increase or decrease in muzzle velocity from the standard velocity.
5. Corrections to elevation to compensate for an increase or decrease in ballistic air Temperature from the standard temperature.
6. Corrections in elevation to correct for ballistic head and tail winds.
7. Corrections in elevation to Compensate for an increase or decrease in ballistic air density.
8. Corrections in elevation to compensate for the rotation of the earth.

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* Identical with Volume I of a comprehensive, two-volume work on fire control theory and design titled ~~Encyclopedia of Fire Control~~ that was prepared by the Massachusetts Institute of Technology for the Air Research and Development Command, USAF.

CHAPTER 3

FUNCTIONAL ELEMENTS OF FIRE CONTROL EQUIPMENT EMPLOYED IN THE SOLUTION OF THE FIRE CONTROL PROBLEM

3-1 INTRODUCTION

Fire control equipment is any equipment used to assist in fire control operations, that is, operations concerned with the solution of fire control problems. As discussed in Chapter 1, such equipment is sometimes classified according to its physical location with respect to the weapon as "on-carriage" or "off-carriage" equipment, where the word "carriage" refers to the weapon and its mount. Some weapons have sufficient on-carriage fire control equipment to aim them, but the position-finding and data-computation phases of fire control operations are performed by off-carriage equipment. When such a fire control system is considered in its entirety, however, it is referred to as an off-carriage fire control system. Some weapons, on the other hand, have all (or substantially all) their fire control equipment on carriage. This is the case for some aircraft gun turrets, certain medium-caliber antiaircraft weapons (see Fig. 3-1, for example*), and such direct-fire weapons as tank and antitank weapons. Such systems are known as on-carriage fire control systems. On-carriage fire control equipment is usually specialized in construction; that is, any one item of equipment can usually be employed with only a particular weapon. Off-carriage fire control equipment, on the other hand, can generally be used with several different weapons.

Fire control equipment can also be classified in accordance with the particular function it is designed to perform in the overall fire control system. It is with this type of classification of fire control equipment that Chapter 3 is primarily concerned.

The discussion utilizes reference to so-called functional diagrams. These diagrams, which are also known as block diagrams or data flow diagrams, can be used to represent in graphic form operating systems of any complexity. They have the advantage of readily indicating (1) the major subsystems and components of the system or equipment under consideration and (2) the signal-flow paths. By convention, the main direction of signal flow through the system from input to output is usually drawn from left to right. For a detailed discussion of the various types of functional diagrams, see Reference 1.

The following paragraphs of this chapter describe and illustrate the various types of functional elements found in fire control equipment. Next, there are discussions of the following related topics:

1. Factors associated with the integration of functional elements into fire control systems.
2. Compatibility problems associated with various types of operating elements.

The chapter concludes with examples of how the various types of functional elements combine to form particular types of fire control systems.

It should be noted that the information presented in this chapter is of a background nature. For details on particular aspects of fire control equipment, the reader should consult the sections of the Fire Control Series referenced in par 3-2.

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* The examples employed in this chapter have been selected solely to illustrate the various kinds of fire control equipment discussed; no attempt has been made to illustrate the most up-to-date equipment.

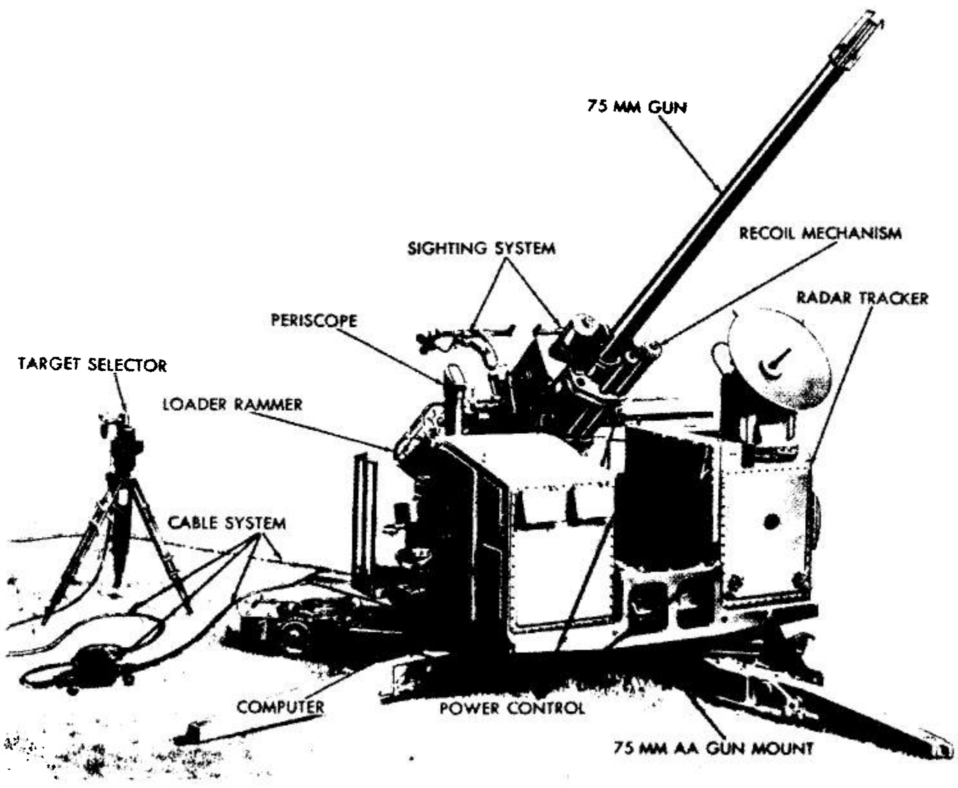


Figure 3-1. The Skysweeper antiaircraft weapon system, which employs an essentially on-carriage fire control system.

3-2 TYPES OF FUNCTIONAL ELEMENTS EMPLOYED IN FIRE CONTROL SYSTEMS

The functional elements into which the most complex fire control system conceivable can be divided are considered to be as follows:

1. An acquisition element.
2. A tracking element.
3. A ballistic-data element.
4. A predicting element.
5. An arbitrary correction element.
6. A compensating element.
7. A pointing element.
8. Data-transmitting elements.
9. A fuze-setting element.
10. A command element.

It should be noted that as the complexity of fire control systems decreases, so usually does the number of functional elements, In the simple case of small arms, for example, the functional elements have all but disappeared, as far as their being representative of actual equipment is concerned. All that exists in the form of fire control equipment is a set of sights, which can be reasonably conceived, from the functional viewpoint, as a combined tracking and pointing element (see par 3-3.2). All other functional elements required are incorporated in the human being who is firing the weapon.

The functional arrangement of these various types of functional elements to form a complete fire control system is shown in Fig. 3-2. As indicated by this figure, certain functional elements can be logically grouped together to form three functional subsystems of the complete fire control system, while other functional elements serve as connecting elements for these subsystems.

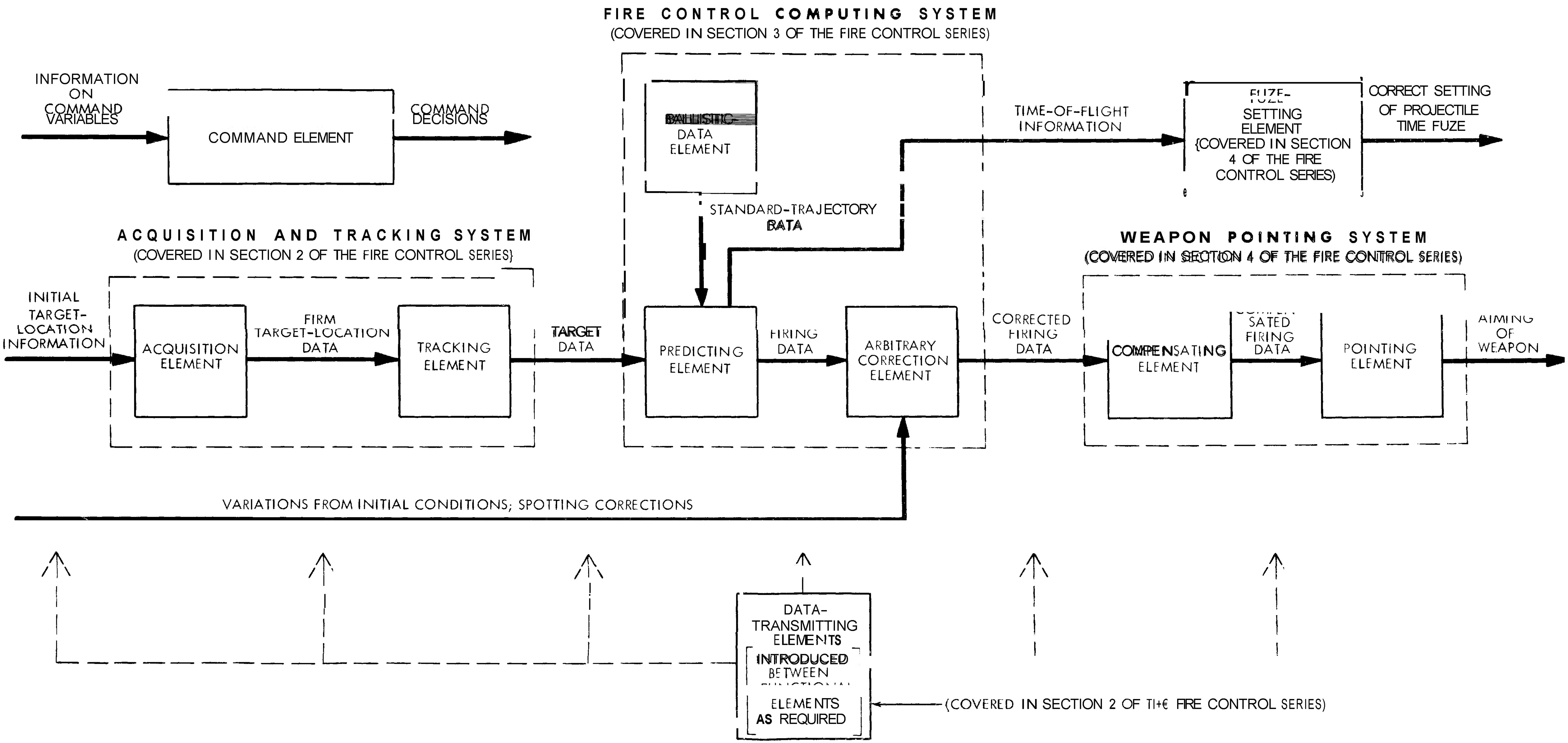


Figure 3-2. Functional diagram of a hypothetical fire control system that contains all of the functional elements associated with fire control equipment.

The three functional subsystems are the following:

1. The acquisition and tracking system. (This subsystem is covered in detail in Section 2 of the Fire Control Series.)
2. The fire control computing system. (This subsystem is covered in detail in Section 3 of the Fire Control Series.)
3. The weapon-pointing system. (This subsystem is covered in detail in Section 4 of the Fire Control Series.)

The first system noted encompasses all equipment used for observing and determining the position of the target, and tracking the target if either it or the weapon is in motion. The second system pertains to all data-computation equipment. The third system relates to all equipment used in the application of firing data to the weapon itself.

3-2.1 ACQUISITION ELEMENT

The basic function of this element of a fire control system is to acquire the target (i.e., to detect its presence by various means and maintain the capability of continued observation) and provide initial information on its position. Related functions are to identify the target's nature (for example, size and shape) and whether it is hostile or friendly (via appropriate IFF equipment).

A typical example of an acquisition element is the acquisition radar used in the type of fire control system that forms an integral part of certain antiaircraft-artillery weapon systems. This acquisition radar works in conjunction with a surveillance radar that is considered to lie outside the bounds of the weapon system proper and, hence, does not comprise an element of the fire control system. The surveillance radar has the functions of maintaining a continuous air watch over an area of land or water of primary significance to the antiaircraft defenses. It supplies to the antiaircraft artillery defense pertinent information on all aerial targets, with sufficient accuracy to localize them to a degree that will permit transference to other more-accurate radars of the antiaircraft defenses and at a sufficiently long range to enable the outermost firing elements to engage the targets at maximum range. The acquisition radar is a radar of shorter range but of greater accuracy than that of the surveillance radar. Its normal function is to acquire targets on direction from a surveillance radar (or by independent search under certain circumstances) and to transfer these targets to the tracking radar.

The acquisition element is usually mechanized in a fire control system as part of the acquisition and tracking system (see Fig. 3-2). Design information relating to this type of element therefore appears in Section 2 of the Fire Control Series.

3-2.2 TRACKING ELEMENT

The basic function of this element of a fire control system is to continuously track the target once it has been acquired and the tracking equipment has been locked onto the target, and to generate tracking data that represents the position (range, elevation and azimuth), the relative speed, and the direction of relative motion of the target with respect to the weapon.

For the situation in which there is no significant relative motion between target and weapon, the term "sighting and ranging" is more applicable than the term "tracking". Tracking denotes the action of keeping target-locating equipment (radar, optics, etc.) continuously pointed at a moving target. Sighting and ranging, on the other hand, denotes the action of determining the position of a stationary target in terms of the direction of the line between weapon and target and the range between weapon and target. (See also par 2-3.2 of Chapter 2.)

A typical example of a tracking element is the tracking radar that would be used in conjunction with the acquisition radar whose function in an antiaircraft-artillery weapon system is described in par 3-2.1. The tracking radar used in such a weapon system has a higher order of accuracy than either the surveillance radar or the acquisition radar. It has the function of supplying accurate position data on aerial targets, so that the required range and rate data can be obtained for gun-laying purposes. (It should be noted that for some applications the acquisition and tracking functions can be carried out by the same piece of equipment. An example of this is an acquisition and tracking radar, which

is a radar set capable of locking onto a strong target signal and then tracking the target that is emitting the signal.)

The tracking element is usually mechanized in a fire control system as part of the acquisition and tracking system (see Fig. 3-2.) Therefore, for design information relating to this type of element, see Section 2 of the Fire Control Series.

3-2.3 BALLISTIC-DATA ELEMENT

The basic function of this element of a fire control system is to supply to other functional elements of the system such data regarding the standard trajectory of the particular projectile and weapon concerned as may be required for them to perform their own particular functions. (See Chapter 2 and references noted therein for source information relating to standard projectile trajectories.)

A typical example of a ballistic-data element is that portion of a complex fire control computer that stores standard trajectory data for the particular weapon system concerned. This data is usually stored as a function of target range and target elevation.

The ballistic-data element is usually mechanized in a fire control system as part of the computing system (see Fig. 3-2). Design information relating to this type of element therefore appears in Section 3 of the Fire Control Series.

3-2.4 PREDICTING ELEMENT

The basic function of this element of a fire control system is to continuously compute -- based on data provided by the tracking and ballistic-data elements -- the direction in which the weapon must be aimed in order to score hits on the target.

For a target that is stationary with respect to the weapon, the computation must take into account the various forces acting on the projectile during its flight to the target position and also the jump effects that can cause the initial projectile velocity direction to differ from the direction in which the weapon is fired. (Any errors that may result from variations from standard conditions present at the time of firing can be corrected by means of the arbitrary correction element described in par 3-2.5.) For a target that is moving, a future target position must be calculated that takes into account target motion during the period in which the projectile is in flight. This establishes a future target position and the future line of site to that position. The computations associated with weapon fire on a stationary target must then be applied to fire against the future target position.

A typical example of a predicting element is that portion of a complex fire control computer that takes the information supplied by the tracking element and the ballistic-data element and derives data for positioning the weapon,

The predicting element is usually mechanized in a fire control system as part of the computing system (see Fig. 3-2). Therefore, for design information relating to this type of element, see Section 3 of the Fire Control Series.

3-2.5 ARBITRARY CORRECTION ELEMENT

The basic function of this element of a fire control system is to introduce into the output of the predicting element either or both of the two following types of corrections:

1. Corrections required because the actual conditions present at the time of firing depart from the standard conditions on which the data supplied from the ballistic-data element are based.

2. Spotting corrections based on observation of actual weapon fire.

A typical example of an arbitrary correction element used for the first type of correction noted is the means through which changes in initial projectile velocity are determined and introduced into the fire control system. The spotting boards used in connection with artillery fire exemplify the second type of correction element.

The arbitrary correction element is usually mechanized in a fire control system as part of the computing system (see Fig. 3-2). Design information relating to this type of element therefore appears in Section 3 of the Fire Control Series.

3-2.6 COMPENSATING ELEMENT

The basic function of this element of the fire control system is to correct for any motion of the system's mechanical reference frames from the basic coordinate frame used for computing purposes. Such a compensating element would be required, for example, between the computing system and the weapon-pointing system if the coordinate system used by the computing system in deriving the firing data required for aiming the weapon differed from the mechanical reference frame associated with the manner used to orient the weapon. An auxiliary function would be in connection with parallax correction.

The compensating element is considered in the Fire Control Series to be mechanized in the fire control system as part of the weapon-pointing system (see Fig. **3-2).** Design information relating to this type of element therefore appears in Section 4 of the Fire Control Series.

It should be noted that the compensating element could just as logically be considered to be part of the computing system. Further, a compensating element may also be needed between the acquisition and tracking system and the computing system, should the mechanical reference frame associated with the manner used to track the target differ from the coordinate frame used by the computing system.

Detailed information on compensating elements is given in Reference 2 which is specifically devoted to this one subject alone.

3-2.7 POINTING ELEMENT

The basic function of this element of a fire control system is to aim the weapon in accordance with the firing data (e. g., azimuth and elevation commands) that have been generated by the predicting element and modified by the arbitrary correction element and the compensating element.

A typical example of a pointing element is a rocket launcher and the associated positioning drive mechanisms. This element is considered to be mechanized in a fire control system as the main element of the weapon-pointing system (see Fig. 3-2). Therefore, for design information relating to such systems, see Section 4 of the Fire Control Series.

3-2.8 DATA-TRANSMITTING ELEMENTS

The basic function of these elements of a fire control system is to transmit data between other elements of the fire control system that are located at some distance from one another. The fact that data-transmitting elements are often utilized at various points in a fire control system is the basis for the method chosen to represent these elements in Fig. 3-2.

Various types of equipment are used to accomplish the data-transmitting function. One of the most accurate and reliable is the well-known synchro system.

Design information relating to data-transmitting devices appears in Section 2 of the Fire Control Series, since it is in connection with acquisition and tracking systems that a need for data transmission is first felt in the passing of information between the several elements of a fire control system.

3-2.9 FUZE-SETTING ELEMENT

The basic function of this element of a fire control system is to set the time fuze of a projectile when such a fuze is employed. Use of time fuzes has now become quite uncommon; instead, proximity fuzes are usually employed.

Inasmuch as there is presently no design effort in connection with fuze setters, nor any contemplated for the future, any coverage on this element will be strictly for historical interest.

For convenience, design information relating to fuze setters is given in Section 4, in connection with weapon-pointing systems.

3-2.10 COMMAND ELEMENT

The basic function of this element of a fire control system is to provide opportunity for the command function to enter into operation of the fire control system. See par **3-3.** 1. 2 for an example of how this type of functional element is employed.

3-3 FACTORS ASSOCIATED WITH THE INTEGRATION OF FUNCTIONAL ELEMENTS INTO FIRE CONTROL SYSTEMS

As already noted, all of the various functional elements represented in Fig. 3-2 will not usually be found in a given fire control system. The factors that determine what elements comprise a particular fire control system, and the complexity of the functional arrangement, include the following:

1. The function of the weapon whose fire is to be controlled.
2. The kind and size of weapon involved.
3. The manner in which the weapons involved are to be used (e.g., single-purpose or multipurpose).
4. The degree of mobility desired for the weapon involved.
5. The degree to which human participation supplies some of the functional elements of a given fire control system.
6. The speed and accuracy requirements of the weapon system concerned.

The paragraphs which follow discuss these factors in turn, and give examples where appropriate.

3-3.1 THE FACTOR OF WEAPON FUNCTION

This is probably the prime factor involved in determining what functional elements are going into a given fire control system. Its effect can be seen by considering the two following examples:

1. The fire control system for a field-artillery weapon being used against a relatively slow-moving target, such as a truck convoy.
2. The fire control system for an antiaircraft weapon, which involves fire against generally fast-moving targets.

3-3.1.1 Field - Artillery Fire Control System Example

For a simple field-artillery fire control system,* the generalized functional diagram shown in Fig. 3-2 would reduce to the diagram shown in Fig. 3-3. Assume that an effectively stationary target is first spotted at an advanced observation post. The acquisition element will then probably consist of a pair of binoculars. The target-location information required for the field-artillery piece to be sighted on the target (a direct-fire situation is being postulated) is transmitted via a field-phone system, which comprises the data-transmitting element at this part of the fire control system.

Using this information, the gunner uses telescopic instruments to sight on the target, thereby establishing the correct azimuth of the gun. Range data is obtained by means of one of the various types of range finders. Since target motion is insignificant, the tracking element of Fig. 3-2 can be considered to be effectively replaced by a simpler sighting-and-ranging element.

Once the range has been determined, this information is used by the gunner in finding the required superelevation angle from the firing tables (which constitute the ballistic-data element), in accordance with the particular types of gun and ammunition being used. No separate prediction element is required since the firing tables give all the information that is needed for the fire control problem under consideration.

The required superelevation angle is achieved by the action of the gunner as he employs the telescope reticle (which comprises part of the pointing element) to sight on the target.

No command element is required since no command function is associated with the fire control problem concerned. No compensating element is involved inasmuch as the gun mount is usually levelled, and hence gun elevation takes place in the same coordinate frame in which the superelevation

* For an example of a more-complex field-artillery fire control system employing automatic computation, see the discussion of the FADAC (Field Artillery Digital Automatic Computer) given in Chapter 13 of Section 3 (Fire Control Computing Systems). The FADAC is also discussed in general terms in par 1-3.4 of Chapter 1 of Section 1.

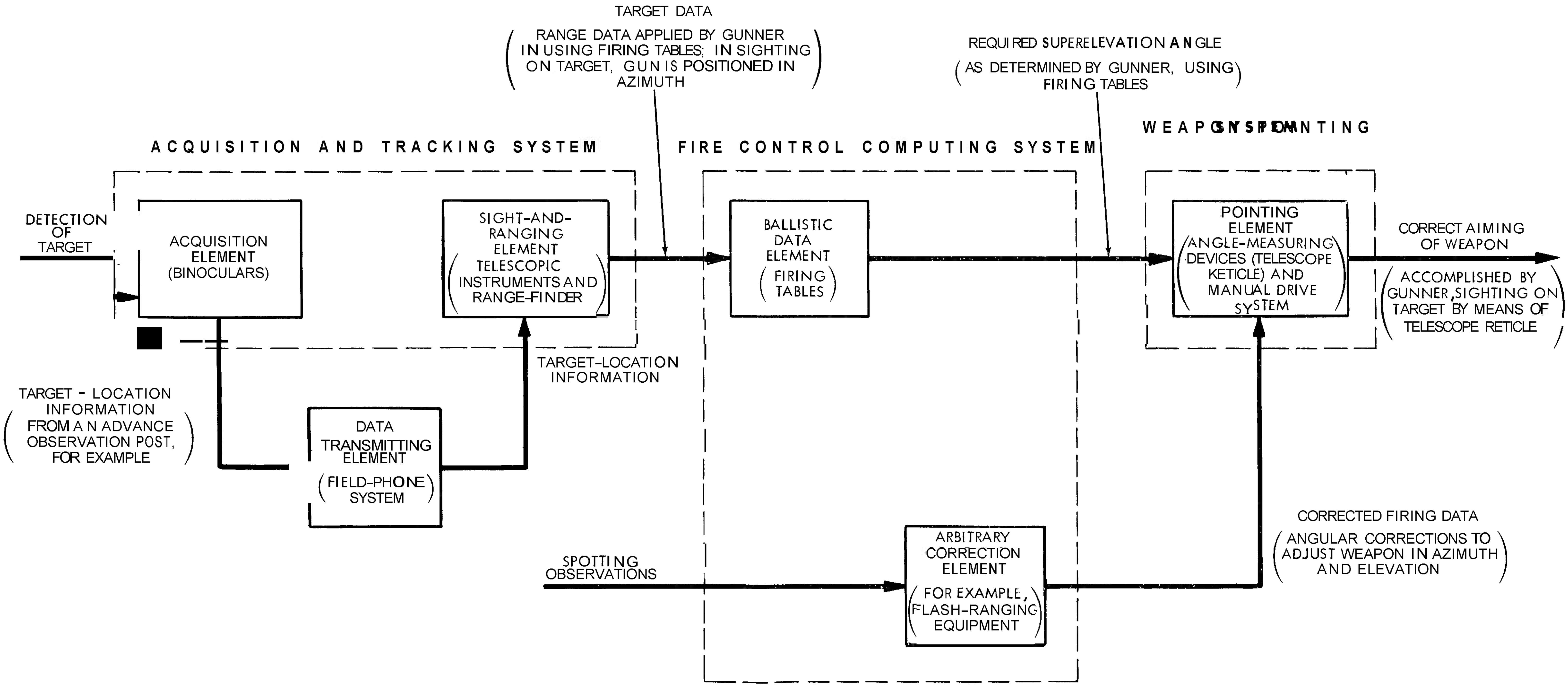


Figure 3-3. Functional diagram of a simple fire control system used with a field-artillery weapon.

angle is computed in establishing the firing tables. No fuze-setting element is required since projectile burst is usually obtained by the use of either impact fuzes or proximity fuzes, rather than time fuzes.

If necessary, the weaponfire can be adjusted by the application of spotting corrections. This can be accomplished by the use of flash-spotting instruments or simply estimated by eye, depending upon the circumstances. The equipment involved in this operation constitutes the arbitrary correction element. The spotting corrections, in mils, are applied to the drive controls by the gunner.

3-3.1.2 Antiaircraft Fire Control System Example

For this example, assumethat the antiaircraft weapon concerned is a fully-integrated, automatic weapon such as the Skysweeper antiaircraft system shown earlier, in Fig. 3-1. The associated fire control system consists of a director that contains (1) a radar tracker, (2) a periscope, (3) a sighting system, (4) a computer, (5) a power-control system, and (6) an off-carriage target selector. Except for the last-noted item, all of the fire control equipment is on-carriage equipment.

For this fire control system, the generalized functional diagram shown in Fig. 3-2 would reduce to the diagram shown in Fig. 3-4.

The acquisition element, in this example, consists of the radar tracker which can search for targets out to a range slightly in excess of 1.5 times the maximum horizontal range of the weapon itself.

The tracking element is provided by this same radar tracker which has the capability of locking onto the detected target and tracking it automatically from a range of about 1.5 times the maximum horizontal range of the weapon into a minimum range of a few hundred yards. Thus, the radar tracker is an acquisition and tracking radar that completely satisfies the functional requirements of the acquisition-and-tracking-system block shown in Fig. 3-2.

If the automatic radar tracker is inoperative, optical tracking can be accomplished by means of the periscope shown in Fig. 3-1. In this type of tracking, the periscope operator keeps his reticle on target by moving a handle-bar unit that is mounted below the periscope. Prisms in the periscope are positioned accordingly, by means of servos that are controlled by the motion of the handle-bar unit.

Target data (which can be monitored on the cathode-ray screens of the tracker console) are transmitted electrically to the computer for its use in calculating the correct azimuth and elevation for firing the gun. The electrical circuits involved constitute the data-transmitting element employed between the acquisition and tracking system and the fire control computing system.

The computing system determines the firing azimuth and the quadrant elevation by means of (1) a ballistic-data element that provides standard-trajectory data, (2) a predicting element that accounts for target motion to determine future target position and computes the required superelevation to hit the target at that future position, and (3) an arbitrary correction element that accounts for such variables as air density, muzzle velocity, trunnion tilt, and the wind.

The power control system, which constitutes the weapon-pointing system, is an electrically controlled hydraulic drive system. The power-control components compare present gun azimuth and elevation with the values of firing azimuth and quadrant elevation developed by the computer. The differences are continually reduced to zero by the operation of the azimuth and elevation drives in response to error signals from the power-control components. No compensating element is required since any trunnion tilt that may be present is accounted for by the computing system.

The target selector shown in Fig. 3-1 is a command element in the fire control system. This selector is mounted on a tripod and is used at a convenient location away from the weapon. With this device, it is possible to survey the sky and horizon for targets that might be overlooked by the radar and periscope operators. When the target selector is switched into the fire control system and a target is chosen by the operator, data on present azimuth and elevation of the selected target are transmitted to the weapon, for use by the radar and periscope

operators, by means of a synchro system. In addition, command signals to the power drive system cause the weapon to slew in azimuth and elevation until it is aligned with the line of site to the selected target.

3-3.1.3 Comparison of the Selected Examples

Comparison of Figs. 3-3 and 3-4 shows that, for the particular examples chosen, the difference in functional elements and their functional arrangement is considerable. This difference stems largely from the basic difference in the functions to be performed. For example, the fact that a target engaged by the antiaircraft fire control system is moving at considerable speed means that a tracking element (to obtain the required target data) and a predicting element (to account for target motion during the projectile time of flight) must be employed. For solution of the fire control problem associated with the effectively stationary target of the field-artillery weapon, however, only a sighting-and-ranging element need be employed to generate the required target data, and no predicting element need be employed in the computing system.

3-3.2 THE FACTOR OF KIND AND SIZE OF WEAPON INVOLVED

The effect of this factor can be readily illustrated by comparing the functional diagram for a small arms weapon (see Fig. 3-5) with the functional diagrams given in Figs. 3-3 and 3-4, respectively, for typical field-artillery fire control and antiaircraft fire control systems.

The sole fire control equipment usually employed on small arms are the sights employed in aiming the weapon. Three different types of sights are currently employed in connection with small arms:

1. Metallic sights.
2. Optical sights.
3. Sniperscopes.

A metallic sight comprises a blade sight at the muzzle end of the barrel and an aperture, open (U-shaped), or folding-leaf sight at the breech end. The blade sight is a thin, flat, metal post. Aperture sights are those that are sighted through, such as a peep sight, a ring sight, etc. Open sights are all those that are sighted over or at, such as a post, bead, notch, etc. Leaf sights are those that can be folded down for their own protection.

An optical sight used for small arms fire is nothing more than a telescope with a reticle that is attached directly to the barrel of the weapon, as on a sniper's rifle.

A sniperscope is a fire-control sighting device that combines a snooperscope and a carbine or other firearm, in order to enable the operator to see and shoot at targets in the dark. The snooperscope is a hand-carried device that combines a source of infrared rays with a viewer, and thereby enables the operator to see in the dark.

Functionally, a sight serves as a combined tracking and pointing element. For the case of a stationary target, the operator merely uses his sight as a means of orienting the weapon directly at the target. (The rifle leaf sight allows corrections for superelevation, drift and windage. The operator estimates range and crosswind and adjusts the sight accordingly.) For the case of a moving target, the operator uses his sight as an aid in tracking the target long enough to determine the lead that training has shown him he should allow in order to hit the target.

Because of the small size of the weapons, no special drive equipment is required to move them. Because of the simplicity of the fire control equipment, no data-transmitting elements are involved.

The example chosen does, of course, also represent the effect of weapon function and the degree to which human participation supplies some of the functional elements of a fire control system.

3-3.3 THE EFFECT ON FIRE-CONTROL-SYSTEM DESIGN OF A MULTIPURPOSE REQUIREMENT OF THE WEAPON SYSTEM

A multipurpose weapon is a weapon that can be used for a number of different purposes such as against ground forces and against aircraft. This implies that the associated fire control system combine the functional elements represented in Figs. 3-3 and 3-4 in order to effect the necessary control. A good example of such a multipurpose weapon system is the 90 mm gun used against tanks and aircraft in World War 11.

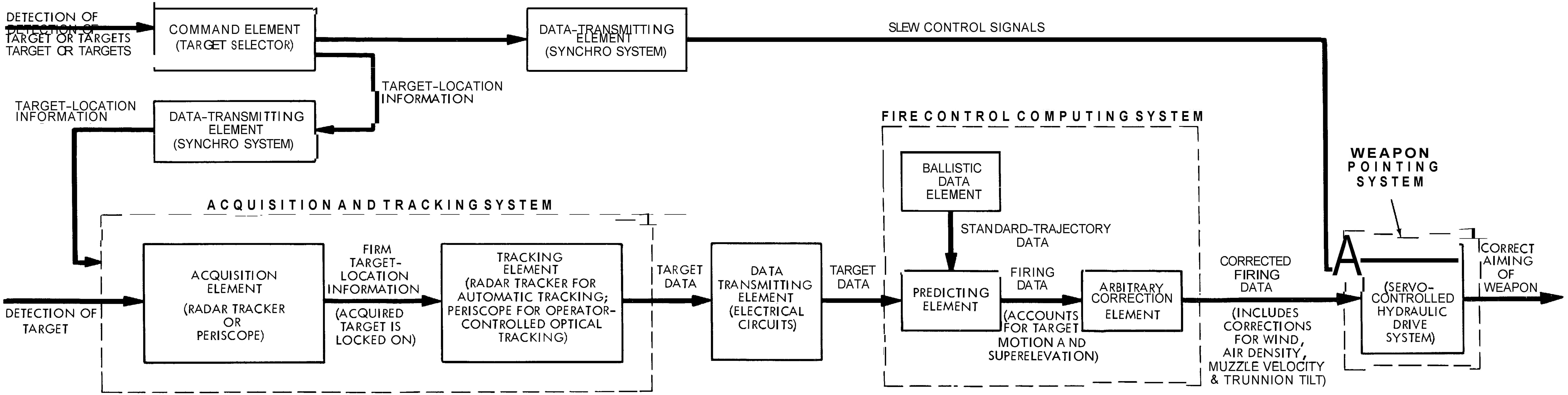


Figure 3-4. Functional diagram of a typical fire control system used with a fully integrated automatic antiaircraft weapon system.

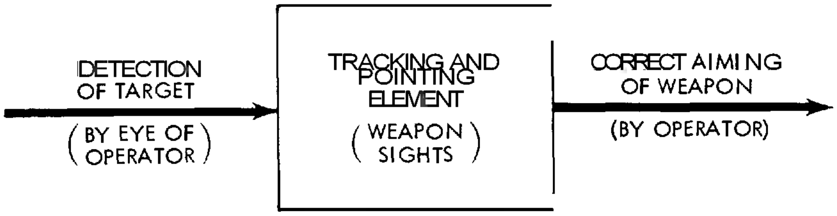


Figure 3-5. Functional diagram of typical fire control equipment for use with small arms.

3-3.4 THE EFFECT ON FIRE-CONTROL-SYSTEM DESIGN OF A WEAPON-MOBILITY REQUIREMENT

A tank fire control system is an example of the factor of weapon mobility. Normally, a tank is not only in linear motion but is also in erratic angular motion as it traverses the terrain in its route. The result is a motion comparable to that to which a ship is subjected on the sea. The fire control problem can therefore be solved by the same means that the Navy employs -- by stabilization techniques. As noted in paragraph 3-5.3, the use of stabilization has increased the accuracy and efficiency of tank guns in battle, while the vehicle is in motion, by a factor of several hundred percent.

Functionally, the stabilization represents an increase in the complexity of the weapon-pointing system. For further information on stabilization systems, see Section 4 of the Fire Control Series.

3-3.5 THE EFFECT ON FIRE-CONTROL-SYSTEM DESIGN OF SPEED AND ACCURACY REQUIREMENTS

Speed and accuracy are characteristics that usually involve a sacrifice in one to achieve excellence in the other. For example, the fire control systems used with the coast-artillery systems of yesteryear achieved an outstanding accuracy. Because of the time required to achieve a fire-control solution of this accuracy, however, the principles on which these systems were based could not be applied to the problem presented by fast-flying aircraft. To achieve the required speed of solution, accuracy had to be sacrificed. The decrease in accuracy was compensated by the natural dispersion of rapid fire, the use of time-fuzed projectiles, and -- as a further refinement -- the use of proximity fuzes. For certain applications, of course, the use of guided missiles has eliminated the need for conventional antiaircraft fire control equipment altogether.

3-4 COMPATIBILITY PROBLEMS OF VARIOUS TYPES OF OPERATING ELEMENTS

3-4.1 GENERAL PRINCIPLES

It is a well-known fact to system designers that for effective design the operating elements of a system must be compatible with one another. This means, for example, that a particular operating element should not be allowed to remain in a system design if its use is going to be detrimental to the overall functioning of the system -- no matter how excellent the performance of the operating element may be as an individual entity or in other applications. Frequently, of course, there is the questionable case of just how adversely one element affects the operation of its companion elements. In such instances, the trade-off of performance between individual components must be carefully evaluated in order to determine the net effect on overall system performance, which is the ultimate criterion.

It is also generally considered inadvisable to employ operating elements whose individual performances are so high in comparison with system needs and the performance of other elements of the system that their full potential will never come close to being utilized. Their inclusion under these conditions would usually be economically unsound.

3-4.2 FACTORS REQUIRING PARTICULAR ATTENTION

The following factors affect the compatibility of one operating component with another:

1. Relative accuracies.
2. Relative speeds of operation.
3. Relative ranges of operation.
4. Types of associated equipment.
5. Interconnecting devices used between system elements,

The effect of each of these factors will be illustrated by general examples in the following paragraphs.

3-4.2.1 Relative Accuracies

The relative accuracies of the operating elements in a system represent a factor of prime importance since the accuracy of the least-accurate element in a chain generally establishes the overall accuracy of the chain. As an example of the importance of this factor, consider a fire control computing system that supplies firing data to a weapon-pointing system whose accuracy capability for positioning the weapon is only one-tenth the accuracy of the firing data itself. Obviously, the two subsystems of the fire control system are mismatched, and hence incompatible. The situation should be corrected by improving the accuracy of the weapon-pointing system, if the overall accuracy specified for the complete weapon system requires this. Otherwise, the computing system should probably be simplified -- with attendant economies -- until the output accuracy of the firing data closely matches the accuracy of the weapon-pointing system.

3-4.2.2 Relative Speeds of Operation

The relative speeds of operating elements in a fire control system frequently comprise a significant factor, particularly for systems used where the target is within the firing range of the weapon for only a brief period of time. Consider, for example, a hypothetical antiaircraft fire control system whose speed of determining firing data is such that the weapon cannot be used during the initial phase of an incoming air attack, even though the target is within firing range of the weapon. If the rest of the fire control system -- aside from the computing elements -- can operate at the required speed, then the computing elements are incompatible with the other elements of the system and with the overall requirements of the weapon system.

3-4.2.3 Relative Ranges of Operation

As an example of how this factor affects the compatibility of operating components in a system, consider an acquisition and tracking radar whose range capability for locking onto the target and commencing the tracking operation is only slightly greater than the effective range of the weapon itself. Inasmuch as a certain amount of accurate tracking is required before usable target data can be generated by the tracking element for use by the computing system, it is clear that the range limitation of the radar makes this operating element incompatible with the remaining elements of the fire control system. A radar whose lock-on range is about 1.5 times the effective range of the weapon, on the other hand, would probably be quite compatible with the system.

3-4.2.4 Types of Associated Equipment

Certain fire-control situations prescribe fixed types of equipment for one or more parts of the system. The type of equipment used in the remainder of the system, on the other hand, may be of various types. The essential requirement here is that this latter equipment be compatible with the equipment that is incapable of modification.

3-4.2.5 Interconnecting Devices

As an example of this factor in the compatibility of system design, consider a complex fire control system that has been set up with a particular type of data-transmission equipment; e.g., synchro-type equipment. No matter how excellent some particular element in the system that received synchro signals might be on its own merits, it would be incompatible with the overall system if it were not adapted to use these signals efficiently.

3-5 EXAMPLES OF FUNCTIONAL ARRANGEMENTS EMPLOYED IN VARIOUS TYPES OF FIRE CONTROL SYSTEMS

3-5.1 INTRODUCTION

As noted, all fire control systems can be considered to comprise three major subsystems (see Fig. 3-2):

1. The acquisition and tracking system which comprises observing, position-finding, and tracking equipment.
2. The computing system which comprises data-computation equipment for the generation of firing data.
3. The weapon-pointing system which comprises equipment for the application of firing data.

Only a relatively few fire control systems would contain all of the functional elements shown in Fig. 3-2, however.

It is the purpose of the present section to provide examples of the manner in which the elements described in paragraphs 3-2.1 through 3-2.10 combine to form functional arrangements in various types of fire control systems. The following types of fire control equipment are briefly described: *

1. Fire control equipment for artillery.
2. Fire control equipment for tanks.

3-5.2 FIRE CONTROL EQUIPMENT FOR ARTILLERY

3-5.2.1 Observing and Position-Finding Equipment

Prior to the invention of radar, the functions of observing and position finding were all performed solely by optical instruments. Azimuth and angular elevations were determined by telescopic instruments functioning in the same manner as a surveyor's transit. The same instruments served for observation purposes. Range finding was also accomplished by optical instruments. All these instruments are still in wide use, but have been replaced in some applications by radar.

The various instruments used for angular measurement and range finding fall into three main categories, which are described in the paragraphs which follow, namely:

1. Optical equipment such as binoculars, aiming circles, and range finders.†
2. Radar equipment.
3. Sound and flash equipment.

See Section 2 of the Fire Control Series for a more-detailed discussion of acquisition and tracking equipment.

3-5.2.1.1 Optical Equipment

Binoculars (see Fig. 3-6) or field glasses are moderate-power instruments used for general observation and spotting. They vary from 6- to 9-power, having individual diopter adjustments for each eye and an interpupillary adjustment. The binocular construction increases the observer's stereoscopic (depth) perception. Some instruments have reticle patterns (see Fig. 3-7) for the estimation of angles.

The aiming circle (see Fig. 3-8) is used for measuring angles in azimuth and site, and for general topographic work in the orientation of the battery. The instrument includes a monocular telescope and magnetic compass.

Range Finders. Optical range finders determine range by solving a right triangle in which one side and two angles are known, range being one of the unknown sides. With reference to Fig. 3-9, the range from a range finder PS to a target T is R. The distance between P and S is the range-finder base length, B. As the base length is increased, by enlarging the range finder, it becomes possible to obtain accurate readings at greater ranges. The lines of site to the target from P and S form the angle of convergence θ at the target. The line IS is parallel to PT; hence the angle IST is equal to θ. The instrument is built *so* that angle TPS, which defines

* For a description of a complex antiaircraft fire control system, see the analysis of the fire control system of the Vigilante Antiaircraft Weapon System that appears in par 4-6 of Chapter 4.

† As noted in Chapter 1, laser range finders are now being developed also.

Figure 3-6. Binocular M8.

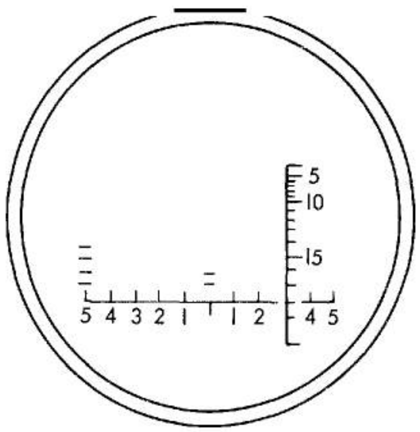


Figure 3-7. Binocular reticle pattern.

the left line of site with respect to the range finder PS, is always 90°. If the left line of site is sighted onthe target, it willbe necessary for the right line of site to move left from I by the angle θ for it to see the target. If this right line of site is controlled by turning a calibrated range knob that traverses an optical sight, the angle θ can be measured. The range-knob scale is calibrated according to the equation $R = B \cot \theta$ and a direct range reading is possible once the angle of convergence has been determined.

There are two types of optical range finders: "coincidence" and "stereoscopic". They are similar in general appearance and both work on the "convergence angle and known base length" principle that has just been described.

The stereoscopic type of optical range finder is based on the principle of natural stereoscopic vision, which enables a human being to detect a difference between the distances to two remote objects. Therefore, the operator must use both eyes. A set of fixed reticles, or optical reference marks, are superimposed in the observer's field of vision and appear to be at some fixed, known distance. The observer's right line of site is swung by a range knob controlling movable optics, while his left line of site remains fixed. This permits him to make the reticles appear to move in or out in range, as he varies his angle of convergence. His problem is to make the reticle appear to be at the same range as the target. The angle of convergence θ that accomplishes this is

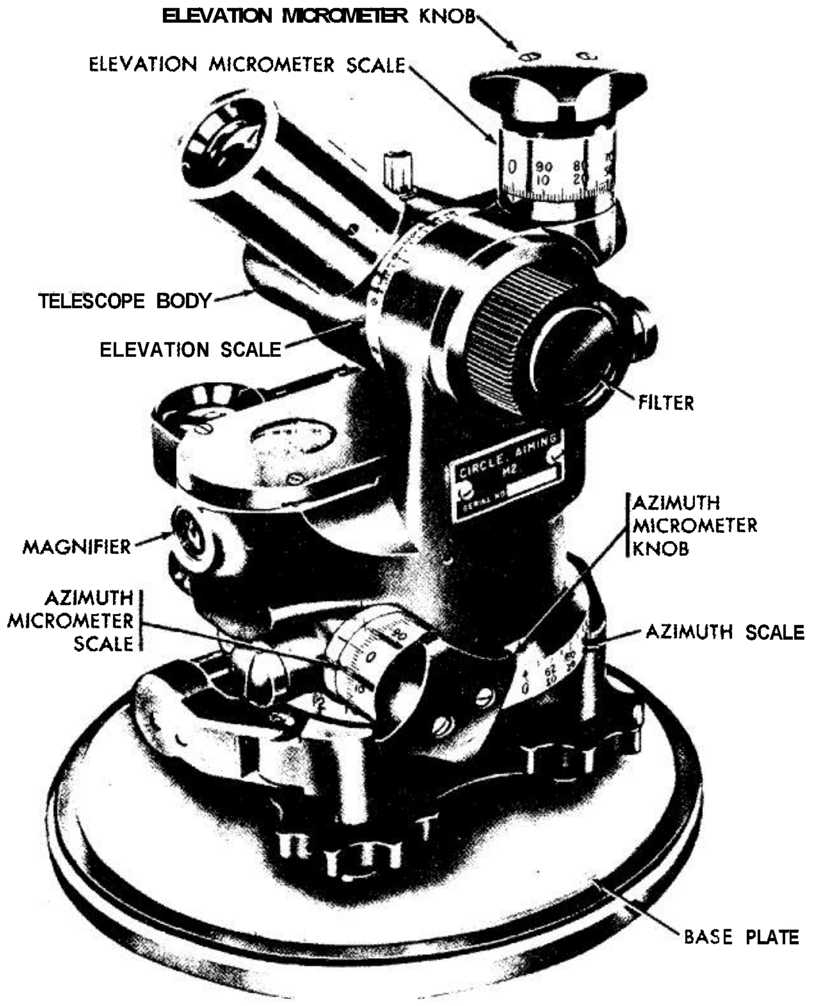


Figure 3-8. Aiming Circle M2,

measured by the range knob. By calibrating the knob (using the aforenoted relationship between the range R, the cotangent function of θ, and the base length B), range can be read off directly.

The coincidence type of optical range finder uses a split field of vision and the operator uses only one eye. It is the same type range finder commonly used on cameras. The one eye sees a target that appears cut horizontally in two by a halving line (see Fig. 3-10). The upper half of the image is coming in from the fixed left line of site and the bottom half from the movable right line of site. When the range knob is turned, movable optics bend the right line of site until the image halves coincide vertically. This measures the angle of convergence θ, and hence, range.

Optical range finders are complex, delicate instruments, difficult to maintain in adjustment, and require skilled operators. This requirement is particularly true of the stereoscopic type and, as a result, coincident-type devices are replacing stereoscopic-type devices at the present time. It should be noted, however, that with proper operator training the stereoscopic type is very useful for ranging on fast-moving, irregularly-shaped targets or for spotting projectile bursts.

3-5.2.1.2 Radar Equipment

The principles of radar (Radio Direction And Ranging) are well documented in readily-accessible technical literature and hence will not be repeated here. For present purposes,

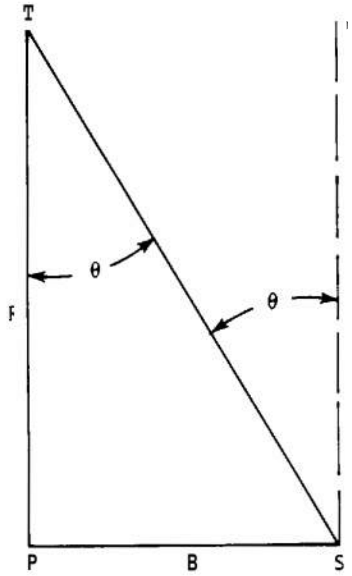


Figure 3-9. Optical range finder; principle of operation.

it is sufficient to note that radar can furnish the range, azimuth and angular elevation of the target's present position. There are two general types of radar equipment: search radars and fire-control radars. A search radar (e.g., the surveillance radar described in paragraph 3-2. 1) sends out a long pulse in a broad beam and cannot measure target data with high precision. A fire-control radar, on the other hand, such as used for acquisition and tracking, sends out a short pulse in a narrow beam; hence, while not being too well suited for searching, it does measure target data with high precision. A special type of fire-control radar sometimes employed for particular applications is the range-only radar.

Radars are not as accurate in making angular measurements as are optical instruments, but they can measure the ranges of moving targets more accurately. In addition, radar has the great advantage over optical instruments that it can be used night or day, in good visibility or poor, and at considerably longer ranges. It has the disadvantages of being large and complex, subject to enemy jamming, and disclosing to the enemy the fact that he is under observation. The radar-reflectivity characteristics of various targets result in another important limitation that must not be overlooked.

With the relative merits of optical instruments and radar being as they are, operational doctrine results in the tendency to use the two types of equipment to complement each other where possible)::, or to use optics alone -- especially in the case of short-range weapons.

3-5. 2. 1. 3 Sound and Flash Equipment

For the purpose of locating hostile guns and adjusting the fire of friendly artillery, both sound and flash ranging may be employed. In these procedures, various microphones, recorders, plotting boards, and spotting instruments are used. These instruments will not be described in detail; however, the general methods in which they are employed will be briefly discussed.

Sound ranging is the procedure of locating the source of a sound, such as a gun report or shell burst, by calculations based upon observations of the propagated sound wave. If two microphones are placed some distance apart and the distance in terms of arrival time at each microphone is recorded, the location of a hyperbola that passes very close to the origin of the sound may be determined. Other combinations of two microphones will provide similar hyperbolas, and from their intersection the source of sound may be located.

Flash ranging is the procedure employed in locating enemy installations or friendly projectile bursts by visual observations and plotting the intersection of the lines of site obtained from two or more observation posts. Each observation post is equipped with an observing instrument for reading horizontal and vertical angles (see Fig. 3-11). Such instruments are oriented to measure angles to points in the target area. When these angles are reported to the plotting center and plotted from the base line connecting the two observation posts, the positions of the points are located.

3-5. 2. 2 Firing-Data Computation Equipment

For fire-control applications in which

* For example, in the Vigilante Weapon System that is described in Chapter 4; see par 4-6.

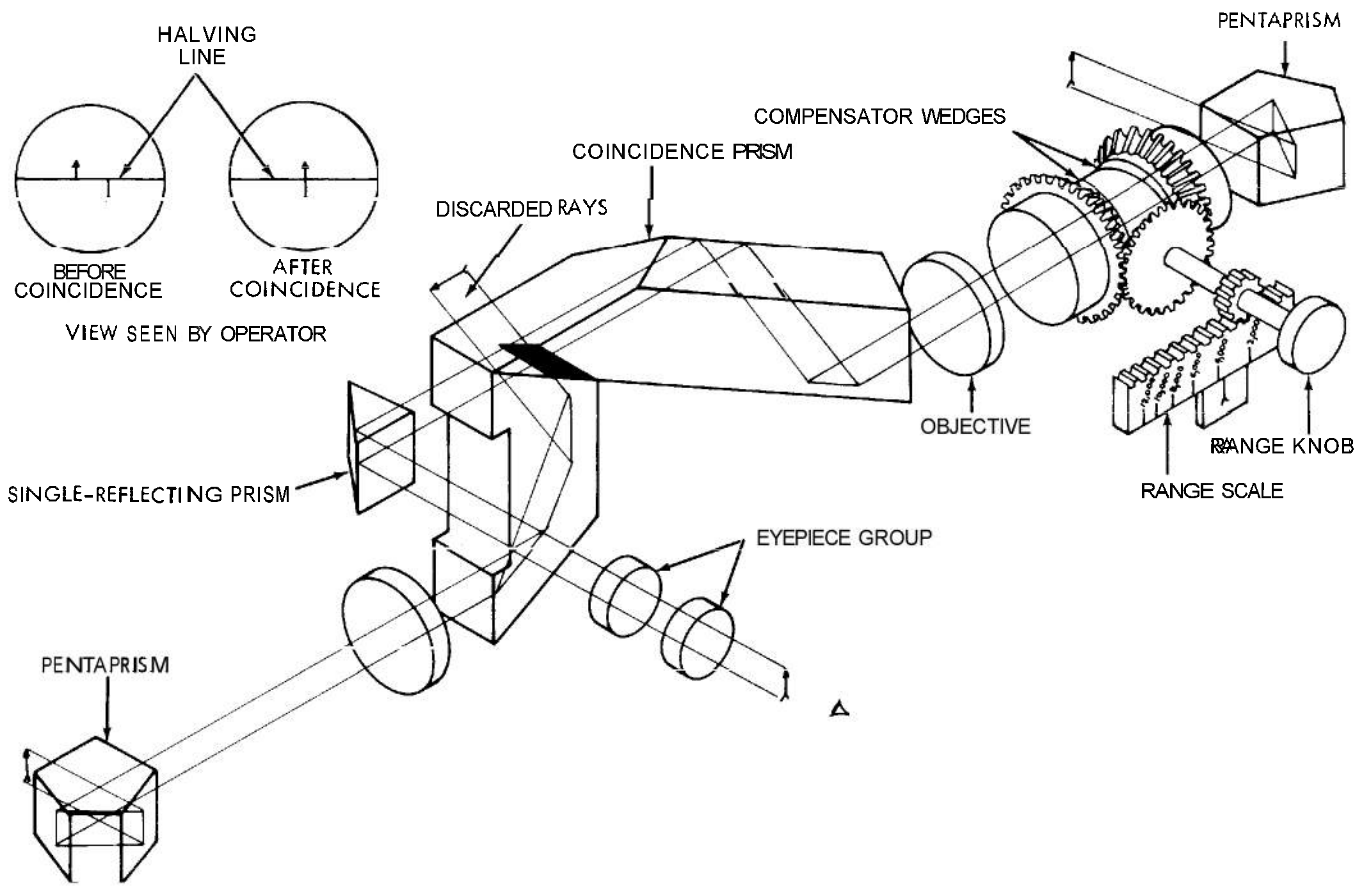
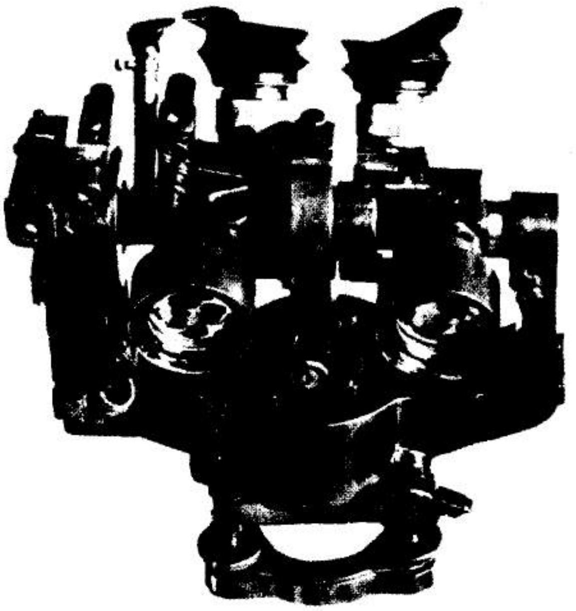


Figure 3-10. Coincidence type of optical range finder.



Note: Instrument is mounted on tripod for use.

Figure 3-11. Flash-spotting instrument.

firing data are computed by human means, such items as firing tables, plotting boards and their accessory equipment, and many other similar aids are used. The use of mechanical, electrical, or electronic computers, on the other hand, occurs when the fire control problem is beyond human-performance capabilities, as in the case of very-fast-moving targets or when both target and weapon are moving. At present, they are used primarily in antiaircraft fire, naval gunfire, and aircraft bombing, gunnery and rocket firing. Automatic computing equipment applicable to artillery fire has now been developed, however.

Some of the equipment needed for data computation is described here briefly for the two broad classifications that apply:

1. Computation by human means.
2. Computation by mechanical, electrical and electronic data computers.

3-5.2.2.1 Human Data Computation*

Firing tables are available in two types for the calculation of firing data: in book form and in graphical form.

The tables prepared in book form are available for every type of weapon and ammunition. Instructions for use are included.

The graphical firing table (see Fig. 3-12) is a slide-rule type of device that speeds the calculation of firing data. Each table is intended for use with one weapon and projectile, and carries scales based on the various charges used with the weapon.

3-5.2.2.2 Mechanical, Electrical and Electronic Data Computation

These are all types of automatic firing-data computers, the input to which is target data and certain ballistic corrections such

* It should be noted that the need for human data computation has now been effectively eliminated; the firing tables of this paragraph are discussed solely for the sake of being complete. For example, the availability of **FADAC** (see par **3-3.1.1**) would eliminate the need for hand computation under normal circumstances in the solution of a field-artillery fire control problem.

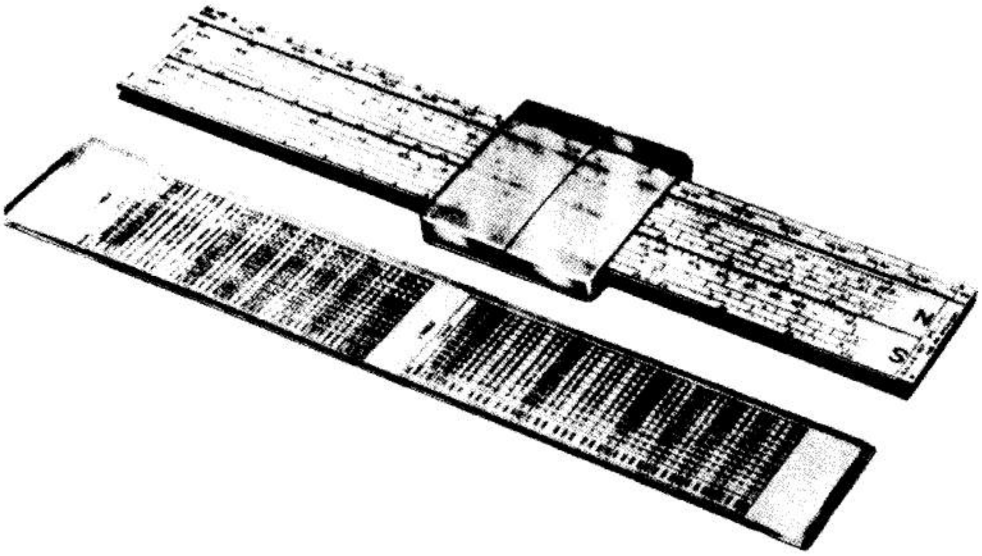


Figure 3-12. Graphical Firing Table M27.

as atmospheric temperature, winds and the like. Essentially, they are nothing more than mathematical computers that are capable of high-speed solution of the moving-target problem. They usually work on either the linear-speed or angular-rate-of-travel principle (see paragraph 2-3.3 of Chapter 2). They contain stored within them -- in cams, linkages, electronic circuits and the like -- the ballistic-performance data of the weapon and ammunition with which they are intended for use. They solve the target-movement problem, cull their ballistic references for the firing data, correct the firing data for nonstandard conditions and send it to the weapon, all almost instantaneously and continuously after going through an initial settling down period -- a matter of several seconds.

In general, computers vary in size and complexity in accordance with the difficulty of the fire control problem involved and the accuracy with which it must be solved. See Section 3 of the Fire Control Series for a more-detailed discussion of fire control computing systems.

3-5.2.3 Equipment for the Application of Firing Data

The firing data generated by the data-computation equipment must be used to aim the weapon in azimuth and elevation, and to set time fuzes when necessary. The manner in which the firing data is applied depends on how and where the data is generated. For example, in the case of an indirect-fire field-artillery weapon-laying situation, elevation firing data could be generated by setting target-range and angle-of-site information directly into a sighting device (such as the range quadrant discussed in par 3-5.2.3.2) located on the weapon being employed. The range drum incorporated into the sighting device would convert the range data into the required elevation offset angle as a rotation of the longitudinal level in the vertical plane. (The cross level and cross-leveling control would be used to maintain this rotation in the vertical plane.) The elevation firing data thus generated in the form of the elevation offset angle would then be applied through the action of elevating the weapon until the bubble of the longitudinal level was centered. With this same equipment, on the other hand, the required elevation angle could be separately determined by various types of computational aids and then applied by setting it into the sighting device by means of the elevation controls. The weapon would then be correctly positioned in elevation with the aid of the longitudinal level, as before.

Computed azimuth firing data would be applied by orienting the weapon with respect to a reference point of known azimuth, employing a device such as the panoramic telescope discussed in par 3-5.2.3.1.

Before firing data can be applied to a weapon, it first must be transmitted from the place of computation to the weapon. In many cases, oral or telephone communications are utilized. When a mechanical or electrical computer is used to generate the firing data, however, this data is usually transmitted electrically to the weapon by a synchro data-transmitting system. Servomechanisms on the weapon receive this data and use it to position the weapon automatically and continuously in azimuth and elevation, and (if applicable) to set the fuze setter to the required time of flight. Electrical data-transmission systems and automatic positioning mechanisms are considered fire control equipment. Such systems are used, for example, with weapons designed to combat high-speed targets such as antiaircraft weapons.

Typical sighting instruments, fuze setters, and transmission apparatus are described briefly in the remainder of this section under the following headings:

1. Optical equipment.
2. Mechanical equipment.
3. Automatic and/or remote-control equipment.

See Section 4 of the Fire Control Series for a more-detailed discussion of weapon-pointing systems.

3-5. 2. 3. 1 Optical Equipment

The optical equipment associated with the application of firing data includes such fire control instruments as the panoramic telescope, the straight-tube telescope, and the telescope mount. The panoramic telescope and telescope mount are normally mounted on the left side of the top carriage. The straight-tube telescope is normally on the left side when only one telescope is employed; when two telescopes are used, it may be on either side.

The panoramic telescope (see Fig. 3-13 and the discussion in par 1-2.4. 3. 3 of Chapter 1) is normally used in indirect fire for laying the weapon in the proper firing azimuth with respect to a reference point of known azimuth. This point may be in almost any visible direction not obscured by the weapon. The panoramic telescope may also be used for direct fire. In panoramic telescopes intended for extensive direct-fire use, the reticle is graduated with a range pattern based on the ammunition, projectile, and charge having the most use (see Fig. 3-14).

The straight-tube telescope (see Fig. 3-15 and the discussion in par 1-2.4. 3. 1 of Chapter 1) is used in direct fire, laying the gun in azimuth or in both azimuth and elevation. The reticle is graduated in a range pattern that is based on the projectile and charge having the most use.

The telescope mount supports the panoramic telescope (see Fig. 3-13). It may have longitudinal and cross leveling (correction for cant) devices for establishing a horizontal plane for the setting in of ranges and elevation. An angle-of-site mechanism may be included to facilitate the application of these data. The mount may include elevation scales graduated in mils for use with any ammunition, or scales graduated in yards for rapid use with one charge.

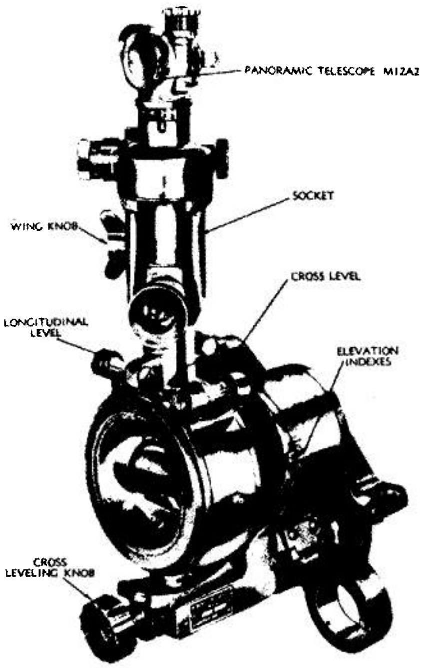


Figure 3-13. Telescope Mount M21A1 with Panoramic Telescope M12A2.

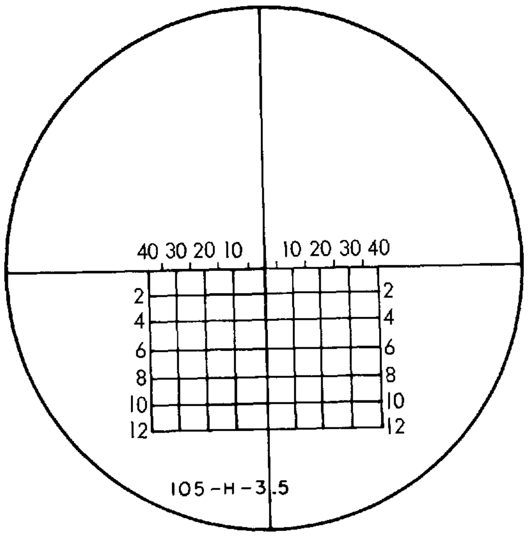


Figure 3-14. Panoramic Telescope M12A2; reticle pattern.

3-5. 2. 3. 2 Mechanical Equipment

The range quadrant (see Fig. 3-16 and the discussion in par 3-5. 2. 3) is mounted on the right-hand side of the top carriage. It includes elevation scales, which are generally graduated in mils, for use with any ammunition, and removable range drums, graduated in yards, for rapid use with one charge. Normally, it is used for two-man indirect sighting. An elbow telescope is provided where two-man direct sighting is sometimes required; however, in this case, the reticle pattern -- rather that the elevation scale -- is used for ranging.

The gunner's quadrant (see the discussion in par 1-2.4. 2. 1 of Chapter 1) is used in bore sighting to level the weapon for aligning the telescope and mount. It can also be used to emplace the weapon in elevation. The instrument includes a quadrant graduated in mils, a level mounted on a swinging arm, and mounting surfaces arranged for two positions of mounting.

Fuze setters are used to set or "cut" a time fuze so that the projectile will explode at the desired time after the gun is fired. Field-artillery fuze setters are of either the wrench type or the hand type. The wrench-type fuze setter has no seal-es. The fuze setter turns a ring in the fuze to set the timing to agree with the range required. The hand-type fuze setter has range or time scales and a corrector scale.

On larger-caliber antiaircraft weapons, automatic fuze setters are employed. The fuze-setting data are received electrically from the computer, and the fuze setter is then positioned continuously by a servomechanism.

3-5. 2. 3. 3 Automatic and/or Remote-Control Equipment

Automatic or remote-control systems receive as input signals the firing data transmitted from the sight or computer and -- through either electrical or hydraulic-drive servomechanisms -- convert these data into mechanical energy to position the weapons. Usually, separate drives are utilized for azimuth and elevation. Automatic positioning systems usually also have manual controls that can be used in case electrical power is lacking.

Figure 3-15. Straight-tube Telescope M6.

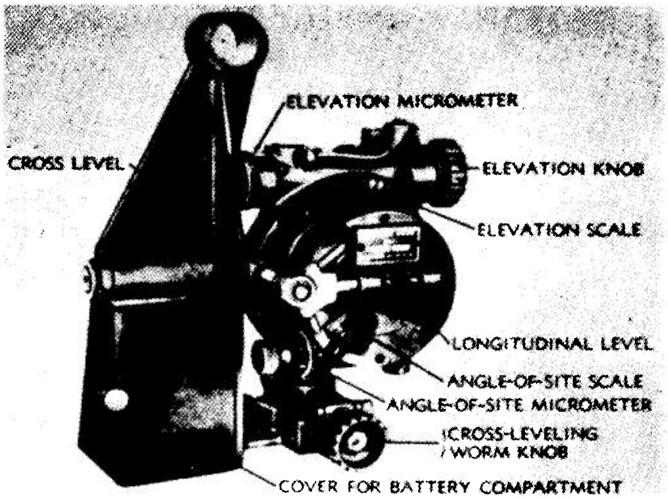


Figure 3-16. Range Quadrant M4A1.

3-5.3 FIRE CONTROL EQUIPMENT FOR TANKS*

A special fire control problem exists in the case of tank-versus-tank combat. A high probability of achieving a first-round kill is of extreme importance since each of the contending tanks usually will have a weapon powerful enough to destroy its opponent with one hit. Increasing the probability of this first-round kill, however, is hindered by several complications, including the small, cramped gun compartment, the speed and maneuverability of the "target" tank, and the erratic pitching and yawing of the "gun" tank as it moves over cross-country terrain.

As a solution to the aforenoted problem, the principle of the integrated fighting compartment has been developed. Such a compartment may include one or all of the following main components:

1. Dual power controls for the turret.
2. Built-in optical range finder.
3. Lead computing sight.
4. Vertical and horizontal stabilization of both gun and fire control equipment.

For power control, the gunner is provided handle bars that actuate power-drive controllers for turret traverse and gun elevation. (Manual traverse and elevation are provided for in the event of power failure.) The tank commander, from a separate station, may assume speed control in traverse in order to slew the gun to a new target. The basic units in a typical turret-traversing and gun-elevating system are shown schematically in Fig. 3-17.

The optical range finder is usually operated by the tank commander. It is a combination direct-fire sight and range-measuring instrument of either the stereoscopic or coincidence type and constitutes the primary sighting system for our current medium tanks. The gunner operates the range finder by setting it for the type of ammunition being employed, measuring the range to the target, tracking the target, and firing. The range is transmitted automatically to the computing sight.

The range-finder sight provides adjustment for the visual characteristics of the observer, a filter for use under bright-light conditions, and a ballistic correction system that allows adjustment for variable conditions affecting velocity (e.g., air density, powder temperature, tube wear, etc.).

Although complex in design, the range

* Refer to par 1-3.2 of Chapter 1 for a discussion of the development of fire control systems for tanks that has taken place subsequent to World War II.

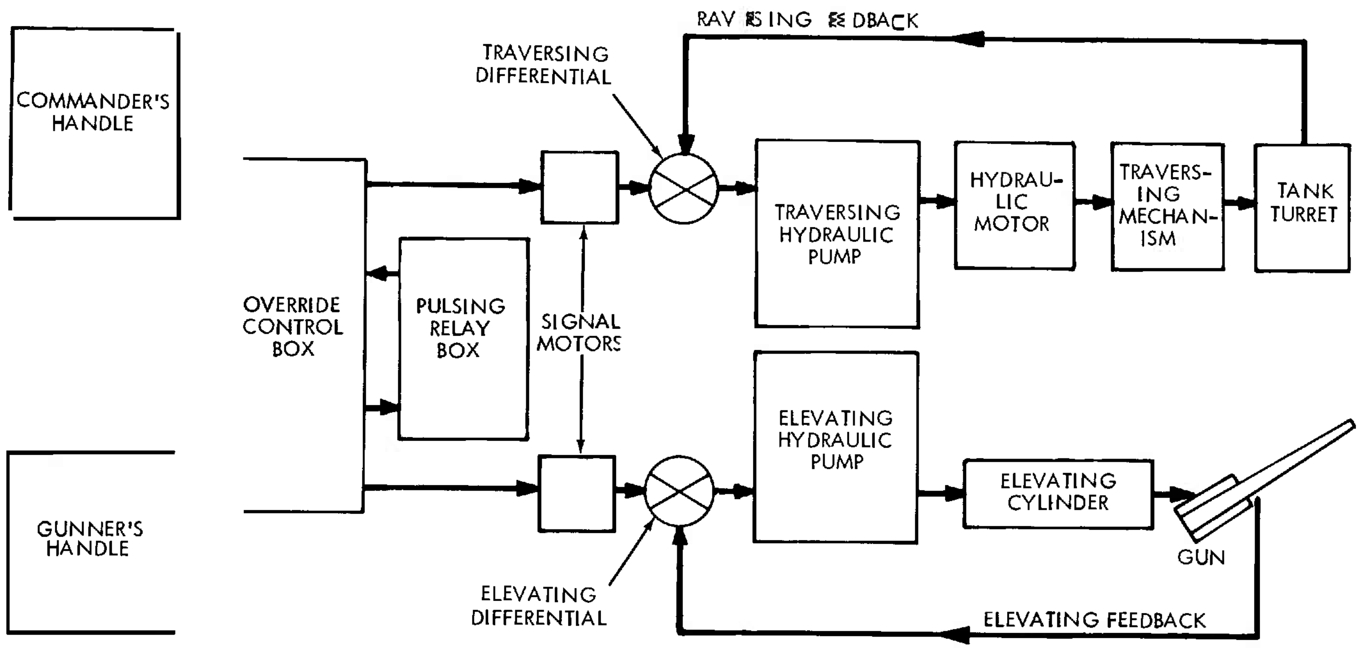


Figure 3-17. diagram of the turret-traversing and gun-elevating system for the 90 mm Gun Tank, M47.

finder is relatively simple to operate when engaging targets. The process of ranging a target requires only a few seconds. Thus, the probability of destroying the target with the first round is greatly increased.

The computing sight is for use against moving targets. It mechanically computes the kinetic lead angle and feeds it to the gunner's telescopic sight, thereby displacing the gun axis from the line of site by the necessary lead angle. The required superelevation is based upon the range data received from the range finder.

The vertical and horizontal stabilization is of the conventional gyrostabilized variety. It should be noted that both the gun and range finder must be stabilized. The stabilizer has increased the accuracy and efficiency of tank guns in battle, while the vehicle is in motion, several hundred percent. It alone, however, does not substitute for a good gunner, since accurate sighting and range corrections must still be maintained. The stabilizer, does, however, dampen movements of the gun due to rough terrain and tank vibrations and helps to achieve effective fire control.

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1. C. S. Draper, W. McKay, and S. Lees, Instrument Engineering. Vol. I. Methods for Describing the Situations of Instrument Engineering, Chapter 2, McGraw-Hill Book Company, Inc,, New York, N.Y. , 1952.
2. AMCP 706-331, Engineering Design Handbook, Compensating Elements.

CHAPTER 4
DESIGN PHILOSOPHY*

4-1 INTRODUCTION**

The achievement of a successful design for a fire control system is likely to result from the observance of two fundamental technical requirements:†

1. The system designer (or team†† of system designers) must obtain a clear understanding of the objectives of the system and a good conception of the functional breakdown of the system into subsystems and of the characteristics required of each of these subsystems. In other words, at least those who are leading the design team must become completely familiar with the fire control problem concerned.

2. The system designer must employ a unified design approach or philosophy if he is to achieve an optimum solution to his problem. This means that his attention must be directed toward the development of a system that is optimum in some clearly stated overall sense. Achievement of an overall optimum frequently means that some of the subsystems will not have as high performance as they might have if attention were focused specifically on achieving maximum performance from each subsystem individually. Realistic performance specifications on the subsystems can be set only in relationship to the role they play in the overall system. For example, the chief results of incorporating a very precise computer in a fire control system in which the basic information-gathering equipment introduces large errors are to increase the cost of the system and, quite probably, to reduce its reliability.

The first requirement will, of course, be met automatically if the system designer is completely familiar with the type of system concerned. This situation occurs only if the system to be designed represents merely a small change from systems that have been built previously. If, however, the system under consideration represents a significant advance over existing systems or is intended to serve some completely new need, the designer must become familiar with the broad aspects of the problem by making a preliminary study. Should such a study be necessary, it should lead to a very clear understanding of (1) the basic functions the fire control system is to perform, (2) the basic physical phenomena involved in the operation of the system, and (3) the form of the system that will be acceptable to the ultimate user and compatible with the conditions imposed by such factors as environ-

* Part of this chapter and of other chapters of the Fire Control Series are based on developmental work performed by the late Dr. John G. Tappert. For a more appropriate acknowledgment and tribute, the reader is referred to the special item following the references at the end of this chapter.

** By W. W. Seifert. Based largely on Seifert and Steeg,[1] this material also incorporates concepts on the subject of system design presented by Draper, McKay and Lees,[2] Jerger,[3] Wrigley and Hovorka,[4] and Floyd and Eisengrein.[5]

† For the purposes of this handbook, only technical requirements are considered. In actual practice, of course, contractual relationships, accounting organization, personnel organization, and policy supervision must all be properly coordinated with the engineering effort in order to achieve a satisfactory result.

†† Among the advantages of a team effort are (1) the increased objectivity obtained by subjecting "judicious" assumptions and decisions to critical review by team members with different areas of specialization as the design and analysis progresses and (2) the availability of designers skilled in a variety of technical disciplines. It should be pointed out that mere numbers do not insure an effective team. Each member must be highly trained in some particular area and in addition must be anxious to communicate with other team members.

ment and economics. Every effort should be made to differentiate between those system characteristics that are essential and those that are accidental. Many pitfalls await the designer who cannot pass quickly over the trivial aspects of the problem and concentrate on those of fundamental significance.

The remainder of this chapter outlines a design philosophy that is useful in meeting the second requirement. This philosophy should be viewed more as a mental approach to the design problem than as a design methodology in which procedures are emphasized. The rapid advance of technology has created situations such that modern systems can be designed efficiently only by the application of a unified design philosophy. As systems have become larger and more complex, system designers have found it increasingly difficult to capitalize on their specific experience. Instead, they have found it necessary to develop systematic procedures, utilizing mathematics to the fullest extent, in order to augment the limited ability ofthe designer to cope withthe complete system design in any but an orderly procedure.

The magnitude of the problem associated with the design and development of a fire control system is exemplified by Fig. 4-1. This figure represents the complexity of information flow required for the development of a typical instrument system, such as a fire control system, as conceived by Draper, McKay and Lees? It shows that the complete development of an instrument system separates into three technical phases:

1. The research and development phase which is initiated by the receipt of performance requirements, culminates in the engineering test model design, and yields as its primary output the information required to carry out the production design. The secondary output is design information for test and inspection equipment.

2. The production design development phase which culminates in the production design and yields manufacturing information for (a) the system as a final product and (b) the test and inspection equipment required during manufacture of the system.

3. The production phase which culminates in the desired product. Ordinarily, test and inspection equipment is used in the production manufacturing procedure and is produced only incidentally as an output of the production phase.

This handbook is concerned primarily with the first of these three phases.

The basis for a fire-control-system design is usually a performance specification of some kind that defines the performance requirements. The particular form of this specification will vary, depending on the originator and the particular type of fire control system concerned. The technique whereby the performance required of the fire control system is determined is called operations research and is outside the scope of this handbook. Rather, the concern here is with the translation of a performance specification into a successful system design.

Because the various steps in the design procedure overlap to a considerable extent, it is impossible to separate them completely on either a chronological or a functional basis. For example, some of the functions outlined in the discussion which follows may be carried out, at least in a superficial manner, during the preliminary study phase. For present purposes, it is assumed that a preliminary study has been completed, and a rather good understanding of the task to be performed by the fire control system and of the fundamental limitations imposed on its design has been obtained. These fundamental limitations are associated with such factors as the mass of the weapon that has to be positioned and the projectile time of flight.

Once a good understanding has been obtained of the functions to be performed by the fire control system, of the accuracy required, and of other specifications to be met, the design process consists in a large part of developing successively more detailed models of the system until, finally, detailed drawings and parts lists are completed and fabrication can be undertaken. This model concept, which is implicit in the discussions of the preceding chapters, is sketched briefly in the sentences below and is developed in detail in par 4-2 of this chapter.

The first model usually employed is a word model, which is a rather inexact description of the major portions of the fire control system and of the manner in which these subsystems will be related. Such a

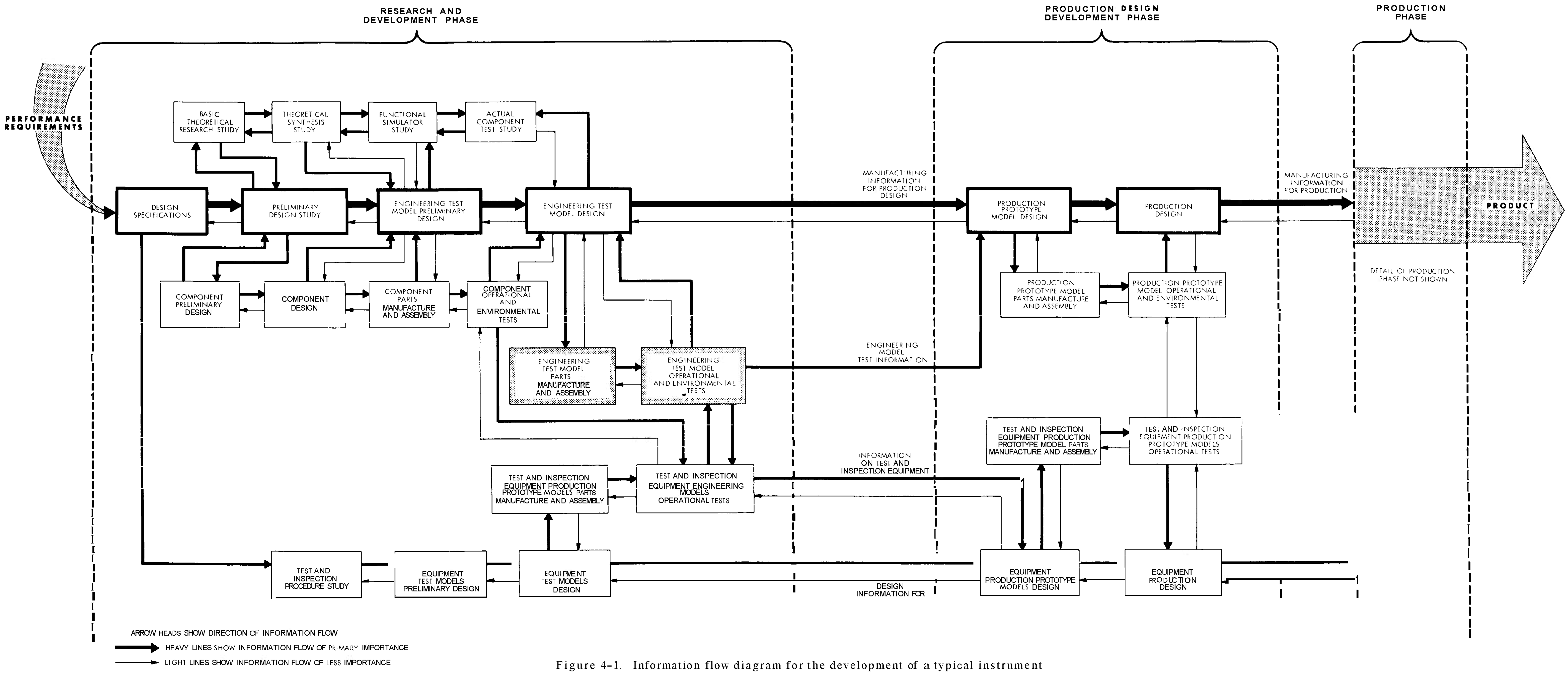


Figure 4-1. Information flow diagram for the development of a typical instrument system such as a fire control system. (From Volume I of INSTRUMENT ENGINEERING by C. S. Draper, W. McKay, and S. Lees. Copyright © 1952 by McGraw-Hill, Inc. Used by permission of McGraw-Hill Book Company.)

crude model can be created relatively easily once an adequate preliminary study has been completed. As a next step, a rudimentary signal-flow or block diagram may be constructed. Next, a series of mathematical models is developed, each one in turn serving as a closer approximation to the actual physical system that is being evolved. Examination of these mathematical models by either analytic or computer techniques permits the designer to predict, to a degree established by the accuracy of the model, the performance of the system before it is actually constructed. Paragraph 4-3 describes the steps involved in the development of mathematical models.

Throughout the development of the mathematical models, the designer must carefully consider the factors that affect the accuracy of the fire control system. From operational requirements on hit probability (as defined in the performance specifications), the allowable error in each subsystem of the fire control system may be apportioned by mathematical techniques. These techniques may then be further applied to deduce the allowable errors in individual components. The application of the mathematical techniques must, of course, be tempered in each instance by the engineering judgment of the system designer as applied to the particular system concerned. System and subsystem accuracy considerations are analyzed in par 4-4.

Once a mathematical model has been evolved that is sufficiently detailed to include all effects that have any significant influence on the performance of the overall fire-control system, attention is turned toward mechanization of this mathematical model. In its broad sense, this phase of the design procedure involves the detailed design, fabrication, and testing of the complete fire control system. Here, the system designer must call upon his knowledge of current progress in fire control systems and components, and computer technology in general, and apply his ingenuity in order to achieve any advantage that might be gained by employing single physical components to combine several separate mathematical functions. The procedure for mechanizing the mathematical model is discussed in more detail in par 4-5. During this mechanization phase of the design procedure, the maintainability of the equipment, its ability to withstand the military environment, and the integration of the human operators who form a part of the system, must all be considered. These additional topics are discussed separately in Chapter 5.

The major task of the system designer is complete once a workable fire control system has been achieved. Nonetheless, it will usually be necessary to continuously modify the design of the components and subsystems in an attempt to improve system performance. The techniques employed in this development phase are not, however, different in nature from those used in the design process.

In order to illustrate the use of the aforementioned general concepts as system design tools, this chapter concludes (see par 4-6) with an example of good system design that has been carried out at Frankford Arsenal. The example chosen is the Vigilante Antiaircraft Weapon System, a complex system whose design exemplifies most of the techniques discussed in this chapter.

4-2 MODELS*

Although Webster defines the word model – as used in the scientific sense – as a miniature representation of a thing, the word is used by system designers in a broader sense to denote any scheme whether it be physical, mathematical, or verbal that assists in the representation or study of individual ideas and of the interrelationships existing within a group of ideas.† In this broader sense, models are useful in representing not only physical objects but also the most abstract notions. For the purpose of

* By W. W. Seifert.

† See Chapter 2 of Reference 1.

discussion, it is appropriate to consider the following categories of models:

1. Word or language models
2. Pictorial representations
3. Direct analogs
4. Mathematical models.

During the course of developing any complex system, the designer will probably make use of models of each of these four types. At the inception of a project, the functions that the system is to perform and its general mode of operation are usually expressed in terms of a word or language model. As the program progresses, these word pictures are further clarified by pictorial representations in the form of block diagrams, graphs, tables, and mechanical drawings.* For example, Fig. 4-2 is a very rudimentary functional block diagram for the Vigilante Antiaircraft Weapon System discussed at some length in par 4-6. This diagram, which is characteristic of those employed in the early stages of a system design, shows the principal portions and aspects of the weapon system; namely, the information-gathering, computing, and weapon-pointing systems (which together comprise the fire-control portion of the weapon system), the weapon itself, and the weapon-target geometry. A representation as fundamental as that shown in Fig. 4-2 is normally arrived at during the broad preliminary study that precedes the start of the actual design process. Such a rudimentary diagram depicts merely the overall functional aspects of the system without making any attempt to define exactly what physical components might be used in the mechanization of the functions of the various boxes shown or to specify the mathematical relationships involved. The designer has not yet gathered sufficient information to define in any detail the dynamic characteristics of (1) the various components or (2) the inputs to the system. Figure 4-2 indicates the use of a pulsed-Doppler radar for target acquisition and of an optical sight for tracking, but before the designer is in a position to set specifications

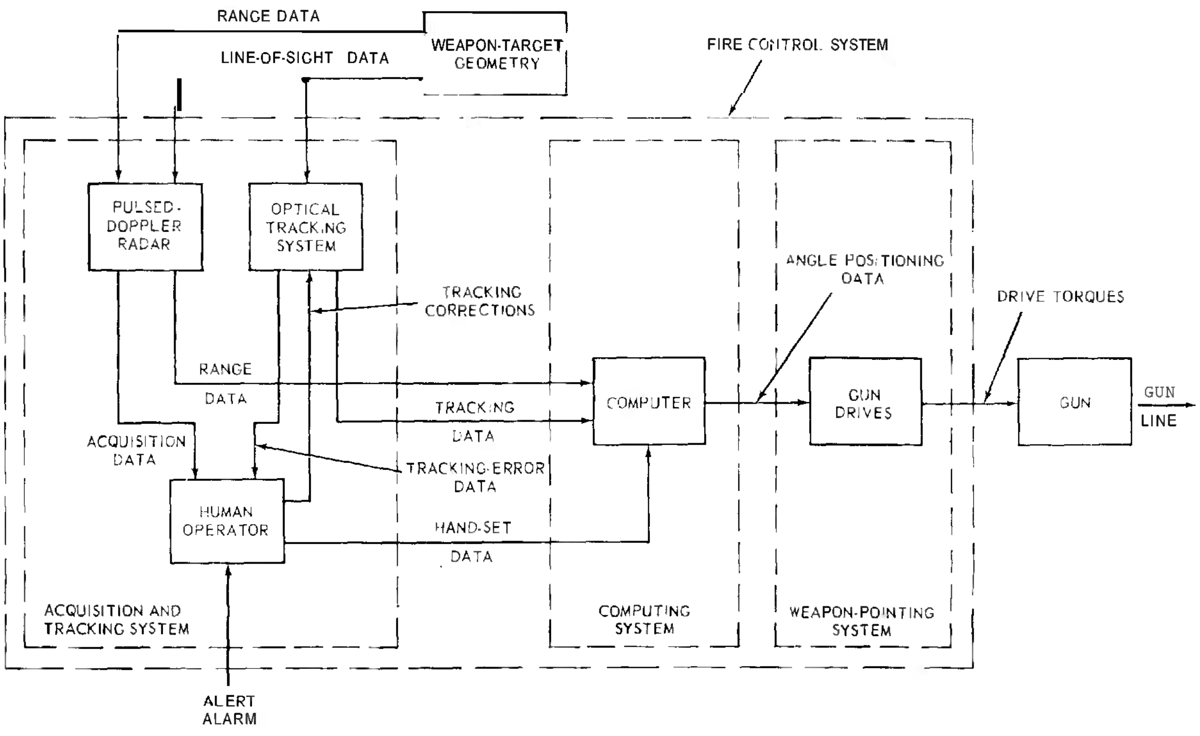


Figure 4-2. Rudimentary functional block diagram of the Vigilante Antiaircraft Weapon System.

* *See* Chapter *2* of Reference *2.*

on the computing and weapon-pointing systemshemust be able to describe, at least in statistical terms, the types of target motions that the system must handle.

It is very important that the overall system be scrutinized critically at this preliminary-study stage because it is here that some of the most important system decisions are made. Unfortunately, serious deficiencies are sometimes inadvertently designed into a system exactly at this point. Here, the designer should be taking a broad look at the overall system to determine whether the prescribed mission is logical and whether the proposed scheme for realizing the system is technically sound. Later, the designer becomes so engrossed in the details that he is apt to lose sight of the broader aspects of the problem. Unless these broad aspects are carefully considered at the very outset, they may never be considered and the effectiveness of the system may suffer seriously as a result.

Unfortunately, the question of whether or not sufficient effort has been devoted to this early phase of tackling the fire control problem can seldom be answered with any degree of certainty. Frequently, the best that can be done is for the design group concerned* to be permitted to present their ideas to their associates, their supervisors, and to whatever outsiders may have an interest in the program and have sufficient background to serve as an effective sounding board. If these various groups all concur that the approach (or approaches) selected for consideration appears to be reasonable and no one is able to cite any significant weaknesses or to present any better alternatives, the design group can feel that they probably have not committed any major blunder.

As an aid in checking a design, in conveying concepts to potential users or to groups who may be appraising the work of the designers, or as a means for deriving needed design data, the designer may make use of the third category of models; namely, direct analogs. These may take the form of a miniature replica - or scale model – of the final system, or of some essential part of it. On the other hand, it may involve the use of extensive test facilities, such as the wind tunnels used to gather aerodynamic data on proposed designs for aircraft and missiles.

This handbook is concerned primarily with the fourth category – mathematical models. As used herein, "mathematical model" is a term used to describe any scheme for the manipulation of ideas in a group wherein the individual ideas are identified by means of more or less abstract symbols, and manipulations are conducted in accordance with precise rules of logic. Paragraph 4-3, following, discusses the steps involved in developing the mathematical models used during the process of designing a complex fire control system.

4-3 DEVELOPMENT OF MATHEMATICAL MODELS †

4-3.1 GENERAL CONSIDERATIONS

Once the overall aspects of a fire control system are thoroughly understood (either through experience gained on similar systems or as the result of a preliminary study) the system designer is in a position to begin a quantitative mathematical analysis of the system, with the ultimate objective being the mathematical model of the most practicable fire control system it is possible to achieve. This phase of the analysis is accomplished by the development and study of a series of mathematical models for the system. The procedure Is discussed only briefly at this point, inasmuch as it is considered in much greater detail in Section 3, in connection with fire control computing systems.

The designer begins the process of developing mathematical models by defining the performance required of the various subdivi-

* In the preliminary-study stage, the fire control system designer will probably be one of a team of systems designers that may include experts in such fields as radar, gun design, carriages, and field-use requirements. In addition, there will probably be at least one member with a broad overall view of all these specialties. The function of this group would be to propose solutions to the fire-control problem at hand, subject them to initial analysis and discussion, and eliminate all but the more promising solutions from further consideration.

† By W. W. Seifert.

sions of the system in rigorous mathematical terms *so* that he can arrive at a complete mathematical model for the fire control system. This mathematical model takes the form of a set of equations that describes the system with sufficient accuracy to permit the designer to (1) evaluate such factors as the dynamic response and accuracy and (2) select appropriate system parameters to be used in the final design. Thus, the mathematical model provides a basis for the study of the simulated system on an analog or digital computer. Such computer studies are usually required at one or more points in the design process and with complex systems must be introduced at an early stage in the design. It is important to note, however, that while the designer must have the accuracy requirements well in mind during the early stages of the formulation of the model, it is only with the completion of a realistic mathematical model that he can effectively analyze the errors of the projected system.

Because neither the selection of the primary variables of the mathematical models nor the method for describing the performance of the various portions of a fire control system is usually unique, these aspects of the mathematical analysis call for considerable insight and good engineering judgment on the part of the designer. Introduced during this phase are many of the simplifications that, on the one hand, may make solution of a mathematical model feasible but, on the other hand, may invalidate the model as an acceptable representation of the actual system. This latter condition can, of course, have very serious consequences since it may lead to a false sense of assurance that the corresponding physical system would perform adequately when, in fact, it would not. At the other extreme, this condition may lead to the false impression that the actual system will not work and thereby cause a designer to abandon an approach that would have been acceptable.

In the course of studying a complex system, the designer will in all probability make use of not just one mathematical model, but of a whole spectrum of models. Although the various mathematical models employed usually represent an evolution achieved in small steps and usually at the expense of some retracing of steps, they can, for present purposes, be classified into the following three groups:

1. Models for idealized systems
2. Models for optimum systems
3. Models for practical systems.

Each of these models is discussed in turn in par 4-3.2 through par 4-3.5.

4-3.2 MODELS FOR IDEALIZED SYSTEMS

It is characteristic of complex systems that there exists not just one design that looks promising (at least in the early stages of a program) but, rather, a number of such designs. Initially, the designer should not limit himself too severely but, instead, should look at as many alternative designs as he can create. As he begins to develop models of the first classification just noted, he should endeavor to keep them as simple as possible while, at the same time, including sufficient detail to demonstrate that success in achieving the desired goals does not hinge upon the violation of any physical laws. Two obvious examples of such basic considerations that will arise in a typical fire-control-system design and must, therefore, be taken into account are (1) the fact that very definite restrictions usually exist limiting the accuracy with which the future path of a moving target can be predicted (see Chapter 2), and (2) the fact that if a fire control system is to contain a computer, a nonzero time is required to perform the necessary computations.

Thus, idealized models serve as a means for defining goals and exploring alternate approaches rather than for describing in detail the hardware that will be used in realizing the final system. It is only after the various alternatives have been examined on this idealized basis that the designer should begin to narrow his sights and look in greater depth at particular systems. Once he has arrived at a few idealized systems for which it appears that no immutable restrictions exist to prevent realization of his goal, the designer can begin to select from this group the system that appears to be optimum.

4-3.3 MODELS FOR OPTIMUM SYSTEMS

The selection of the optimum system from a family of approximately ideal systems that do not violate physical limitations

is termed the optimization procedure. An obvious but essential step in this procedure is the choice of appropriate criteria for comparing the different systems. Although mathematically precise criteria – such as least-mean-square error – may be employed, it should be clearly understood that the final selection of the optimum system will probably be based not upon a single criterion but upon the designer's application of his engineering judgment to a number of criteria ranging from those that are purely mathematical to those involving economics, procurement schedules, size, reliability, etc.

The optimization procedure entails the use of several mathematical models, with an increasing degree of realism and concern with the practical aspects of final production of the system. In carrying out this procedure and the associated detailed study and specification of the various subsystems that comprise the complete fire control system, the system designer will undoubtedly wish to study his mathematical models on a modern computing machine. Important factors to be borne in mind in applying computers to this end are summarized in par 4-3.4.

4-3.4 APPLICATION OF COMPUTERS TO THE STUDY OF MATHEMATICAL MODELS

The computer used in the study of the mathematical models chosen to represent a fire control system may be either of the analog type or the digital type, with the choice between the two types dependent on the exact nature of the problem. With the present state of the computer art, it can be stated that, in general, an analog computer offers some advantage when studying actual physical devices, such as when a computer is used to represent an analog of a physical system whose components are to be realized in actual hardware. The study of the dynamic response of a servo system is a typical case in which an analog computer would be used. On the other hand, if the mathematics involved represents the description of physical relationships – e.g., vector resolution – that are automatically satisfied in nature, a digital machine may offer decided advantages. This will be particularly true if high-accuracy calculations must be carried out for a wide range of problem variables, especially if real-time simulation* is unnecessary. The calculation of the trajectory of a long-range projectile is illustrative of this situation. Here, the mathematical model might involve several coordinate systems in order to permit introduction of drag and gravity effects and the associated vector resolution. Whereas these effects are inherently included in the trajectory followed by the actual projectile, inclusion of them in a mathematical model requires the use of several Coordinate systems, with transformations from one system to another required in order to compute the trajectory. No attempt will be made to discuss the design or operation of computers at this point since Section 3 of the Fire Control Series is devoted largely to a discussion of this subject.

Regardless of which type of computer is employed, a number of factors must be considered in the process of preparing to study a mathematical model on a computer. Some of the more important of these factors are the following:

1. Information to be computed
2. Degree of sophistication necessary
3. Accuracy required
4. Solution time
5. Choice of parameter ranges.

Each of these five factors is discussed briefly in the paragraphs which follow,,

4-3.4.1 Information To Be Computed

The first step a designer should take before plunging into the work of simulating a mathematical model on a computer is to define clearly the type of information being sought. A clear definition of what is to be computed will determine to a large extent the complexity of the computer study and the number of different computer setups that may be required. In addition, it may dictate particular quantities that should be recorded or computed in order that the problem of analyzing the computer results and arriving at engineering-design decisions based upon

*Real-time simulation is the computer solution of a problem, or event, in which the computer produces the solution in the same time that the event requires for completion in the actual physical system concerned.

these results may be minimized.

4-3.4.2 Degree of Sophistication Necessary

Obviously, there is no point in studying a mathematical model that is more complex than is required to yield the information being sought. The computer programming becomes more difficult as the problem complexity increases and at the same time, the accuracy tends to deteriorate. Furthermore, with a digital computer, the solution time increases with problem complexity. Consequently, much is to be gained by employing the simplest model that still retains the essential characteristics of that particular aspect of the system under study. For example, in computing the trajectory of a projectile (see Chapter 2), it may be perfectly adequate in the case of relatively short-range fire to consider the projectile as a single point mass moving in space; whereas, for a longer-range projectile, it may be necessary to simulate the dynamic response of the projectile as an aerodynamic body in order to determine its trajectory with sufficient accuracy. Generally speaking, it is preferable to gather one type of data using one model and another type using a different model, than to utilize a single model with the complexity necessary to yield both types of information. In making simplifications of this type one must, of course, determine that each model is adequate for the particular purpose for which it is used.

4-3.4.3 Accuracy Required

Basically, the computer setup on which the mathematical model of a system is to be studied must, to be useful, provide an accuracy sufficient to permit engineering decisions to be made from the solutions obtained. Several different considerations are involved. The most exacting of these is concerned with the absolute accuracy of the results. Here, one might be interested, for example, in actual miss distances and might consider the accuracy insufficient if a computed result were 28 ft, whereas the correct solution to the mathematical problem actually set up on the computer was 17 ft. In some situations, on the other hand, even though the absolute accuracy may be less than desired, the solutions obtained may be entirely adequate for predicting the influence of particular system parameters on the overall performance. As a minimum, however, the computer must produce solutions that are reproducible to a precision greater than the variations that are to be attributed to parameter changes. For example, if the computer is capable of reproducing a miss-distance solution to within an error dispersion of ±5 ft, it is ridiculous to use this particular computer to evaluate the effect of parameter changes that cause only 1-foot changes in the result.

The question of predicting solution accuracy on either an analog or a digital computer is difficult – so difficult, in fact, that it is generally not possible to specify the accuracy mathematically. Certain indications do exist, however, that frequently prove useful in allowing the system designer to appraise the quality of solutions obtained on computers, namely:

1. For digital computers, these indications are variations in the computer results that are obtained when there is either
   a. a change in the time increment and thus the number of steps in a given integration interval, or
   b. a change in the word length used to represent data.
2. For analog computers, on the other hand, the useful indications are variations in the computer results obtained when there is either
   a. a change in the time scale at which the solution is run, or
   b. a small change in selected gain settings at various points within the computer setup.

Although the absence of such indications by no means provides absolute proof that the solutions obtained are accurate, it is a sign that no major difficulty exists if the results obtained are not sensitive to any of these changes.

A great deal of effort can be expended (particularly on an analog computer) in attempting to achieve accuracies higher than those of which the equipment is basically capable, and often higher than those needed for the engineering-design purposes at hand. Also, a great deal of time can frequently be wasted in trying to appraise small comput-

ing errors when, in fact, some major error has been introduced in problem formulation or computer programming, or when the design data desired can be derived just as well from somewhat inaccurate solutions as they could from mathematically precise results. The important point to bear in mind is that one should not blindly accept the results obtained from a computer as being correct, nor should one become preoccupied in attempting to achieve a solution accuracy much higher than that required for the study being conducted.

4-3.4.4 Solution Time

The time required to run a solution on a computer may be greater than, equal to, or less than the time required for the event to take place in the actual physical system. If the entire physical system is simulated on the computer, then the choice of solution time, or time scale, is arbitrary. If the computer is capable of operating with a compressed time scale – i.e., if the computer produces a solution in less time than the event takes in the actual physical system (real time) – considerable overall tirne may be saved if the number of solutions to be examined is large. This situation occurs frequently when analog computers are used. Some analog computers are, in fact, designed to obtain solutions at the rate of 15 to 30 per second. Such machines are particularly well adapted for making statistical studies. On the other hand, the solution of a high-order dynamic system on a digital computer may require much longer than real time. This situation may be inconvenient but is still acceptable for many studies.

The only case in which no choice in time scale exists is when it is desired to include some of the physical components from the actual system in the simulation. In this case, meaningful results can be obtained only if the solutions are run in real time. The programming of a digital computer to run in real time may be impossible, depending on the complexity of the problem and on the characteristics of the machine. In any event, such programming for a digital computer represents a more difficult task than exists if no fixed solution time is specified.

4-3.4.5 Choice of Parameter Ranges

The fact that a computer is capable of producing a large number of solutions to a problem in a relatively short time tends to be a trap. There is little point in generating a much larger number of solutions than can be analyzed because this merely ties up computer time and increases the problems of adequately identifying solutions so that particular ones can be found readily. Nonetheless, the system designer is usually inclined to ask for more solutions than he really needs because he wishes to be sure he has covered all cases that might be of interest. A ready availability of the cornputer to the designer is helpful in reducing this tendency. The important point is for the designer to be realistic in regard to the number of solutions for which he asks. Although it may be easy for him to specify that he wishes to have solutions for 20 combinations of 20 different parameters, the running of the resulting 400 solutions and his evaluation of them (if he is to be at all critical) may require an exorbitant amount of effort. Frequently, much of this effort can be saved if the designer spends just a little more time deciding what he really wants. It is more effective to survey a problem rather roughly in a first set of runs and then examine regions of real interest in a second, more-detailed series than to try to do the whole job in one operation. The first method has the added advantage of permitting the designer to change the course of the study before too much effort is expended, in case a whole new approach is indicated by the first survey.

4-3.5 MODELS FOR PRACTICAL SYSTEMS

As a result of the analytic and computer studies just summarized, the system designer should be able to arrive at a mathematical model for his system that is optimum in some sense. Although construction of an actual system patterned after this optimum model should be within the realm of physical possibility, provided the optimum model was formulated correctly, it will probably be desirable to make a number of compromises that will facilitate production, reduce cost, or achieve some other desired result. At this stage, in particular, the system designer

and the equipment designers should exchange information freely so that, on the one hand, the system designer will be fully informed as to limitations faced by the equipment designers and can therefore attempt to evolve a system that will be least affected by these limitations and, on the other hand, the system engineer can keep the equipment designers informed as to areas where component improvements would pay substantial dividends in system performance.

As more and more of the practical aspects of realizing the physical system are defined, additional information must be gathered to indicate the manner in which deviations from the system as defined by the optimum mathematical model will affect the performance of the final system. This phase of the work will usually involve the formulation of another model incorporating these changes. The performance of the optimum-system model (as has been determined by simulation studies) then becomes a kind of yardstick against which the performance of the model for the practical system can be measured. The evaluation will usually be obtained by computer simulation and will be governed by the same criteria discussed in connection with models for optimum systems. If this evaluation shows that the performance of the practical system will be essentially as good as that of the optimum system, then the job of mathematical formulation in essentially complete and detailed drawings can be completed and fabrication of the physical system can be undertaken. However, if the compromises that it is felt must be made in the optimum system to permit its practical realization degrade its performance seriously, it may be necessary to retrace the design steps and look at other idealized models and their optimum and practical counterparts.

4-3.6 CONCLUSIONS

The discussion presented thus far in connection with the development of mathematical models might lead one to conclude that the mathematical analysis of a system would lead to a unique solution for any particular problem. This is not a correct inference. As technology has developed, the result has been that fewer and fewer cases exist where it is technically impossible to build any one of several systems to accomplish a stated task, providing the task itself is realistic. Consequently, the real test of good engineering involves the development of the particular system that will meet the established specifications most economically and with reliability sufficient to meet the system requirements.

The designer striving to develop a system that is optimum in this sense must not only consider the specific components with which a design might be realized, but must consider the basic approach as well. For example, the computer in a proposed fire control system might be realized on either an analog or a digital basis; early in the analysis of the system, each of these possibilities should be examined and the advantages and limitations of each should be carefully weighed. Under some conditions, the wise approach to system design may be to refine existing proven techniques so as to meet new requirements. In other cases, it may be better to attempt a totally new approach. At the initiation of the design of a complex system, it is most important that the system designer be given the freedom necessary to examine the alternatives and an opportunity to present his findings to those who ultimately will decide what approach should be taken. A small expenditure of effort at this stage in a design may result in very substantial savings later.

The next topic of this chapter concerns some of the techniques employed in describing system accuracy (see par 4-4). Following that (see par 4-5), the discussion returns to some of the considerations involved in the mechanization of the mathematical model derived by the procedure just described.

4-4 SYSTEM AND SUBSYSTEM ACCURACY CONSIDERATIONS *

* By E. St. George, Jr. The basic overall source material for par 4-4 consists of References 1 and 6 through 10. Other references are given where specifically applicable.

4-4.1 INTRODUCTION

A complete projectile - firing weapon system* – of which a fire control system is a major part; see Fig. 4-2, for example – is designed to possess the capability of destroying a hostile target. In turn, the fire control system of this weapon system is designed to point the projector (gun or launcher) in such a way as to direct the projectile toward a point sufficiently close to a selected target that the target may be destroyed. The terms "target" and "destroy" are not subject to simple mathematical definition. However, in a given military situation, targets – which may be troop formations, buildings, aircraft, etc. – can be defined with more or less precision. Also, the degree of target destruction required, and the destructive effect of individual projectiles of various types, can be specified. Such problems are generally handled by specialists in operations research and military strategy, and the results are transmitted to the fire-control-system designer in the form of military requirements, or specifications, on the accuracy of fire.

The destructive effect of a weapon system is determined by the destructive effect of the individual projectile, the accuracy with which the projectile can be delivered to the vicinity of the target, and the number of projectiles that can be delivered to the vicinity of the target during an engagement. The number of projectiles delivered will be determined by the rate of fire and the length of the engagement; the engagement length is primarily determined by target characteristics, the range of the weapon, and user requirements, but may be somewhat affected by the fire-control-system design.

The overall characteristics of the projectile, the projector (gun or launcher), and the fire control system will be determined during the preliminary design of the weapon system, based on a balancing of the factors involved. The main objective is to maximize the destructive power of the weapon system. Secondary objectives may be to maximize the range at which the engagement commences, to minimize the time required for the engagement by increasing the rate of fire or by decreasing the settling time of the computer, to minimize the amount of ammunition fired without destructive effect, and to prevent overkill, i.e., expenditure of ammunition beyond the minimum required for target destruction.

Accuracy is the primary factor under the control of the fire-control-system designer. In fact, fire-control-system accuracy is the basic specification from which he must determine subsystem accuracies and speeds of response. The remainder of par 4-4 will consider the accuracy-specification problem in some detail. First to be discussed will be the specification of system accuracy. This will be followed by a description that shows how the subsystem accuracies are determined.

As previously stated, the fire control system determines the orientation that the gun or launcher must have in order for the projectile to destroy the target. Most of the presently existing fire control systems are capable of solving this problem only for a fixed target or a target moving in a straight line.† Since conclusions drawn for the case of a moving target can readily be reduced to the simpler fixed-target case, only moving targets are discussed.

If the target is not taking evasive action, the fire control system can track the target and, from this tracking information, can determine the velocity and direction of the straight-line motion. The system must then compute the orientation of the gun or launcher that will cause the projectile trajectory to intersect the target path at a common point in time. (Such an intersection defines a perfect hit.) In other words, an ideal fire-control computer must continuously compute the intersection in space of the target path – extrapolated as a straight line – and a ballistic trajectory. Because of the complexity of the ballistic computation, all practical fire-control computers approximate the exact solution to some degree. Errors due to the approximations employed

* The restriction to projectile-firing weapon systems excludes guided-missile fire control systems, in accordance with the corresponding restriction of the subject matter of the Fire Control Series of Engineering Design Handbooks (see Chapter 1).

† However, the M33 fire control system[11] provides a curvilinear solution and considerable theoretical investigations of this approach have been made[12].

can be computed by straight-forward techniques and would, of course, be designed to be small compared with the physical errors of the system (see par 4-4.1.1).

The target and projectile are not, of course, simple points, as assumed in the preceding paragraph. From the standpoint of destruction, or kill,* the target has certain vulnerable sections which do not necessarily correspond to its geometrical outline.** The impact of a projectile (or portion thereof) upon any of these vulnerable sections will have a destructive effect. For practical purposes, such an impact defines a hit.

The effectiveness of a hit is a function of the kill probability. In turn, the probability of a hit is a function of the errors of the weapon system involved. In the normal course of the design of a weapon system, initial studies concerning the required weapon-system effectiveness result in a specification covering the hit probabilities required for specific situations. If we assume the contributing errors from other parts of the weapon system than the fire control system to be beyond the control of the fire-control-system designer and therefore to be considered as constants, the objective of the fire-control-system designer is to meet these hit-probability requirements through control of the errors in the fire control system.

4-4.1.1 Systematic and Random Errors†

The various physical errors in a fire control system, as distinguished from the errors inherent in the mathematical model that are discussed in par 4-3, may be characterized as being either systematic (bias type) or random (dispersion or noise type). Systematic errors are caused by such factors as malalignment or slow drift in components. Random errors are caused by such factors as radar noise, uncertainties in bearings and gearing, and ammunition variations. The effect of any error is, of course, to produce a displacement (measured in a plane perpendicular to the trajectory in the immediate vicinity of the target) of the burst point from the target center, where the burst point is considered to be the center of the destructive effect of the projectile. In the case of uniformly random errors, the burst points will have a circular random dispersion centered at the target. The circular random distribution is discussed in greater detail in par 4-4.2. If systematic errors are also present, the center of the distribution will be displaced from the target by a distance defined as the bias. Frequently, a weapon system will incorporate two axes, each of which is subject to random errors but in differing degree. In this case, the burst points will have an elliptical, rather than circular, distribution.

All practical fire control systems have both dispersion and bias errors, although in field-artillery problems the bias may be reduced to a small value through the use of spotters. It is also useful to note that an amount of dispersion approximating the amplitude of the bias is helpful in increasing the engagement hit probability (see par 4-4.1.2) by ensuring that at least a few hits out of a salvo will be on target, as illustrated by Fig. 4-3.

For diagrammatic simplicity, Fig. 4-3 shows the target as a circular area and the projectiles as points, as would be the case if one were firing a rifle at a bull's-eye target. As shown in Fig. 4-3(B), an increase in the dispersion error increases the probability of a hit when a bias error is present.

4-4.1.2 Engagement Hit Probability††

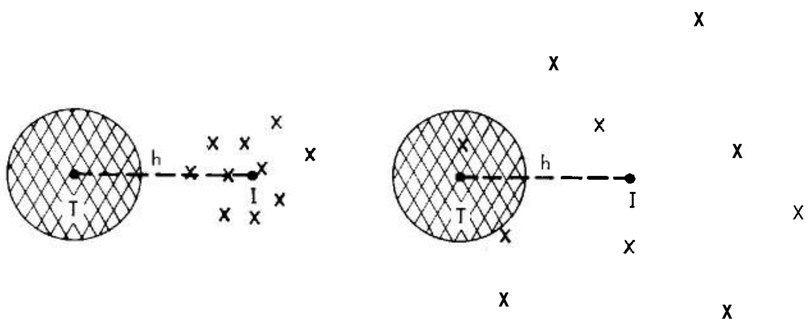
Paragraph 4-4.1.1 introduces the idea of the engagement hit probability, as opposed

* As explained in par 4-4.3, a kill is usually defined in Army technical terminology as the damaging of a target to such an extent that it is no longer capable of effective offensive action.

** One useful concept, that of the diffuse target, is discussed in par 4-4.3.1. Simply by modification of the "hardness" coefficient, the diffuse-target concept can accomodate the distribution of shrapnel from a projectile burst, or the effects of machine-gun bullets, or contact-fused projectiles in arriving at a measure of target destruction that is termed the kill probability. Still greater precision in the description of target and projectile characteristics is possible, but not often required.

† See pages 47-98 of Reference 7, Reference 13, and Chapter 29 of Reference 14.

†† See pages 47-98 of Reference 7, Reference 13, and Reference 15.



(A) Dispersion less than the bias

(B) Dispersion greater than the bias

DEFINITIONS:

T = ASSUMED AREA OF TARGET

I = CENTER OF DISPERSION, OR MEAN VALUE, OF BURSTS (ALSO KNOWN AS CENTER OF IMPACT, C.I.)

h = BIAS

x = BURSTS

Figure 4-3. Dispersion and bias in an engagement.

to the single-shot hit probability.* Rarely is a target engagement limited to a single shot; on the other hand, ammunition certainly cannot be expended without limit. Essentially, the number of shots permitted in an engagement will be determined by such factors as the lengthof time the target remains in range, the firing rate of the weapon, and the supply of ammunition available.

In the accuracy evaluation of a newly developed weapon system, an attempt is usually made to reproduce the situation present in an actual engagement. Therefore, in such an evaluation, the proper criterion is the engagement hit probability rather than the single-shot hit probability. It is important to note that cases have been recorded in which the designers of a weapon system have so reduced the dispersion in an attempt to improve the single-shot errors, that the dispersion was finally less than the bias. In subsequent engagement trials, the "improved" system had poorer performance than a less-elaborate weapon in which the systematic and random errors, while larger, were better balanced.

4-4.1.3 An Outline of the Procedure for Designing a Fire Control System of Prescribed Accuracy

The capability of achieving a specified engagement kill probability† is the fundamental military requirement that is usually imposed on a weapon system. From this military requirement, a requirement for the engagement hit probability is developed through

* The single-shot hit probability (see par 4-4.3.3) is defined as the probability of obtaining a hit on a given target with a single shot. The engagement hit probability (see par 4-4.3.4) is defined as the probability of obtaining a hit during the course of firing on a target in a given engagement.

† The engagement kill probability is defined as the probability of obtaining a kill during the course of firing on a target in a given engagement.

studies of the weapon system's effectiveness. (In this development, the special discipline of operations research plays the major role.* Inasmuch as the subject of operations research lies outside the scope of the Fire Control Series, however, its use in arriving at a prescribed engagement hit probability will not be discussed here. It should be noted though that the fire-control-system designer should be as familiar as possible with the concepts of operation research – as well as the concepts of hit and kill probability – because the development of the required engagement hit probability is most effective when he is in a position to cooperate with the operations research specialist in this development.) The problem of the fire-control-system designer, stated in its simplest terms, is to derive from this prescribed engagement hit probability the accuracy requirements of the fire control system, then of its various subsystems, and finally of the components that make up the subsystems. In actuality, the fire-control-system designer may find that a particular component is required to have a better accuracy than the current state-of-the-art permits. Sometimes it is imperative to break through the state-of-the-art for the component concerned, provided, the time schedule permits. In other cases, the designer must lay out the system so as to minimize the effect of this component on the overall error of the fire control system. Other problems requiring judicious balancing of component and system errors will also arise.

Since the purpose of Chapter 4 is to describe and exemplify design principles, the basic, straightforward procedure will be presented. With such a straightforward procedure, the task of the fire-control-system designer in determining accuracy requirements would be carried out in the following steps:

1. Determine the required single-shot hit probability of the weapon system from the value specified for the engagement hit probability.

2. Determine the allowable overall error of the weapon system from required single-shot hit probability.

3. Determine those errors of the weapon system that are inherently beyond the control of the fire-control-system designer. (These errors are primarily those associated with the input and output portions of the weapon system.)

4. Establish the allowable error for that portion of the fire control system whose errors are not beyond the control of the designer. (The part of the fire control system concerned is primarily the computing system.)

5. Determine the allowable errors of each of the subsystems and components of the fire control system that are under the designer's control, in accordance with the contribution of the particular subsystem or component to the total error.

It is important to note that, in actual practice, the determination of accuracy requirements, like the design process itself, is generally an iterative process. Therefore, in an actual situation, none of the steps given is likely to be a straightforward procedure.

Because of the fundamental importance of probability concepts in the fire-control accuracy problem, a review of the basic concepts of probability theory is presented first in par 4-4.2.

Despite the fact that kill probabilities are not directly involved in fire-control-system design, it is important that this fundamental concept be clearly understood by the fire-control designer, as already noted. Accordingly, in par 4-4.3 and par 4-4.3.1, kill probability and hit probability are defined, and their interrelationships are developed. Then, in par 4-4.3.2 through par 4-4.3.4, an expression that relates single-shot and engagement hit probabilities in the presence of bias and dispersion errors is developed. This expression is given by Eq. 4-73 in par 4-4.3.4; because of its importance, this equation is boxed. Given the value specified for the engagement hit probability, this expression allows the single-shot hit probability to be determined.

Once the required single-shot hit probability of the weapon system has been determined, the allowable overall error of the

* This stems from the fact that the general relationship that links hit probability and kill Probability is as follows: the probability of obtaining a kill on a target is equal to the probability of obtaining a hit times the probability that the hit will result in a kill. This latter factor, termed the terminal ballistic vulnerability of the target, accounts for the way in which the target vulnerability enters the overall study of the weapon system's effectiveness and is the special province of the operations research specialist.

weapon system can be determined in terms of (1) the bias, or systematic error and (2) the variance of the shot-pattern dispersion (see Fig. 4-3), which is a measure of the random error. This determination of the overall error can be achieved by using the relationships that are derived for the single-shot hit probability in par 4-4.3.3. These relationships show how the bias and the variance are combined to compute the hit probability. For the most general case, in which a normal distribution must be assumed for the bias in an engagement as well as for the dispersion, the pertinent relationships are developed in par 4-4.3.4. These equations directly relate the variance of the shot-pattern dispersion and the variance of the bias to the engagement hit probability.

The overall error of a weapon system is determined by a number of error-producing effects, some of which are not under the control of the fire-control-system designer. These effects can be conveniently classified as (1) those associated with the input part of the weapon system, (which is synonymous with the input part of the fire control system), (2) those arising in the fire control computing system, and (3) those stemming from the output part of the weapon system (which includes the gun and the weapon-pointing system),

In order to determine the allowable error for each of these three parts of the weapon system, based on the allowable overall error of the weapon system, the designer must follow an iterative process. First, errors are allotted to the three parts on the most reasonable basis available. These errors are then combined by the procedure described in par 4-4.4 to give a computed value of the overall error of the weapon system. Several reassignments of the errors allotted to the three parts may be necessary in order to achieve a computed value of the overall error of the weapon system that is within the allowable limit. It should be noted that various combinations of the bias and the variance of the overall weapon-system error can be employed to achieve the required hit probability – single-shot or engagement – depending on the procedure followed.

Usually, the output part of the weapon system – i.e., the weapon itself and sometimes the weapon-pointing system also – is specified and therefore, like the target characteristics, beyond the control of the fire-control-system designer. Also, although the tracking system comes under the control of the fire-control-system designer, this system may be subject to physical constraints that force the designer to treat it as a more or less fixed element in the design. For example, the beamwidth of a radar antenna (and hence the assignable error) decreases with increasing frequency and increasing diameter. Since the diameter may be restricted by considerations of portability and the frequency may be restricted by the available microwave equipment, the designer may find that the accuracy of the tracking system is to some degree predetermined. The design of the computing system must then at least be consistent with the tracking-system accuracy and, if possible, compensate for some of the error in the tracking system. This could be accomplished, for example, by smoothing of the tracking data by the computing system.

Because the accuracy characteristics of the input and output parts of the weapon system are relatively fixed in comparison with those of the computing system, they will be considered first (see par 4-4.5). Then, from a knowledge of the errors associated with the input and output parts of the weapon system, the allowable error of the computing system can be established. Following this, the allowable errors in the various elements of the computing system can be established (see par 4-4.6).

Some of the error-producing effects associated with the input part of a fire control system are determined by the target. Thus, an airborne target flying in turbulent air or a land vehicle traversing rough terrain exhibits a motion that may be considered to be made up of a signal component and a noise component that is caused by the rough motion.

Other error-producing effects associated with the input part of a fire control system are due to the nature of the tracking device employed, including the characteristics of a human operator, if such is involved. For example, if target tracking is being accomplished by radar means, additional noise is introduced by random variations of the reflectivity of the target. These variations

have numerous causes, some of which can be correlated with motions of the target; in general, however, the noise effects must be considered to be completely random. Passive tracking systems, such as infrared, on the other hand, are less subject to noise of this type. The effects of noise in the input portion of a fire control system are described more completely in par 4-4. 5.2.

The error-producing effects stemming from the output part of the weapon system include such effects as random variations in ammunition characteristics, vibration of the gun tube, and changes in atmospheric characteristics from those set into the computer. These effects, which are among the major contributors to the dispersion, are discussed in par 4-4. 5.3.

The error-producing effects arising in the fire control computing system result from systematic errors and random errors in the computing-system elements. These errors may be extremely difficult to establish if the fire control computing system is complex. If the description of the computing system does not require differential equations, an exact error analysis may be performed by the methods described in par 4-4.4.1 and par 4-4.4.2. The error analysis becomes much more difficult if (as may well be the case for a fire control computing system) the solution of a differential equation must be obtained. An approximate method for determining the propagation of errors when the solution of a differential equation is involved has been worked out at **Frankford** Arsenal and is described in par 4-4.4. 3. A brief discussion of the types of error that occur in practical computing systems and useful methods of specifying them is presented in par 4-4.6.1 through par 4-4. 6.3. It is quite clear that it is in the fire control computing system that the designer has the greatest opportunity for modification and thereby accomplishment of the overall accuracy goals of the weapon system. This modification is based on an iterative procedure in which the error analysis described in par 4-4.4. 1 through par 4-4.4.4 is employed to arrive at the optimum computing-system configuration.

This completes the summary of the steps involved in determining the accuracy requirements of a fire control system associated with a given weapon system. The remainder of par 4-4 treats in detail the topics just referenced. For convenience, the main topics covered in the remaining subparagraphs of par 4-4 are summarized in Table 4-1.

TABLE 4-1. SUMMARY OF TOPICS COVERED IN PAR 4-4.2 THROUGH PAR 4-4.6.

Paragraph	Topics Covered
4-4.2	The basic concepts of probability theory that are applicable to the design of fire control systems.
4-4.3	The basic concepts of hit and kill probability theory; included are the following: 1. The nature of the probability-of-kill function. 2. The use of the diffuse-target mathematical model to approximate the probability-of-kill function for computational purposes. 3. Identification of the term "kill probability" with the probability of a kill on a diffuse target. 4. Derivation of expressions for the single-shot hit and kill probabilities, based on the concepts presented in par 4-4.2. 5. Derivation of the relationship between the engagement hit probability and the single-shot hit probability. (It is this relationship that provides the means for determining the required single-shot hit probability for a weapon system, given a specified engagement hit probability for that system.)

TABLE 4-1. SUMMARY OF TOPICS COVERED IN PAR 4-4.3 THROUGH 4-4.6 (cont.)

Paragraph	Topics Covered
4-4.3	6. Derivation of equations that directly relate the variance of the shot-pattern dispersion and the variance of the bias to the specified engagement hit probability, for the general case in which a normal distribution must be assumed for the bias in an engagement. The relationships derived in par 4-4.3 provide a means of determining the allowable overall error in a weapon system, given a specified engagement hit probability and the number of shots in an engagement. This allowable error is usually expressed in terms of the systematic and random components which are respectively specified by the variance of the bias σ_b^2 and the variance of the dispersion σ_d^2.
4-4.4	The steps involved in carrying out an error analysis of a fire control system. Useful error-propagation equations are derived for the two following basic types of analog systems: 1. Systems whose operation is describable by other than differential equations. 2. Systems whose operation is describable by differential equations. These equations are required in analyzing and synthesizing a fire control system from the standpoint of error considerations.
4-4.5	The sources of error in those portions of the weapon system that are largely beyond the control of the fire-control-system designer. These portions of the weapon system may include parts of the fire control system itself, specifically, those parts included in the input and output parts of a weapon system.
4-4.6	The sources of error in those portions of a fire control system that are under the control of the fire control system designer. The fire control computing system is the principal entity coming under this category.

4-4.2 BASIC CONCEPTS OF PROBABILITY THEORY

Before proceeding to a detailed analysis of hit probability, it is important to introduce as background information the basic mathematical concepts of probability theory. These concepts are presented without any attempt at rigorous derivation. The necessary understanding of the fundamental concepts can be obtained from an intuitive approach which avoids the introduction of additional mathematical theory. Those interested in a rigorous derivation are referred to the extensive discussions of probability theory that are found in many excellent books.*

In par 4-4.2.1, the basic ideas of chance and probability are first defined (on an intuitive basis) and then developed in connection with discrete, i. e., individually identifiable, events. These basic ideas are more or less familiar to everyone from their application to games of chance. For conven-

* See, for example, pages 406-582 of Reference 1, pages 47-98 of Reference 7, pages 9-89 of Reference 16, and References 17 through 20. Reference 17 is particularly recommended for its approach from the engineering point of view. Reference 18, on the other hand, warrants recognition for its highly theoretical and rigorous treatment of probability.

ience, the analogy of simple dice games will be employed in developing the definitions and axioms associated with the probability of discrete events.

The basic concepts of probability are then extended, in par 4-4.2.2, to continuous functions of a random variable. Here, the derivations become directly applicable to the fire-control problem.

In par 4-4.2.3, a number of convenient statistical parameters are defined. These parameters provide a numerical measure of the important statistical characteristics of a random event.

In the standard tests on probability theory, a large number of commonly encountered probability distributions are discussed in some detail. In fire-control technology, however, the Gaussian, or normal, distribution and the bivariate normal distribution are the distributions that are most commonly employed. These two distributions are described, respectively, in par 4-4.2.4 and in par 4-4.2.5.

4-4.2.1 Probability Applied to Discrete Events[21, 22]

1. Probability of a Single Event

A discrete event is one that is individually identifiable. The probability of occurrence of a specified discrete event can be defined in either of two ways. If it can be assumed that all events, — i.e. all outcomes of a particular experiment – are equally likely, then the probability of a particular event – designated as A for this discussion – can be defined as the ratio of the number of ways in which A can occur to the number of all events that can occur. This definition is represented by the expression

$$\Pr[A] = \frac{n(A)}{N} \tag{4-1}$$

where

$\Pr[A]$ = probability that event A occurs at any trial

$n(A)$ = number of ways in which event A can occur

N = number of all events that can occur.

For example, if a discrete event A is described as a four showing on the throw of a die, then

$$\Pr[A] = \frac{1}{6}$$

where

$\Pr[A]$ = probability of throwing a four

1 = one way only for event A to occur

6 = total events obtainable from one die.

The concept of probability as defined in this way is sometimes called a priori probability.

A second way in which the concept of probability can be defined is as an "empirical" probability. If an experiment is performed a large number of times, the ratio between the number of occurrences and the number of trials will be assumed to approach a limit which is defined as the probability of the occurrence. The defining statement for an empirical probability is

$$\frac{n(A)}{N} \rightarrow \Pr[A] \text{ as } N \text{ becomes very large} \tag{4-2}$$

where

$n(A)$ = number of occurrences of event A

N = total number of trials

$\Pr[A]$ = probability that event A occurs at any trial.

If, in the previous example, the die is thrown a large number of times, it is known from experience that the empirical probability approaches the a priori probability, or, in this case, 1/6.

2. Probability of Mutually Exclusive Events

Two or more events are said to be mutually exclusive if the occurrence of one precludes the occurrence of the other. For example, if a coin is tossed, heads and tails are mutually exclusive since it is possible to get one or the other but never both.

If two events, A and B, are mutually exclusive, the probability that either A or B will occur is given by the following rule:

If A and B are mutually exclusive, then

$$\Pr[A + B] = \Pr[A \text{ or } B] = \Pr[A] + \Pr[B] \tag{4-3}$$

where $Pr[A + B]$ and $Pr[A \text{ or } B]$ are alternative notations for "the probability that either [A] or [B] will occur", $Pr[A]$ is the probability that A will occur, and $Pr[B]$ is the probability that B will occur. For example, if A is the event of tossing a four with a single die and B is the event of tossing a five, these events are mutually exclusive and

$$Pr[A + B] = Pr[A] + Pr[B] = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

This states that on any single roll of the die the probability of either a four or a five showing is 1/3.

3. Probability of Independent Events

Two or more events are said to be independent if the occurrence or non-occurrence of one in no way affects the occurrence of any of the others. For example, if A and B stand respectively for getting heads in two successive flips of a coin, then A and B are independent since the outcome of the second flip is in no way affected by what happened in the first flip.

The probability that any two events, A and B, will both occur, which is called the joint probability of A and B, is given by the following rule:

If A and B are independent, then

$$Pr[A, B] = Pr[A \text{ and } B] = Pr[A] \cdot Pr[B] \quad (4\text{-}4)$$

where $Pr[A, B]$ and $Pr[A \text{ and } B]$ are alternative notations for "the probability that A and B both occur", i.e., the joint probability of A and B. For example, if A is the event of tossing a four on one die, and B is the event of tossing a four on a second die, and the dice are thrown simultaneously, these events are independent and

$$Pr[A, B] = Pr[A] \cdot Pr[B] = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}.$$

This states that on any single roll of a pair of dice the probability of two fours showing is 1/36. This can be intuitively understood since there are 36 possible combinations that could occur and there is only one combination that yields two fours.

4. Probability of Events that are Not Mutually Exclusive

Two or more events are said to be "not mutually exclusive" if the Occurrence of one does not preclude the occurrence of the other, and if the two events can occur jointly. If two events, A and B, are not mutually exclusive, the probability that either A or B occurs is given by the following rule:

If A and B are not mutually exclusive, then

$$Pr[A + B] = Pr[A] + Pr[B] - Pr[A, B]. \quad (4\text{-}5)$$

For example, if A is the event of tossing four on one die, and B is the event of tossing four on a second die, and the dice are thrown simultaneously, the probability of either event A or event B occurring when the events are not mutually exclusive is

$$Pr[A + B] = Pr[A] + Pr[B] - Pr[A, B]$$

$$= \frac{1}{6} + \frac{1}{6} - \frac{1}{36} = \frac{11}{36}.$$

This states that on any single roll of a pair of dice, the probability of a four showing on one die or the other is 11/36. The term $-Pr[A, B]$ obviates the possibility of fours showing on both dice; thus $Pr[A + B]$ denotes the probability of either A or B, but not both.

Another example of "not mutually exclusive events" would be as follows:

Suppose that $Pr[A]$ is the probability that it will rain (on a certain day in a certain place) and that $Pr[B]$ is the probability that it will snow, and that it is desired to determine the probability that it will either rain or snow. In order to compensate for the days on which there is both rain and snow, it is necessary to subtract the proportion of the days on which rain or snow occur simultaneously; hence the need for the term $-Pr[A, B]$.

5. Conditional Probability

In the case of events that are to some extent interdependent, an important concept is that of "conditional probability". The probability that event B will take place provided that event A has taken place (is taking place or will for sure take place) is called the conditional probability of B relative to A, and is shown symbolically as $Pr[B|A]$. By using

conditional probabilities, it is now possible to formulate a more general rule for the probability that two events A and B will both occur:

$$Pr[A, B] = Pr[A] \cdot Pr[B|A]$$

or (4-6)

$$Pr[B|A] = \frac{Pr[A, B]}{Pr[A]}$$

For example, suppose that two cards are selected at random from a pack and it is desired to know the probability of drawing two aces. If the first card is replaced and the pack shuffled, then the two events (of drawing an ace on the first or second draw) are independent. If A is the event of drawing an ace on the first draw, and B is the event of drawing an ace on the second draw, then the probability of drawing two aces is the joint probability, $Pr[A, B]$. In this case,

$$Pr[A, B] = Pr[A] \cdot Pr[B]$$

and

$$Pr[A] = Pr[B] = \frac{4}{52}$$

Therefore,

$$Pr[A, B] = \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}.$$

If the first card is not replaced, however, then the two events are not independent. $Pr[A]$ remains 4/52, but the conditional probability of B, given the prior occurrence of A, $Pr[B|A]$, is equal to 3/51, there being one card, an ace, missing from the pack. In this case, the joint probability is

$$Pr[A, B] = Pr[A] \cdot Pr[B|A] = \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}.$$

6. Multiple Probability

The probability concepts that have been developed for two events can be extended to three or more events.* Various combinations of the basic relationships are also possible. For example, a desired result might be obtained by different arrangements of two events. Thus, if the desired event E can be obtained by the joint occurrence of A and B, or by the joint occurrence of C and D, the following relationship applies:

$$Pr[E] = Pr[A, B] + Pr[C, D] \quad (4\text{-}7)$$

Since

$$Pr[A, B] = Pr[A] \cdot Pr[B] \quad (4\text{-}8)$$

and

$$Pr[C, D] = Pr[C] \cdot Pr[D] \quad (4\text{-}9)$$

then

$$Pr[E] = Pr[A] \cdot Pr[B] + Pr[C] \cdot Pr[D] \quad (4\text{-}10)$$

For example, if the desired result is an eleven showing on a roll of a pair of dice, there are two possibilities, a six and a five, or a five and a six. Use of Eq. 4-10 shows that

$$Pr[11] = \left(\frac{1}{6} \cdot \frac{1}{6}\right) + \left(\frac{1}{6} \cdot \frac{1}{6}\right) = \frac{1}{36} + \frac{1}{36} = \frac{1}{18}$$

where

A is the event of a six showing on the first die; $Pr[A] = \frac{1}{6}$

B is the event of a five showing on the second die; $Pr[B] = \frac{1}{6}$

C is the event of a five showing on the first die; $Pr[C] = \frac{1}{6}$

and

D is the event of a six showing on the second die; $Pr[D] = \frac{1}{6}$

This example can also be solved intuitively since out of 36 possible combinations with the dice, there are two combinations that provide the desired result.

* *See* Chapter 2 of Reference 16.

7. Application of Probability to Fire Control Problems

Example 4-1 in the Appendix to Chapter 4 contains two problems that illustrate the application of the foregoing principles of probability for discrete events to simple fire-control situations.

4-4.2.2 Continuous Functions of a Random Variable

The discussion on probability in this handbook will now employ the concept of a random variable (sometimes known as a chance variable), which is generally defined as any variable that may have a probability function.* A variable is defined mathematically as a quantity to which an unlimited number of values can be assigned in an investigation. Similarly, a random variable is the numerical expression of the outcome of a random experiment. It should be noted that a random variable may be either a discrete or a continuous function.

1. Continuous Random Variables

Thus far, only the probability of discrete random events has been considered. Consider now the case of a continuous random variable. A random variable is actually a random function of an independent variable, such as time. General usage has established the name "random variable", however, and this terminology will be employed. A function is said to be continuous in an interval if it is continuous at all points in that interval, i. e., if the value of the function can be made as nearly equal to the value at the point as one pleases by restricting the value of the independent variable to values sufficiently near the given value.† In the case of continuous random variables as functions of time or of some other independent variable, statistical properties may be determined from an ensemble, or set, of random variable functions, or from a study of the statistical properties of a single member of the ensemble that is observed through a particular range (or interval) of the independent variable.

As an example of the ensemble method, consider a set (or ensembly) of simultaneously conducted experiments whose results (x_1, x_2, . . . , x_n) can be plotted as functions of time (see Fig. 4-4). Each of these random time functions can be sampled at time intervals such as t_1, t_2, etc. Each sample, at a specified time, say t_1, can be averaged to obtain the ensemble average. Alternatively, any member of the set of random time functions can be averaged with time. If the random variable is a stationary function of time, the ensemble average and the time average will be the same. In general, whenever a random variable is a stationary function of time, either the statistical parameters of the ensemble or the corresponding statistical parameters of the time function can be employed interchangeably.

Most statistical problems associated with fire control systems can be considered to be stationary since the parameters do not change rapidly with time. In contradistinction, problems associated with ground-to-air guided missiles frequently require the use of nonstationary parameters since such rapidly-changing factors as the range and altitude cause continual change in the statistical parameters.

2. Probability Density Function

A continuous random variable x(t) may be quantized into a large number of increments Δx (see Fig. 4-5). Let the symbol x_i denote the event that a random time sample of the random variable lies in the increment $i\Delta x < x < (i + 1)\Delta x$, where i is any positive integer. Then $Pr[x_i]$ denotes the probability of the event x_i and can be expressed, analogous to Eq. 4-2 in par 4-4. 2. 1, as follows:

$$\frac{n_i}{N} \rightarrow Pr[x_i] \text{ as } N \rightarrow \infty \qquad (4\text{-}11)$$

where

n_i = number of random time samples of x(t) falling in the increment $i\Delta x < x < (i + 1)\Delta x$

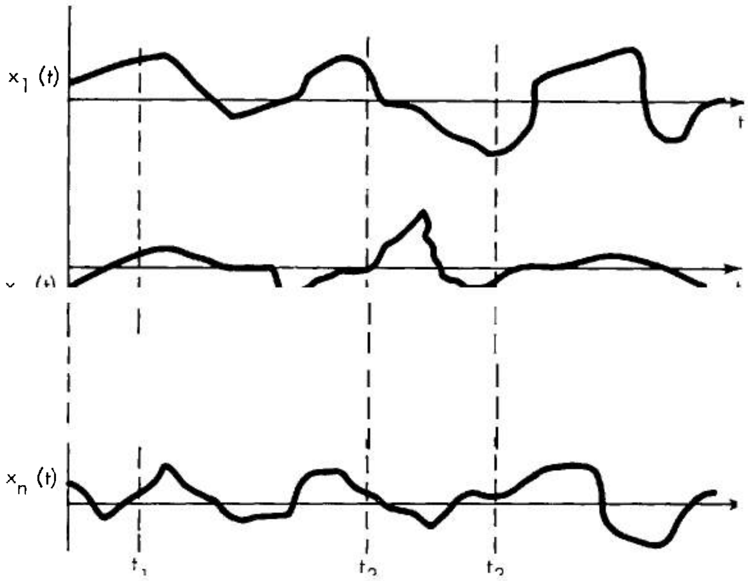
N = total number of random time samples of x(t) falling in all increments.

Figure 4-6 illustrates the concepts involved by means of a simple example.

As Δx becomes smaller, $Pr[x_i]$ tends toward zero. This can be seen intuitively since

* See pages 43 and 294 of Reference 24.

† See page 73 of Reference 24.



NOTE:
ANY OF THE RANDOM VARIABLES $x_1, x_2, \ldots, x$ CAN BE AVERAGED WITH TIME, OR THE ENSEMBLE CAN BE SAMPLED AT SOME PARTICULAR TIME, SUCH AS t_1, t_2, or t_3, AND THE RESULTING TIME SAMPLES AVERAGED.

Figure 4-4. An ensemble of random time variables.

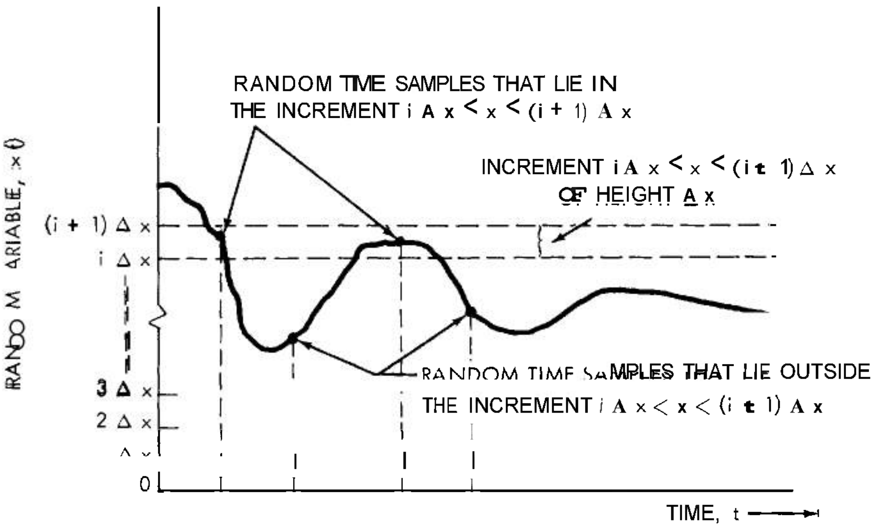
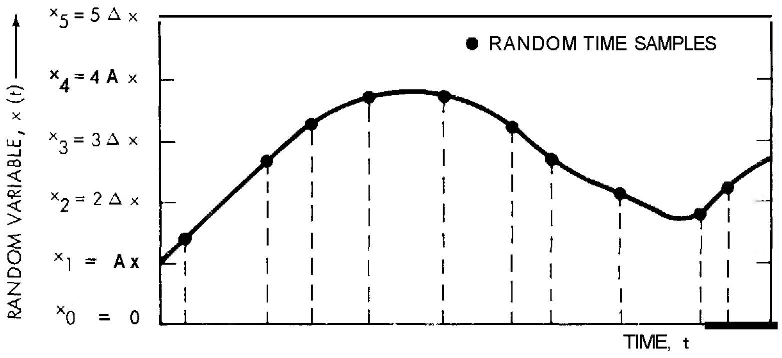


Figure 4-5. The quantization of a random variable into a large number of increments.



$n_0 = 0$ NUMBER OF RANDOM TIME SAMPLES FALLING IN THE INCREMENT $0 < x < x_1$

$n_1 = 2$ NUMBER OF RANDOM TIME SAMPLES FALLING IN THE INCREMENT $x_1 < x < x_2$

$n_2 = 4$ NUMBER OF RANDOM TIME SAMPLES FALLING IN THE INCREMENT $x_2 < x < x_3$

$n_3 = 4$ NUMBER OF RANDOM TIME SAMPLES FALLING IN THE INCREMENT $x_3 < x < x_4$

$n_4 = 0$ NUMBER OF RANDOM TIME SAMPLES FALLING IN THE INCREMENT $x_4 < x < x_5$

$N = 10$ TOTAL NUMBER OF RANDOM TIME SAMPLES FALLING IN ALL INCREMENTS OF THE RANDOM VARIABLE

$P(x_0) = \frac{n_0}{N} = 0$ = PROBABILITY THAT A RANDOM TIME SAMPLE WILL FALL IN THE INTERVAL $0 < x < x_1$

$P(x_1) = \frac{n_1}{N} = 0.2$ = PROBABILITY THAT A RANDOM TIME SAMPLE WILL FALL IN THE INTERVAL $x_1 < x < x_2$

$P(x_2) = \frac{n_2}{N} = 0.4$ = PROBABILITY THAT A RANDOM TIME SAMPLE WILL FALL IN THE INTERVAL $x_3 < x < x_2$

$P(x_3) = \frac{n_3}{N} = 0.4$ =PROBABILITY THAT A RANDOM TIME SAMPLE WILL FALL IN THE INTERVAL $x_4 < x < x_3$

$P(x_4) = \frac{n_4}{N} = 0$ = PROBABILITY THAT A RANDOM TIME SAMPLE WILL FALL IN THE INTERVAL $x_5 < x < x_4$

NOTE:
THE PROBABILITIES GIVEN IN THIS EXAMPLE ARE CALLED OUT AS SUCH FOR ILLUSTRATIVE PURPOSES ONLY. STRICTLY SPEAKING, THEY CANNOT BE CONSIDERED TO BE TRUE PROBABILITIES INASMUCH AS (SEE EQ. 4-11) THE TOTAL NUMBER OF RANDOM TIME SAMPLES N CAN BY NO MEANS BE CONSIDERED TO APPROACH INFINITY.

Figure 4-6. Illustrative application of the probability concepts associated with a continuous random variable.

as the increment is decreased, the probability of the event x_i becomes smaller because of the fluctuation of the random variable x(t) (see Fig. 4-5). In the limit, as Δx approaches zero, the ratio $Pr[x_i]/\Delta x$ remains finite. This ratio is called the probability density function of a random variable and is expressed as follows:

$$p(x_i) = \lim_{\substack{\Delta x \to 0 \\ N \to \infty}} \left[\frac{1}{\Delta x}\left(\frac{n_i}{N}\right)\right] = \lim_{\Delta x \to 0} \left[\frac{Pr[x_i]}{\Delta x}\right] \tag{4-12}$$

where

$p(x_i)$ = probability density function of x evaluated at $x = x_i$.

The probability density function is, then, a measure of the relative likelihood of the random variable having a value that falls in a particular increment $i\Delta x < x < (i + 1)\Delta x$. It corresponds to the probability of occurrence of a discrete event A as defined by Eq. 4-1 in par 4-4.2.1.

3. Probability Distribution Function

Frequently, the knowledge of the probability of particular amplitudes of a random

variable is less important than knowledge of the probability of exceeding a certain value. For example, the designer of a breakwater would want to know the probability of a wave's exceeding a certain height. The probability that the amplitude of the random variable x* does not exceed a particular value X is given by the probability distribution function of X, P(X), which is obtained by integrating the probability density function over all values of $x \le X$; i.e.,

$$P(X) = \int_{-\infty}^{X} p(x)\, dx \qquad (4\text{-}13)$$

An alternative definition first defines a probability distribution function P(X) as the probability that $x(t) \le X$, where x(t) is again a random variable. Then the probability density function is defined by the relationship

$$\frac{dP(X)}{dx} = p(X) \qquad (4\text{-}14)$$

The probability of the occurrence of all values of the random variable can be obtained by summing the probability distribution over all possible values as follows:

$$P(\infty) = \int_{-\infty}^{\infty} p(x)\, dx = 1 \qquad (4\text{-}15)$$

where the integral $\int_{-\infty}^{\infty} p(x)\, dx$ is the area under the entire probability density function. This means that the probability that x will lie between $-\infty$ and $+\infty$ is equal to unity, which is expected since all possibilities are encompassed. Also, since $P(-\infty) = 0$, the probability distribution function takes on only values that lie between 0 and 1.

Example 4-2 in the Appendix to Chapter 4 shows a practical application of the probability density function and the probability distribution function.

Curves of a typical probability density function and a typical probability distribution function are shown in Fig. 4-10 (see par 4-4.2.4). It should be noted that the probability density function (Fig. 4-10(A)) tends to peak at the most likely value, although it can, however, have more than one peak or be completely flat, depending on the physical situation concerned. Also, the probability density function need not be symmetrical. The probability distribution function (Fig. 4-10(B)) on the other hand, always increases steadily from zero to unity.

4. Joint Probability Density Functions

When two or more random variables are involved, joint and conditional probability density functions can be defined similar to the joint and conditional probabilities developed in par 4-4.2, 1. The joint probability density function is a measure of the relative likelihood that any random time sample† of a particular random variable x_A will have a given value and that any random time sample of a second random variable x_B will have some other given value. Figure 4-7 shows two random variables x_A and x_B quantized into increments Δx. It is possible to count the number of occurrences of the event in which the random variable x_A lies in the increment for which $i\Delta x < x_A < (i + 1)\Delta x$, for any random time sample. Let this event be designated x_i and the number of occurrences be designated as n_i. Similarly, for a random variable x_B in an increment for which $j\Delta x < x_B < (j + 1)\Delta x$, let the event of x_B falling in the increment at any random time sample be designated as x_j, and the number of occurrences of the event be designated as n_j. It should be noted that an equal number of random time samples for the random variables x_A and x_B must be taken. Let the total number of samples taken be denoted by the symbol N, and the total number of joint occurrences of events x_i and x_j for the N samples be designated as n_{ij}; i.e.,

$$n_{ij} = n_i + n_j \qquad (4\text{-}16)$$

* In the aforenoted example, **x** would be the height of random waves.

† A random sample is obtained by a selection of points from a set in a manner such that any point has an equal chance of being selected (see page 294 of Reference 24).

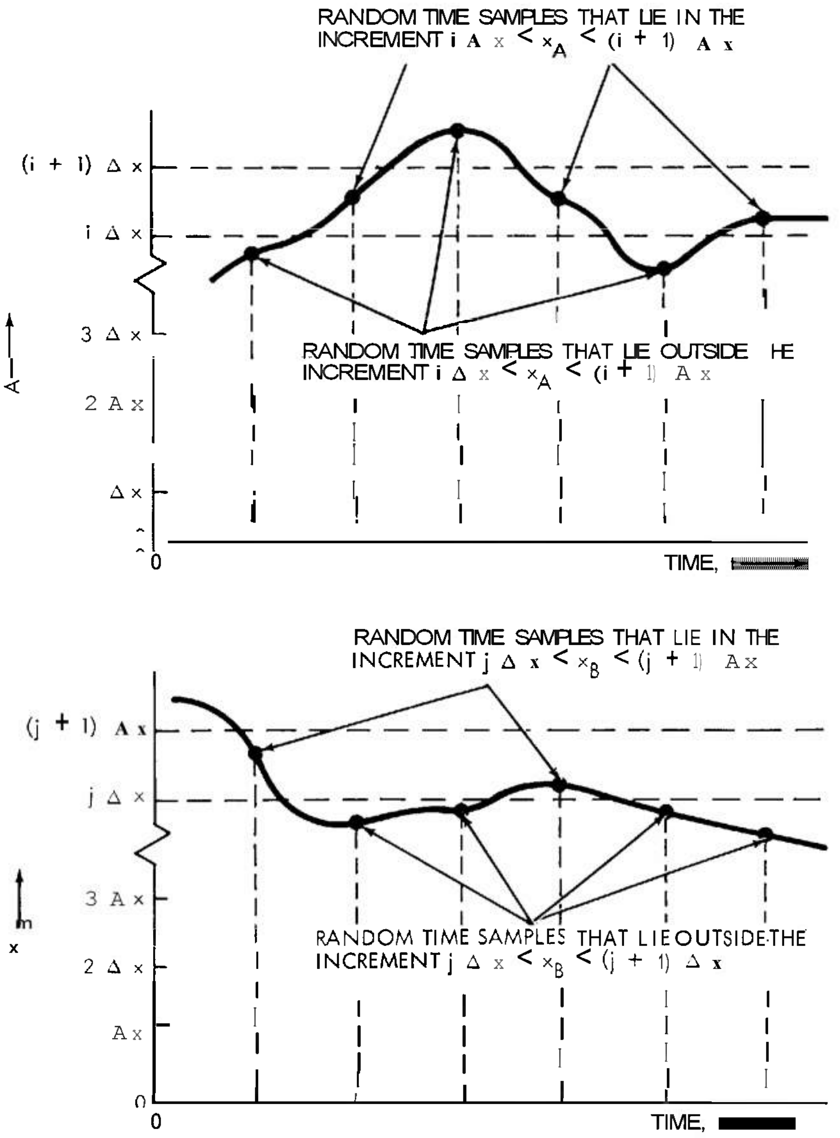


Figure 4-7. Illustrative representation of joint probability density.

The joint probability of the two events x_i and x_j – i.e., the probability that any random sample of x_A will be x_i and that any random sample of x_B will be x_j – is therefore defined by the relationship

$$Pr[x_i, x_j] = Pr\left[i\Delta x < x_A < (i+1)\Delta x,\ j\Delta x < x_B < (j+1)\Delta x\right] = \lim_{N\to\infty}\left[\frac{n_{ij}}{N}\right] \tag{4-17}$$

where

N = total number of random time samples of x_A and x_B falling in all increments.

If Δx is allowed to approach zero and N is allowed to approach infinity, the joint probability density function is defined by the relationship

$$p(x_i, x_j) = \lim_{\Delta x\to 0}\left[\frac{Pr[x_i, x_j]}{(\Delta x)^2}\right] = \lim_{\substack{\Delta x\to 0\\ N\to\infty}}\left[\frac{1}{(\Delta x)^2}\ \frac{n_{ij}}{N}\right] \tag{4-18}$$

in a manner similar to the definition of the probability density function in Eq. 4-12.

For a practical example of how joint probability density functions can be employed, see Example 4-3 in the Appendix to Chapter 4.

5. Conditional Probability Density Function

The probability of the conditional event $j\Delta x < x_B < (j+1)\Delta x$, given the prior occurrence of the event $i\Delta x < x_A < (i+1)\Delta x$, is given in a manner analogous to Eq. 4-6 by the relationship

$$Pr[x_j | x_i] = Pr\left[j\Delta x < x_B < (j+1)\Delta x \,|\, i\Delta x < x_A < (i+1)\Delta x\right]$$

$$= \frac{Pr[x_i, x_j]}{Pr[x_i]} = \lim_{N\to\infty}\left[\frac{\frac{n_{ij}}{N}}{\frac{n_i}{N}}\right] = \frac{n_{ij}}{n_i}. \tag{4-19}$$

Again taking the limit as $\Delta x \to 0$ and $N\to\infty$, the conditional probability density can be defined as

$$p(x_j | x_i) = \lim_{\Delta x\to 0}\left[\frac{Pr[x_j|x_i]}{\Delta x}\right] = \lim_{\substack{\Delta x\to 0\\ N\to\infty}}\left[\frac{1}{\Delta x}\left(\frac{n_{ij}}{n_i}\right)\right] \tag{4-20}$$

in a manner similar to the definition of the probability density function in Eq. 4-12 and the definition of the joint probability density function in Eq. 4-18. Dividing both the numerator and the denominator of Eq. 4-20 by $N(\Delta x)^2$ yields

$$p(x_j | x_i) = \lim_{\substack{\Delta x\to 0\\ N\to\infty}}\left[\frac{1}{\Delta x}\left(\frac{n_{ij}}{n_i}\right)\frac{\frac{1}{N(\Delta x)^2}}{\frac{1}{N(\Delta x)^2}}\right] = \lim_{\substack{\Delta x\to 0\\ N\to\infty}}\left[\frac{\frac{1}{(\Delta x)^2}\left(\frac{n_{ij}}{N}\right)}{\frac{1}{\Delta x}\left(\frac{n_i}{N}\right)}\right] \tag{4-21}$$

Substitution of the relationships given by Eqs. 4-12 and 4-18 into Eq. 4-21 shows that

$$p(x_j | x_i) = \frac{p(x_i, x_j)}{p(x_i)}. \tag{4-22}$$

For a practical example of how conditional probability density functions can be employed, see Example 4-4 in the Appendix to Chapter 4.

If x_i and x_j are statistically independent, then the prior occurrence of x_i can have no effect on the probability of occurrence of x_j. Accordingly, for this circumstance

$$p(x_j \mid x_i) = p(x_j). \tag{4-23}$$

Substitution of Eq. 4-23 in Eq. 4-22 shows that

$$p(x_j) = \frac{p(x_i, x_j)}{p(x_i)} \tag{4-24}$$

Therefore, for independent random variables

$$p(x_i, x_j) = p(x_i)\, p(x_j). \tag{4-25}$$

This relationship for the joint probability density function is particularly useful in the discussion of hit and kill probabilities that appears in par 4-4. 3.

4-4.2.3 Averages of Random Variables

Instead of dealing with random variables as such, a considerable simplification is obtained by employing various statistical parameters, or averages, that provide a numerical measure of the important characteristics. The basic parameters are introduced here, while certain more specialized parameters will be introduced where they are needed, notably in par 4-4. 4. 2.

One of the most useful statistical parameters is the <u>time average.</u>* The time average of a random variable (or of any variable, actually) is defined by the equation

$$\widetilde{x(t)} = \lim_{\Delta t \to \infty} \left[\frac{1}{\Delta t} \int_t^{t+\Delta t} x(t)\, dt \right] \tag{4-26}$$

where

$\widetilde{x(t)}$ = time average of $x(t)$

At = a time interval.

In practical work, it is sufficient to make At of a sufficient duration that the average does not change significantly for a further increase in Δt.

It should be noted that: (1) the symbol $\widetilde{x(t)}$ denotes the time average of the random time variable $x(t)$; (2) this time average is a real number that may be positive, negative, or zero; and (3) the time average is not itself a function of time. The time average can be computed analytically if an analytic expression for $x(t)$ is available. If a time record of $x(t)$ is available, the time average can be computed either mechanically or numerically. Figure 4-8 is an example of the time average of a random variable and illustrates a graphical technique that may be used to compute the time average. Note that if the random variable should have negative values, the corresponding areas would be subtracted.

Another useful statistical parameter is the <u>mean-square time average.</u> This quantity is utilized to express the average power of a random variable. Its usefulness stems from the fact that when – as is usually the case – the random variable represents a voltage, displacement, or similar type of physical quantity, the average power in the physical system

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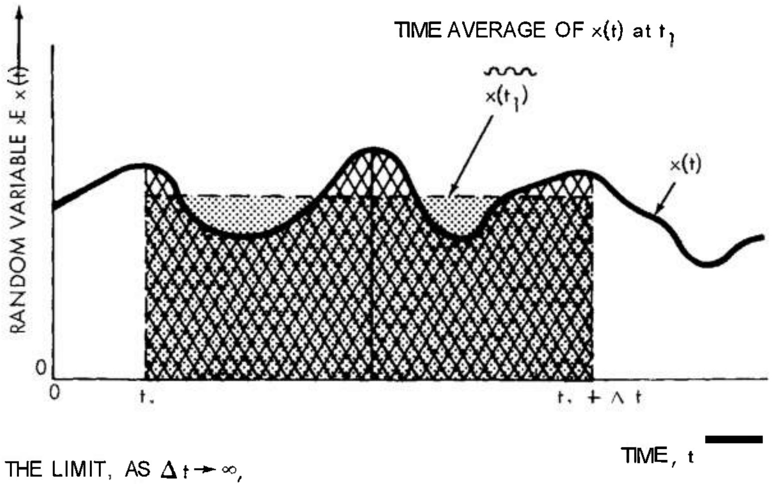
* See par 4.2 (Time Averages) of Chapter 10 (Basic Statistical Theory) of Reference 1.



IN THE LIMIT, AS $\Delta t \to \infty$,

$$\left\{ \begin{array}{l} \text{AREA UNDER } \widetilde{x}(t_1) \\ \text{BETWEEN } t_1 \text{ AND } t_1 + \Delta t \end{array} \right\} = \left\{ \begin{array}{l} \text{AREA UNDER } x(t) \\ \text{BETWEEN } t_1 \text{ AND } t_1 + \Delta t \end{array} \right\}$$

THEREFORE,

$$\widetilde{x}(t_1) = \lim_{\Delta t \to \infty} \left[\frac{1}{\Delta t} \int_{t_1}^{t_1 + \Delta t} x(t)\, dt \right]$$

= TIME AVERAGE OF x(t) AT AN ARBITRARILY CHOSEN TIME, t_1

SINCE t_1 IS AN ARBITRARY TIME, THE TIME AVERAGE OF x(t) MAY BE WRITTEN IN THE GENERAL FORM

$$\widetilde{x}(t) = \lim_{\Delta t \to \infty} \left[\frac{1}{\Delta t} \int_{t}^{t + \Delta t} x(t)\, dt \right]$$

Figure 4-8. The time average of a random variable.

concerned is then expressed by the mean-square time average. The mean-square time average is defined as the time average of the square of the random variable; i.e.,

$$\widetilde{x^2}(t) = \lim_{\Delta t \to \infty} \left[\frac{1}{\Delta t} \int_{t}^{t + \Delta t} x^2(t)\, dt \right] \tag{4-27}$$

where

= mean-square time average of x(t).

For example, assume a random instantaneous voltage, e(t), that is applied to aone-ohm

resistor. The instantaneous power dissipated in the resistor is $e^2(t)$. A time average of the instantaneous power yields the average power and, as shown by Eq. 4-27, turns out to be the mean-square time average of the voltage; i. e.,

$$P_{(av)} = \lim_{\Delta t \to \infty} \left| \int_t^{t+\Delta t} e^2(t)\, dt \right] = \widetilde{e^2(t)} \tag{4-28}$$

A final type of time average of importance is the root-mean-square (or RMS) time average. This statistical parameter is simply the square root of the mean-square time average. It is convenient in that it is expressed in the same units as the random variable itself.

Also important are the averages associated with an ensemble of random variables (see Fig. 4-4, for example). The most significant ensemble averages are the *so*-called moments. The main usefulness of these moments lies in their ability to completely specify any probability density function. The n^{th} moment of a probability density function $p(x)$ is defined by the general relationship

$$m^{(n)} = \int_{-\infty}^{\infty} x^n\, p(x)\, dx \tag{4-29}$$

where

x = random variable of probability density function $p(x)$

$m^{(n)}$ = moment of order n.

The particular moments that are of significance to the fire-control hit-probability problem are discussed in the paragraphs which immediately follow.

A comparison of Eq. 4-29 with Eq. 4-15 shows that the zero-order moment is the area under the entire probability density function and is, therefore, equal to unity; i. e.,

$$m^{(0)} = \int_{-\infty}^{\infty} x^0\, p(x)\, dx = \int_{-\infty}^{\infty} p(x)\, dx = 1 \tag{4-30}$$

The first moment, $m^{(1)}$ or simply m, is called the statistical mean, or average. It is also denoted by the symbol $\overline{x}$. Equation 4-29 shows that m is defined by the equation

$$m = m^{(1)} = \bar{x} = \int_{-\infty}^{\infty} x\, p(x)\, dx. \tag{4-31}$$

The time average given by Eq. 4-26 and the ensemble average given by Eq. 4-29 are identical when the random variable is a stationary function of time.

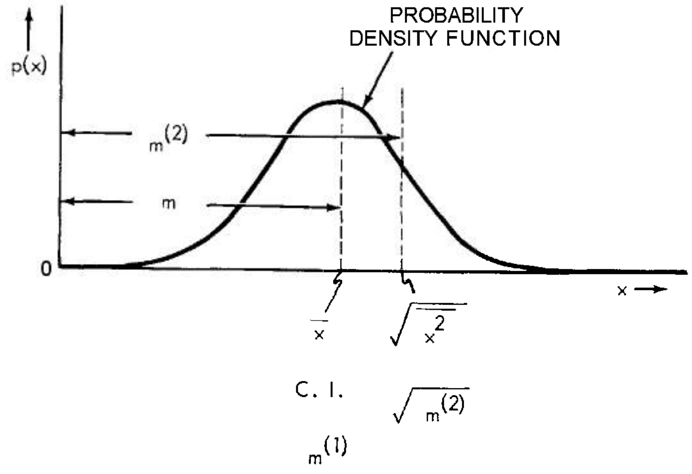
The second moment $m^{(2)}$ is called the mean-square value of x. In accordance with Eq. 4-29, it is given by the equation

$$m^{(2)} = \overline{x^2} = \int_{-\infty}^{\infty} x^2\, p(x)\, dx \tag{4-32}$$

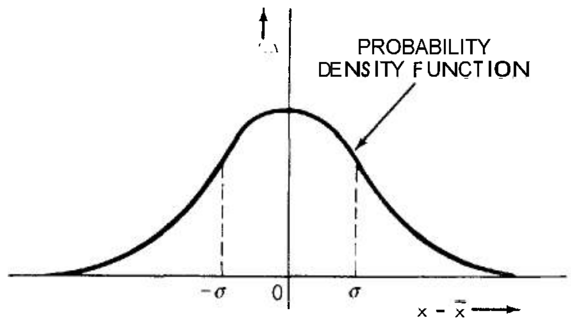
where the symbol $\overline{x^2}$ is also employed to represent the mean-square value, or second moment, of x. The time average given by Eq. 4-27 and the ensemble average given by Eq. 4-32 are identical when the random variable is a stationary function of time.

In the fire-control hit-probability problem, the value of the mean of a set of firing errors is the bias error, i. e. , the distance between the center of impact* and the target (see Fig. 4-3).

* The center of impact is defined as the center of gravity of the points of fall of a large number of rounds fired with a given laying of a weapon.



(A) Moments about the zero axis



(B) Moments about the mean

Figure 4-9. Moments about the zero axis and the corresponding moments about the mean.

Inasmuch as the dispersion* errors tend to scatter about the center of impact, a useful measure of dispersion would be the second moment of a set of firing errors from which the mean value (the bias) had been subtracted. Such moments about the mean (or center of impact), rather than about the zero axis (see Fig. 4-9), are called central moments and are defined by the general mathematical relationship

$$\mu^{(n)} = \int_{-\infty}^{\infty} (x - \bar{x})^n \, p(x) \, dx \tag{4-33}$$

where

$\mu^{(n)}$ = central moment of order n.

The most significant central moment is the second, $\mu^{(2)}$. This moment, which is called the variance, is the mean-square value of x about the mean. Since the units in which the variance are measured will be the square of the units in which the random variable is measured

* Dispersion is defined as the scattering of shots due to unavoidable variations. These are chiefly the variations in the initial yaw of the projectile and in the initial projectile velocity (see Chapter 2).

(see Eq. 4-32), it is convenient to introduce a quantity that is the square root of the variance, This quantity is called the <u>standard deviation</u> and is given the symbol σ. Accordingly, the variance is usually denoted by the symbol σ^2 and is given by the equation

$$\sigma^2 = \mu^{(2)} = \int_{-\infty}^{\infty} (x - \bar{x})^2 \, p(x) \, dx. \tag{4-34}$$

From this mathematical definition of the variance, it is possible to derive as follows the important relationship that the variance is the difference of the <u>mean-square</u> value of x, i. e., $\overline{x^2}$, and the <u>square of the mean</u>, i. e., $\bar{x}^2$. Expansion of Eq. 4-34 shows that

$$\sigma^2 = \int_{-\infty}^{\infty} (x - \bar{x})^2 \, p(x) \, dx = \int_{-\infty}^{\infty} (x^2 - 2x\bar{x} + \bar{x}^2) \, p(x) \, dx \tag{4-35}$$

$$\int_{-\infty} x^2 \, p(x) \, dx - 2\bar{x} \int_{-\infty} x \, p(x) \, dx + \bar{x} \int_{-\infty} p(x) \, dx. \tag{4-36}$$

A comparison of this equation with Eq. 4-32 shows that the first term on the right-hand side of Eq. 4-36 is identical with $\overline{x^2}$, the mean-square value of x. A similar comparison with Eq. 4-31 shows that the second term on the right-hand side of Eq. 4-36 is identical with the product $-2\bar{x}^2$. From a comparison with Eq. 4-30, it is apparent that the third term on the right-hand side of Eq. 4-36 is equal to $\bar{x}^2$. Accordingly,

$$\sigma^2 = \overline{x^2} - 2\bar{x}^2 + \bar{x}^2 = \overline{x^2} - \bar{x}^2. \tag{4-37}$$

As noted earlier, the primary value of the moment concept is that any probability density function can be completely specified by its moments. In most cases of interest, including fire control, the mean and the standard deviation are sufficient. For example, in fire control, the mean is identical with the bias and the standard deviation is a measure of the dispersion.

4-4.2.4 Gaussian Distribution

1. <u>Usefulness of the Gaussian Distribution</u>

While there are a number of common probability distributions, the one of the greatest practical importance is the Gaussian, or normal, distribution. The Gaussian distribution is obtained in a large number of situations as follows:

a. It has been empirically observed that most continuous random processes in nature can be described approximately by a Gaussian distribution.
b. Discrete random variables are often described by a binomial distribution, but for a large number of trials the binomial distribution is approximated by the Gaussian distribution.
c. In cases where a random variable is derived from a sum of a number of individual random variables, each of which may have any distribution, the resultant random variable is found to have a Gaussian distribution.

2. <u>Definition of the Gaussian Distribution</u>

The probability density function of a Gaussian distribution is defined by the general relationship

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x - \bar{x})^2}{2\sigma^2}\right] \tag{4-38}$$

where

$\overline{x}$ = mean value (see Eq. 4-31)

σ^2 = variance (*see* Eq. 4-34).

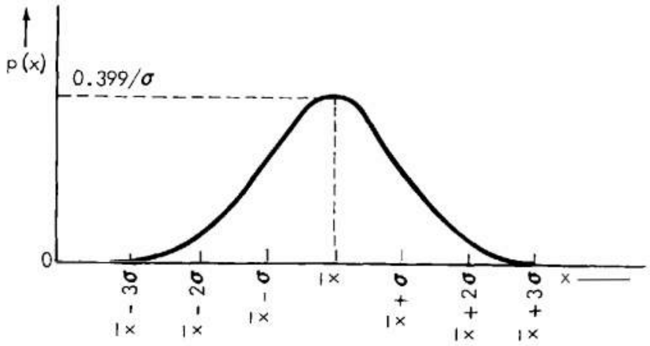
From Eq. 4-13, it is apparent that the associated Gaussian probability distribution function is given by the expression

$$P(X) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{X} \exp\left[-\frac{(x-\bar{x})^2}{2\sigma^2}\right] dx . \qquad (4\text{-}39)$$

The factor $1/\sigma\sqrt{2\pi}$ normalizes the expression for p(x) *so* that the total area beneath a plot of p(x) is unity. The Gaussian probability density function and the associated Gaussian probability distribution function are shown in Figs. 4-10(A) and 4-10(B), respectively. It should be noted that 68.3% of all values of the variable fall within $\pm\sigma$ of the mean, and practically all values (99.7%) fall within $\pm 3\sigma$.

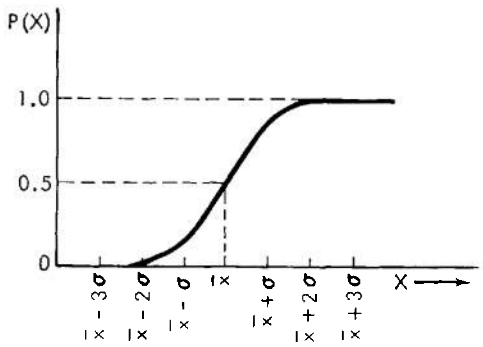
3. The Error Function

For numerical work with normal distributions, reference may be made to tabulations of



$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\bar{x})^2}{2\sigma^2}\right]$$

(A) Probability density function



$$P(X) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{X} \exp\left[-\frac{(x-\bar{x})^2}{2\sigma^2}\right] dx$$

(B) Probability distribution function

Figure 4-10. The probability density function and the corresponding probability distribution function for a Gaussian or normal distribution.

a standardized Gaussian distribution, called the error function. (The use of the error function is illustrated in the example of par 4-4.4). The error function is obtained by a change of variable in which the origin is shifted and a scale factor is applied, thus providing a normal distribution having zero mean and unity variance. The new variable is

$$y = \frac{x - \bar{x}}{\sigma} \tag{4-40}$$

Equation 4-38 then has the form

$$p(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} . \tag{4-41}$$

The corresponding distribution function may be tabulated in several ways. One method, which is convenient because finite limits of integration are employed, is obtained as indicated below.

Substitute Eq. 4-40 into Eq. 4-39 to yield

$$P(X) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{X} e^{-\frac{y^2}{2}} dy \tag{4-42}$$

The error function of $X/\sqrt{2}$ is defined by the relationship

$$\text{erf}\left(\frac{X}{\sqrt{2}}\right) = \frac{1}{\sqrt{2\pi}} \int_{-X}^{X} e^{-\frac{y^2}{2}} dy = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{X} e^{-\frac{y^2}{2}} dy - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-X} e^{-\frac{y^2}{2}} dy \tag{4-43}$$

where $\text{erf}(X/\sqrt{2})$ is the error function. Equation 4-43 is obtained simply by the subtraction of areas under a probability distribution curve such as Fig. 4-10(B).

Since $e^{-y^2/2}$ is an even function, i.e., $f(x) = f(-x)$, the integral from $-X$ to X is twice the integral from zero to X. Therefore,

$$\text{erf}\left(\frac{X}{\sqrt{2}}\right) = \frac{2}{\sqrt{2\pi}} \int_{0}^{X} e^{-\frac{y^2}{2}} dy. \tag{4-44}$$

A change of variable, $t = (y/\sqrt{2})$, yields the form

$$\text{erf}\, X = \frac{2}{\sqrt{\pi}} \int_{0}^{X/\sqrt{2}} e^{-t^2} dt. \tag{4-45}$$

The error function is tabulated in published mathematical tables; e.g., see pages 129 and 206 of Reference 23 and pages 116-120 of Reference 26.

4. The Central-Limit Theorem

In a system whose output is affected by a number of random inputs, it is found that, even though the input functions may individually depart greatly from normal distributions, the out-

put is approximately a normal distribution. The mathematical justification for this observation is called the central-limit theorem. Use of this theorem in connection with fire control systems enables the designer to combine errors as though all were Gaussian, without the necessity for detailed examination of the individual probability distributions.

The central-limit theorem states that the sum of a set of random variables is a random variable whose probability density function approaches the Gaussian form as the number of terms in the sum increases without limit. The theorem is valid, with minor restrictions – e.g., the first and second moments of the individual probability density functions must exist – regardless of the form of the individual probability density functions.

In symbolic form, assume a set of random variables $x_1, x_2, \ldots, x_i, \ldots, x_n$, each of which may have an arbitrary probability density function. The sum of these is another random variable X which is defined by

$$X = \sum_{i=1}^{n} x_i . \tag{4-46}$$

The central-limit theorem states that

$$p(X) = \frac{1}{DIG} \exp\left[-\frac{(X-\overline{X})^2}{2\sigma^2}\right] \tag{4-47}$$

where

$$\overline{X} = \sum_{i=1}^{n} \overline{x}_i \tag{4-48}$$

$$\sigma = \sum_{i=1}^{n} \sigma_i \tag{4-49}$$

and $\overline{x}_i$ is the mean, and σ_i the standard deviation, associated with the random variable xi.

4-4.2.5 The Bivariate Normal Distribution

A final concept, which is termed the <u>bivariate normal distribution</u>, will be introduced before proceeding to a discussion of the hit-probability problem. If independent normal distributions exist along each of two orthogonal axes x and y, then the joint probability density function p(x, y) is found from Eq. 4-25 to be

$$p(x,y) = \text{[illegible]}\; p(y) . \tag{4-50}$$

In accordance with Eq. 4-38, the independent density functions may be expressed as

$$p(x) = \frac{1}{\sigma_x\sqrt{2\pi}} \exp\left[-\frac{(x-\overline{x})^2}{2\sigma_x^2}\right] \tag{4-51}$$

and

$$p(y) = \frac{1}{DY\sqrt{2\pi}} \exp\left[-\frac{(y-\overline{y})^2}{2\sigma_y^2}\right] \tag{4-52}$$

where

$p(x)$, $p(y)$ = probability density functions of the random variables x and y, respectively

σ_x^2, σ_y^2 = variances of the x and y distributions, respectively

$\bar{x}$, $\bar{y}$ = mean values of the x and y distributions, respectively.

The substitution from Eqs. 4-51 and 4-52 into Eq. 4-50 shows that

$$
\begin{aligned}
p(x,y) &= \frac{1}{2\pi\sigma_x\sigma_y} \exp\left[-\frac{(x-\bar{x})^2}{2\sigma_x^2} - \frac{(y-\bar{y})^2}{2\sigma_y^2}\right] \\
&= \frac{1}{2\pi\sigma_x\sigma_y} \exp\left\{\frac{1}{2}\left[\frac{(x-\bar{x})^2}{\sigma_x^2} + \frac{(y-\bar{y})^2}{\sigma_y^2}\right]\right\}
\end{aligned}
\tag{4-53}
$$

The joint probability density function $p(x,y)$ is termed the bivariate normal distribution because there are two variables involved. As shown by the representative plots in Fig. 4-11, the bivariate normal distribution may be visualized in three dimensions as a "hill" that is based on the x, y-plane and has its center at $(\bar{x}, \bar{y})$. The bivariate normal distribution is referred to frequently in the following paragraphs on hit and kill probability theory, where the distribution of shots along each of two perpendicular axes is of prime significance.

4-4.3 HIT AND KILL PROBABILITY THEORY*

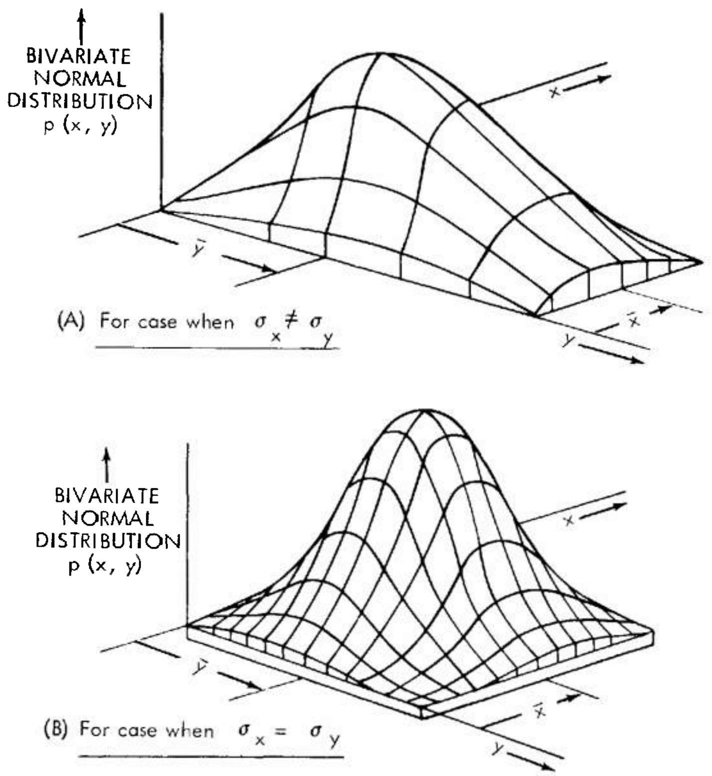
The subject of hit and kill probability theory presented in this handbook concerns the means of determining the probability of obtaining a hit on a given target that is capable of killing that target. As used in this context, the terms hit and kill are defined as follows, in accordance with Army technical terminology:[29]

1. hit. A blow or impact on a target by a bullet, bomb, or other projectile. (Thus, for example, a bullet would actually have to strike a target in order to effect a hit, whereas an exploding projectile would technically effect a hit if the force of its explosion created a blow or impact on the target, even if no physical part of the projectile actually struck the target itself.)

2. kill. (1) As a noun, kill is a term used alternatively with the destructive- damage category known as K damage. (Two damage-category classifications applying to combat material subject to attack have been accepted for use in evaluating damage and the damage potential of ammunition. The first classification, which applies to aircraft targets, employs the following definition of K damage: damage such that the aircraft will fall out of control immediately after the damage occurs. The second classification, which applies to armored-vehicle targets, employs the following definition of K damage: damage that will cause the vehicle to be destroyed.) (2) As a verb, kill means to destroy an aircraft or other vehicle to the extent defined by the K-damage category.

Hit probability is generally defined in Army technical terminology as the probability of a hit or hits being made on a target out of a given number of projectiles directed at the target. A particular type of hit probability that is of importance is the single-shot hit probability which is defined as the probability that a single projectile fired against a target will hit that target under a given set of conditions. No implication of a kill is implied in these hit-probability definitions. In standard Army technical terminology, which will be employed here, kill probability is generally defined as the probability that, given a hit, a single projectile will kill the target against which it is fired.

*The treatment of this subject that is provided here is based primarily on References 27 and 28. It should be particularly noted that Reference 27 provides (1) an excellent summary of the work done on the subject of kill probability over a number of years and (2) an annotated bibliography of some useful early references on the subject.



$$p(x, y) = \frac{}{\pi \sigma \sigma} \exp \left\{ -\frac{1}{2} \left[\frac{(x - \bar{x})^2}{\sigma_x^2} \quad \frac{(y - \bar{y})^2}{\sigma^2} \right] \right\}$$

= BIVARIATE NORMAL DISTRIBUTION

where

$\bar{x}$ = MEAN OF THE x DISTRIBUTION

$\bar{y}$ = MEAN OF THE y DISTRIBUTION

σ_x^2 = VARIANCE OF THE x DISTRIBUTION

σ_y^2 = VARIANCE OF THE y DISTRIBUTION

Figure 4-11. Representative plots of the bivariate normal distribution.

In order to avoid confusion, the relationship between hit probability and kill probability will be discussedin detail. Any target, considered for the moment as two-dimensional, can be represented by a vulnerable area. In the simplest case, this vulnerable area is simply the presented area times the probability that a hit on the presented area is a kill. More precisely, the vulnerable area A_v is given by the relationship

$$A_v = \iint_A (Pr)_{(kill)} (x,y)\, dx\, dy \tag{4-54}$$

where

x and y = coordinates centered at the center of gravity of the target and in a plane perpendicular to the projectile trajectory

$(Pr)_{(kill)}(x, y)$ = probability of a kill on the target for a hit at the point (x, y)

$\iint_A$ = a surface integral over the presented area A.

The probability-of-kill function $(Pr)_{(kill)}(x,y)$ assigns a probability that the target will be killed if there is a hit at the point (x,y). It is important that this probability-assignment function not be confused with a probability density function.

From Eq. 4-54, it is apparent that the vulnerable area is always less than the presented area **A.** Some modification of the definition is necessary in situations where the blast effect is important because the vulnerable area may then be greater than the presented area.

A complex target such as an aircraft or tank can be considered as made up of individual vulnerable components. If the components do not mask one another and are not redundant, the total vulnerable area is merely the sum of the vulnerable areas of the components. The contributions of masked and redundant components can also be determined with some additional complications. This technique of summation of individual vulnerable areas is in effect a method of integrating Eq. 4-54 for a situation in which the vulnerability cannot be expressed by a mathematical function.

Of course, vulnerability studies of targets are not a primary concern of the fire-control-system designer. Weapon-system designers, however, whose concern is with weapon-system effectiveness, must take into account target vulnerability, ammunition characteristics, and the accuracy of the launcher and fire control system in determining overall weapon-system effectiveness. The brief outline of the weapon-system design process which follows, insofar as it relates to fire-control accuracy, will be helpful to an understanding of fire-control accuracy specifications.

First, a preliminary study of the weapon system is performed, in which preliminary ammunition characteristics and preliminary launcher and fire-control accuracies are chosen. Then, using probable targets and the first choice of ammunition, target vulnerability studies are performed. In certain cases, these studies might include experimental work; in general, however, the target vulnerability would be obtained from existing component vulnerability data. The vulnerable area of the whole target would then be computed by summation of the component vulnerable areas, taking into account masking and redundancy.

The vulnerable area is a region of total lethality. However, account must be taken of the distribution of component vulnerable areas over the projected area of the target. If this distribution is random, such as that indicated diagrammatically in Fig. 4-11(A), one might consider the use of a Gaussian probability distribution as a model of the vulnerability of the target. In fact, it turns out that such a model, known as a diffuse-target model, is more tractable mathematically then the assumption of an area of total lethality. The diffuse-target model, originally suggested by Von Neumann,[30] is discussed in the paragraph which follows.

4-4.3.1 The Probability of Kill and the Diffuse Target

The diffuse target is a probabilistic mathematical model in which the likelihood of a kill increases the closer the burst point* is to the target center. Specifically, the probability that a burst at or near the actual target will produce a kill is given by the following relationship which is derived in Derivation 4-1 in the Appendix to Chapter 4:

$$(Pr)_{d(kill)} = e^{-\frac{r^2}{2\sigma_c^2}} \tag{4-55}$$

where

$(Pr)_{d(kill)}$ = probability-of-kill function for the diffuse target
= probability of kill of a burst at or near the actual target

* The burst point will be considered to be the center of the destructive effect of the projectile; it does not necessarily imply an exploding projectile.

r - distance of the burst point from the target center (r is also known as the miss distance)

σ_c = constant depending on size and armoring ("hardness") of the target, and on ammunition characteristics; σ_c is called the vulnerable radius of the target for a particular type of ammunition and must have the same units of length as the distance r.

As noted previously, the kill probability function is not a density function but a probability assignment function, i. e. , a function that assigns a probability to a given event. Equation 4-55 may be stated as follows: Given a burst located at the point (r, θ, ϕ) (see Fig. 1 of Derivation 4-1), the probability of a kill on a target centered at the origin is a function of r – i. e., $\exp[-r^2/2\sigma_c^2]$ – and is independent of ϕ and θ.

As shown by Derivation 4-1, the diffuse target is characterized by a vulnerable area a whose radius is $\sqrt{2}\sigma_c$. Bursts that fall outside the vulnerable area have relatively low kill probabilities compared with those of bursts that fall inside that area. There, the kill probability ranges from 0.368 for a burst falling on the circumference of the vulnerable area to 1.0 for bursts falling at the center of the vulnerable area.

The validity of the diffuse-target concept can be appreciated from the fact that a burst close to the target center will be more likely to cause K damage to the target than will one that is more remote from the target center. The concept is of such general usefulness that it will be employed throughout the following discussion. This concept, incidentally, is an excellent example of a useful mathematical model.

4-4. 3. 2 Single-Shot Kill Probability

Before proceeding to a consideration of hit probability, the kill probability on a diffuse target will be derived since the resulting relationships can be readily adapted under certain conditions to hit-probability problems.

In a number of fire-control problems, the model of the target may be reduced to two dimensions. For example, if the time of impact is not important, as in the case when the target is stationary, then if one axis is chosen tangent to the path of the projectile, an error in position along this axis would have no effect on the hit probability. Therefore, the initial discussion of single-shot kill probability will employ, for simplicity, a set of two orthogonal axes, x and y, located at the target center and in a plane normal to the path of the projectile. It can be assumed that the distributions of bursts along these two axes are both normal and independent. As a result, the burst pattern may be expressed by a simple bivariate normal distribution, such as represented by Eq. 4-53 and Fig. 4-11.

Because of the presence of systematic error, the burst pattern during an engagement is, in general, biased with respect to the target (see Fig. 4-3, for example)? In order that the kill-probability expressions may be as simple as possible, the x- and y-axis system is chosen to be oriented in such a way that the x-axis passes through the center of the dispersion; thus, the total bias h is measured along this axis (see Fig. 4-12). A second assumption, which is justified in many practical situations, is that the variance of the burst-pattern dispersion is the same in both axes.

Derivation 4-2 in the Appendix to Chapter 4 derives the single-shot kill probability $(Pr)_{ssk}$ in two dimensions for the case of zero bias. The resulting expression for the zero-bias case (h = 0) is

$$(Pr)_{ssk} = \frac{\sigma_c^2}{\sigma_c^2 + \sigma_d^2} \quad (h = 0) \tag{4-56}$$

* The error of an individual burst is the displacement of the burst from the target center. The systematic, or bias error in a burst pattern is the mean value of these individual errors. The random, or dispersion, error associated with each individual burst is the difference between the mean value and the position of the burst, regardless of whether or not a bias error is present. (See par 44.1. 1 and the discussion of systematic and random errors given in par 44.4. 3).

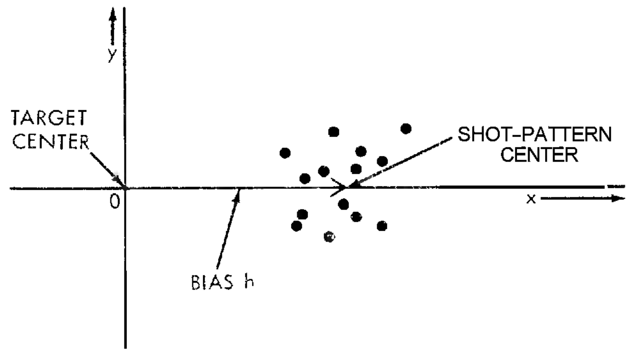


Figure 4-12. Selected orientation of the x, y coordinate system.

where

σ_c = vulnerable radius

σ_d^2 = variance of the burst pattern.

An alternative expression in terms of the vulnerable area a is

$$(Pr)_{ssk} = \frac{a}{a + 2\pi\sigma_d^2} \quad (h = 0). \tag{4-57}$$

For the case of nonzero bias (h = 0), the single-shot kill probability is, from Derivation 4-3 in the Appendix to Chapter 4

$$(Pr)_{ssk} = \frac{\sigma_c^2}{\sigma_c^2 + \sigma_d^2} \exp\left[-\frac{h^2}{2(\sigma_c^2 + \sigma_d^2)}\right] \tag{4-58}$$

If the lethal radius is much less than the standard deviation of the dispersion, a simpler expression may be employed; i. e.,

$$(Pr)_{ssk} = \frac{a}{2\pi\sigma_d^2} \exp\left[-\frac{h^2}{2\sigma_d^2}\right] (\sigma_c << \sigma_d). \tag{4-59}$$

In three dimensions, Eq. D4-3. 15 of Derivation 4-3 applies. In this case

$$(Pr)_{ssk} = \left(\frac{\sigma_c^2}{\sigma_c^2 + \sigma_d^2}\right) \exp\left[-\frac{h^2 + k^2 + \ell^2}{2(\sigma_c^2 + \sigma_d^2)}\right] \tag{4-60}$$

where h, k, and ℓ are values of bias along the x-, y-, and z-axes, respectively.

The case of zero bias in three dimensions (h = k = ℓ = 0) is obtained readily from Eq.

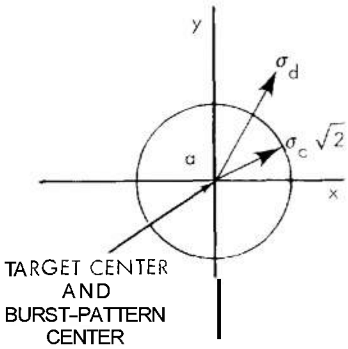
4-60. The expression is

$$(Pr)_{ssk} = \left(\frac{\sigma_c^2}{\sigma_c^2 + \sigma_d^2} \right)^{3/2} \quad (h = k = \ell = 0). \qquad (4\text{-}61)$$

All of the foregoing expressions that have been derived for kill probability are summarized in Fig. 4-13.

4-4. 3. 3 Single-Shot Hit Probability

The determination of hit probability is simpler in concept than that for kill probability. The target is a physical object and has a known volume. The probability of a hit is then the probability that the path of the projectile will intersect this volume. If consideration is given



AS SHOWN BY THE DERIVATIONS OF EQS. D4-2.19 AND D4-2.20 OF DERIVATION 4-2 AND EQ. D4-3.23 OF DERIVATION 4-3 (SEE THE APPENDIX TO CHAPTER 4),

$$(Pr)_{ssk} = \frac{\sigma_c^2}{\sigma_d^2 + \sigma_c^2} = \frac{a}{a + 2\pi\sigma_d^2}$$

WHERE

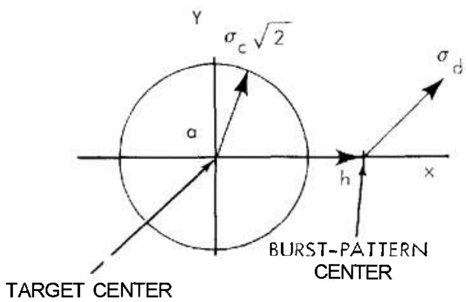
IS THE SINGLE-SHOT KILL PROBABILITY,

σ_c IS THE VULNERABLE RADIUS,

σ_d IS THE STANDARD DEVIATION OF THE BURST-PATTERN DISPERSION

AND a IS THE VULNERABLE AREA = $2\pi\sigma_c^2$.

(A) For a two-dimensional zero-bias burst pattern



AS SHOWN BY THE DERIVATION OF EQ. D4-3.20 OF DERIVATION 4-3 (SEE THE APPENDIX TO CHAPTER 4),

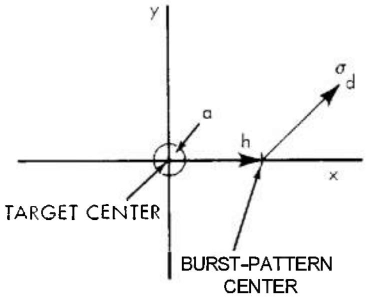
$$(Pr)_{ssk} = \frac{\sigma_c^2}{\sigma_d^2 + \sigma_c^2} \exp\left[- \frac{h^2}{2(\sigma_d^2 + \sigma_c^2)} \right]$$

WHERE

h IS THE BIAS, AND THE OTHER SYMBOLS ARE THE SAME AS DEFINED IN PART a.

(B) For a two-dimensional fixed-bias burst pattern; with bias measured along the x axis

Figure 4-13. Summation of the expressions for single-shot kill probability.



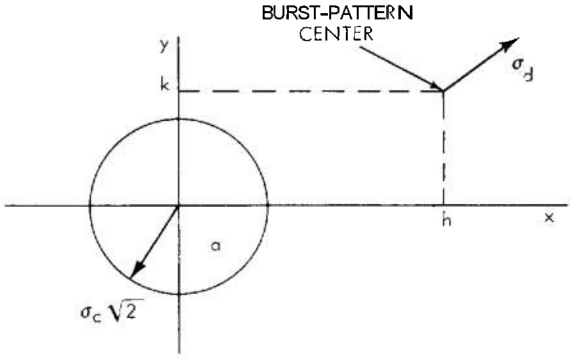
AS SHOWN BY THE DERIVATION OF EQ. D4-3.21 OF DERIVATION 4-3 (SEE THE APPENDIX TO CHAPTER 4),

$$(P_r)_{ssk} = \frac{a}{2\pi\sigma_d^2}\exp\left[-\frac{h^2}{2\sigma_d^2}\right]$$

FOR $a << 2\pi\sigma_d^2$,

WHERE THE SYMBOLS ARE AS PREVIOUSLY DEFINED IN PARTS (A) AND (B).

(C) For a two-dimensional, fixed-bias burst pattern; approximation when the vulnerable radius is much smaller than the dispersion



AS SHOWN BY THE DERIVATION OF EQ. D4-3.19 IN DERIVATION 4-3 (SEE THE APPENDIX TO CHAPTER 4),

$$(P_r)_{ssk} = \frac{\sigma_c^2}{\sigma_c^2 + \sigma_d^2}\exp\left[-\frac{h^2 + k^2}{2(\sigma_c^2 + \sigma_d^2)}\right]$$

WHERE

h IS THE COMPONENT OF THE BIAS ALONG THE x AXIS,

k IS THE COMPONENT OF THE BIAS ALONG THE y AXIS,

AND THE OTHER SYMBOLS ARE AS DEFINED IN PART (A).

(D) For a two-dimensional, fixed-bias burst pattern; with both x and y components of bias

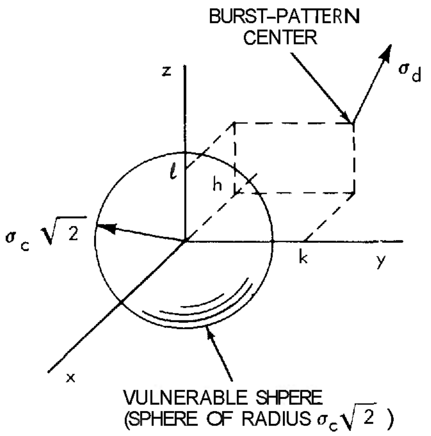
Figure 4-13. Summation of the expressions for single-shot kill probability (cont.).

to a hit-probability assignment function, analogous to the kill-probability assignment function, then this assignment function will be unity inside the target volume and on its surface, and zero everywhere else.

Since the computational labor involved in the use of a complex target volume is rarely justified, it is common practice to assume a simple target model such as a sphere or ellipsoid. A further complication arises in the case of explosive proximity-fuzed projectiles since the projectile does not have to intersect the target volume in order to secure a hit. In this case, the target-model dimensions can be increased by an amount equal to the detection radius of the fuze. (This is because the burst radius is presumably at least as great as the detection radius.)

An analysis as general as that given in Derivation 4-3 for kill probability is not possible for hit probability, for reasons noted in the ensuing discussion. Therefore, the hit probabilities for certain simplified cases will be derived instead.

As discussed for kill probabilities, many hit-probability problems can be reduced to two dimensions. Consider first a two-dimensional, circular target model and a zero-bias shot pattern with a dispersion variance of σ_d^2. As shown in Fig. 4-14, the target is a circle of

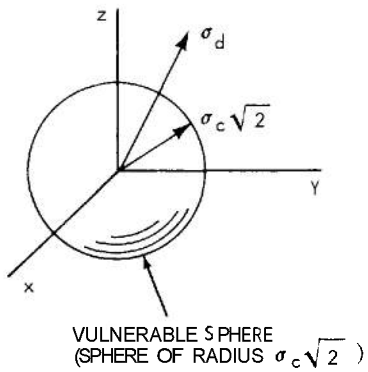


AS SHOWN BY THE DERIVATION OF EQ. D4-3.15 IN DERIVATION 4-3 (SEE THE APPENDIX TO CHAPTER 4),

$$(Pr)_{ssk} = \left(\frac{\sigma_c^2}{\sigma_c^2 + \sigma_d^2}\right)^{3/2} \exp\left[-\frac{h^2+k^2+\ell^2}{2\left(\sigma_c^2+\sigma_d^2\right)}\right]$$

WHERE
ℓ IS THE COMPONENT OF THE BIAS ALONG THE z AXIS, AND ALL THE OTHER QUANTITIES ARE AS DEFINED IN PARTS (A) AND (D).

(E) For a three-dimensional, fixed-bias burst pattern



AS SHOWN BY THE DERIVATION OF EQ. D4-3.22 IN DERIVATION 4-3 (SEE THE APPENDIX TO CHAPTER 4),

$$(Pr)_{ssk} = \left(\frac{\sigma_c^2}{\sigma_c^2 + \sigma_d^2}\right)^{3/2}$$

WHERE THE QUANTITIES ARE DEFINED IN PART (A).

(F) For a three-dimensional, zero-bias burst pattern

Figure 4-13. Summation of the expressions for single-shot kill probability (cont.).

radius R. In terms of polar coordinates (r,θ), the hit-probability assignment function $(Pr)_h$ is given by

$$\left.\begin{aligned} (Pr)_h &= 1 \quad r \leqq R \\ (Pr)_h &= 0 \quad r > R \end{aligned}\right\} . \tag{4-62}$$

The probabilitydensity $p(r,\theta)$ of the shot pattern – which can be obtained from Eq. D4-2. 12 of Derivation 4-2 – is

$$p(r,\theta) = \frac{1}{2\pi\sigma_d^2} \exp\left[-\frac{r^2}{2\sigma_d^2}\right] \tag{4-63}$$

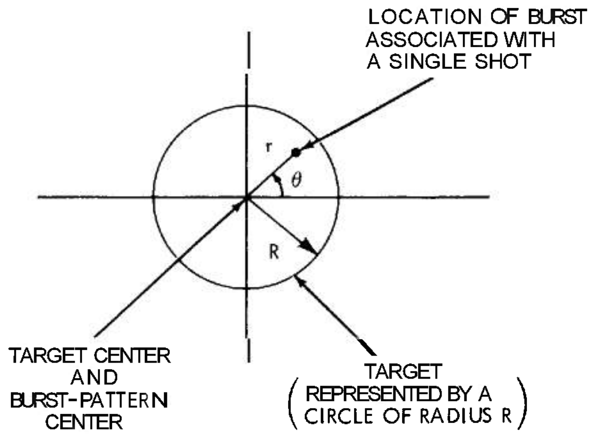


Figure 4-14. A circular target model with a zero-bias shot pattern.

The single-shot hit probability $(Pr)_{ssh}$ is the probability that a burst occurs in the differential area $rdrd\theta$, located at the point (r,θ), times the hit-probability assignment function $(Pr)_h$ for that point; i.e., in general terms

$$(Pr)_{ssh} = \int_0^{2\pi} \int_0^{\infty} p(r,\theta)\,(Pr)_h\, r dr d\theta. \tag{4-64}$$

The derivation of Eq. 4-64 is exactly analogous to that of Eq. D4-2.13 of Derivation 4-2, and, therefore, will not be repeated here.

The solution of Eq. 4-64 for the specific target model under consideration is quite straightforward. First, substitution of Eq. 4-63 into Eq. 4-64 yields

$$(Pr)_{ssh} = \frac{1}{2\pi\sigma_d^2} \int_0^{2\pi} \int_0^{\infty} (Pr)_h\, r \exp\left[-\frac{r^2}{2\sigma_d^2}\right] dr d\theta. \tag{4-65}$$

All terms in the integral are invariant with θ. Therefore, integrating with respect to θ yields

$$(Pr)_{ssh} = \frac{1}{\sigma_d^2} \int_0^{\infty} (Pr)_h\, r \exp\left[-\frac{r^2}{2\sigma_d^2}\right] dr \tag{4-66}$$

Equation 4-66 is integrated by making the substitution $\xi = r^2/2\sigma_d^2$. Then

$$(Pr)_{ssh} = \int_0^{\infty} (Pr)_h\, e^{-\xi} d\xi = \int_0^{R^2/2\sigma_d^2} e^{-\xi} d\xi \tag{4-67}$$

since $(Pr)_h = 0$ for $r > R$, or $\xi > R^2/2\sigma_d^2$. Evaluation of the integral yields

$$(Pr)_{ssh} = 1 - \exp\left[-\frac{R^2}{2\sigma_d^2}\right] \tag{4-68}$$

Note that as the dispersion becomes small compared with the target radius, the hit probabil-

ity approaches unity, whereas for large dispersions it approaches zero.

Next, consider a second simplified case that involves a two-dimensional target model, a dispersion as in Eq. 4-63, and a fixed bias h. If the target radius R is much smaller than σ_d, the distinction between the vulnerable area a and the target area A becomes negligibly small, and Eq. D4-3.21 of Derivation 4-3 can thus be directly converted to a hit-probability expression;* i. e.,

$$(Pr)_{ssh} = \frac{A}{2\pi\sigma_d^2} \exp\left[-\frac{h^2}{2\sigma_d^2}\right] \tag{4-69}$$

where

$A = \pi R^2$ = the target area

$A \approx a$ = the vulnerable area.

If h = 0, Eqs 4-68 and 4-69 rapidly approach coincidence as R becomes much smaller than σ_d.

For more complex cases than the two just discussed, analytic solutions become almost impossibly difficult to achieve. If an analytic solution is desired, however, a worthwhile approximation is available from the expressions previously derived for kill probability by simply replacing the lethal radius σ_c with the target radius R. Since the circular or spheroidal target model is in any case a gross approximation to the dimensions of the real target, it is not likely that this change in the model will decrease the accuracy of representation. In fact, by judicious adjustment of the radius R of the circular target model, this accuracy may even be improved.

It should be noted that if computers are available, techniques of numerical analysis (see Section 3 of the Fire Control Series) may be employed to carry out the hit-probability integrals. With such aids to computation, much more elaborate target models can be employed.

4-4.3.4 Engagement Hit Probability

The engagement hit probability $(Pr)_{eh}$ – i.e., the probability of obtaining a hit with at least one shot during the course of an engagement† – can be stated in terms of the single-shot hit probability $(Pr)_{ssh}$ just derived and the number of shots per engagement n. Let the single-shot probability of not hitting be designated Q_{ssh}, where, by definition

$$Q_{ssh} = 1 - (Pr)_{ssh} . \tag{4-70}$$

Then, since the probability of not hitting can logically be assumed to be the same for each shot and to be statistically independent, the probability of not hitting during the engagement Q_{eh} is given by the relationship

$$Q_{eh} = Q_{ssh}^n . \tag{4-71}$$

This equation is based on an extension of Eq. 4-4 to n variables, with all n variables in this case being equal. By definition

$$Q_{eh} = 1 - (Pr)_{eh} \tag{4-72}$$

* Equation 4-69 can also, of course, be derived independently by using a procedure similar to that of Derivation 4-3.

† An engagement is defined as the firing of more than one shot from a given weapon at a given target under essentially the same conditions.

where

$(Pr)_{eh}$ = probability of obtaining a hit with one or more shots during the course of an engagement.

Thus, a combination of Eqs. 4-70, 4-71 and 4-72 shows that the engagement hit probability is given by the relationship

$$\boxed{(Pr)_{eh} = 1 - Q^n_{ssh} = 1 - \left[1 - (Pr)_{ssh}\right]^n}^{*} \tag{4-73}$$

As discussed in the introductory material relating to accuracy considerations (see par 4-4. 1), the design of a fire control system from the accuracy standpoint is usually based on a required single-shot hit probability which can be derived from a specified engagement hit probability. Given the number of shots n in the engagement, Eq. 4-73 can be used to compute the required single-shot hit probability from the specified engagement hit probability. Conversely, Eq. 4-73 can also be used to determine the engagement hit probability from the computed or experimentally determined single-shot hit probability of a given weapon system. A third use of Eq. 4-73 is illustrated by Example 4-5 in the Appendix to Chapter 4. Because of its particular importance, Eq. 4-73 is shown boxed.

The number of shots in an engagement may be specified from tactical Considerations, or may be computed from the known values of the average rate of fire of the weapon, the maximum range of the weapon, and the average speed of the target. In certain cases, the number of shots in an engagement is determined by the characteristics of the weapon employed in the weapon system under consideration. For example, in the Vigilante Antiaircraft Weapon System, (See par 4-6), the number of shots in an engagement is determined by the capacity of the weapon's magazine.

While the engagement hit probability $(Pr)_{eh}$ is of basic importance in the design of a fire control system, it is a quantity that is diffucult to obtain analytically. A simplified case that lends itself to analysis, however, has been treated by Tappert.[28] This case, which has great practical significance, assumes a target area that has linear dimensions that are small compared with σ_d†. If, as a first step, the case of a fixed bias h is assumed, the engagement hit probability $(Pr)_{eh}$ can be obtained by substituting from Eq. 4-69 into Eq. 4-73. This gives

$$(Pr)_{eh} = 1 - \left\{1 - \frac{A}{2\pi\sigma_d^2} \exp\left[-\frac{h^2}{2\sigma_d^2}\right]\right\}^n . \tag{4-74}$$

In general, however, the bias error does not have a fixed value during the course of an engagement as has been assumed in the analysis thus far. Accordingly, a probability must be assigned to each possible value of the bias. As in the case of dispersion, it can be assumed that the bias is normally distributed. In addition, the variance of the bias σ_b can be assumed to be the same for both axes. Therefore, the probability density function of the

---

*Note that when n = 1, the relationship for the engagement hit probability reduces to the relationship for the single-shot hit probability.

† In those cases where the target radius R is small with respect to the standard deviation of the bias σ_b, but not necessarily small compared with the standard deviation of the dispersion σ_d, the derivation that follows is still applicable, provided that the roles of σ_b and σ_d are reversed.

It should also be noted that, for the assumption made, the diffuse target model becomes indistinguishable from a target model that consists of an area within which a burst produces a kill and outside of which a burst does not produce a kill; i. e., the kill probability assignment function is unity for bursts within the area and is zero for bursts outside the area. Accordingly, the terms "hit-probability" and "kill-probability" can be used interchangeably in this case.

bias $p_b(x, y)$ is given by the bivariate normal density function of Eq. 4-53 with zero means and equal variances, i. e.,

$$p_b(x,y) = \frac{1}{2\pi\sigma_b^2} \exp\left[-\frac{x^2+y^2}{2\sigma_b^2}\right] \tag{4-75}$$

where

$p_b(x, y)$ = probability density function of the bias in rectangular-coordinate form

σ_b^2 = variance of the bias.

This density function can be converted to the polar-coordinate form $p_b(r, \theta)$ in a manner similar to the procedure followed in the derivation of Eq. D4-2. 12 of Derivation 4-2, giving

$$p_b(r,\theta) = \frac{1}{2\pi\sigma_b^2} \exp\left[-\frac{r^2}{2\sigma_b^2}\right] \tag{4-76}$$

For each value of the bias, there exists an individual engagement hit probability assignment function, denoted $(Pr)_{ieh}(r, \theta)$, that has the form of Eq. 4-74, with the fixed bias h replaced by the variable r, i. e.,

$$(Pr)_{ieh}(r,\theta) = 1 - \left[1 - \frac{A}{2\pi\sigma_d^2} \exp\left[-\frac{r^2}{2\sigma_d^2}\right]\right]^n . \tag{4-77}$$

The engagement hit probability can now be found for any point (r, θ) by determiningthe probability that the bias has the particular value (r,6) and weightingthis probability by the assignment function $(Pr)_{ieh}$(r, @). The engagement hit probability for all points $(Pr)_{eh}$ is then obtained by summing the joint probabilities thus obtained over the entire r, θ-plane containing the target. This procedure can be carried out by the use of joint probability density functions in the same manner used to derive Eq. D4-2. 9 of Derivation 4-2 in the Appendix to Chapter 4. The result is

$$(Pr)_{eh} = \int_0^{2\pi} \int_0^{\infty} (Pr)_{ieh}(r,\theta) p_b(r,\theta) r\,dr\,d\theta \tag{4-78}$$

since

$$(Pr)_b(r,\theta) = p_b(r,\theta) r\,dr\,d\theta. \tag{4-79}$$

The derivation of the engagement hit probability, adapted from Ref. 28, is given in Derivation 4-5 in the Appendix to Chapter 4. The expressions derived (see Eqs. D4-5. 12through D4-5. 17 of Derivation 4-5) are not repeated in the text because of their length.

In Ref. 27, a method of determining the engagement kill probability is presented. The integrals in this method may be evaluated by a number of alternative series expansions, or by tables of the incomplete gamma function. It should be readily possible to adapt the derivations of Ref. 27 to the determination of engagement hit probabilities.

For a simple illustrative application of hit and kill probability theory, consider the instance of a mortar that is firing on an enemy strong-point that is located at the edge of a river (see Fig. 4-15)* Assume that the following conditions hold true:

*For a practical, far more complex, example of the application of hit probability theory, see par 4-6 on the Vigilante Weapon System. Among other things, this example illustrates the effect of such factors as target range, speed, and aspect on the hit probability.

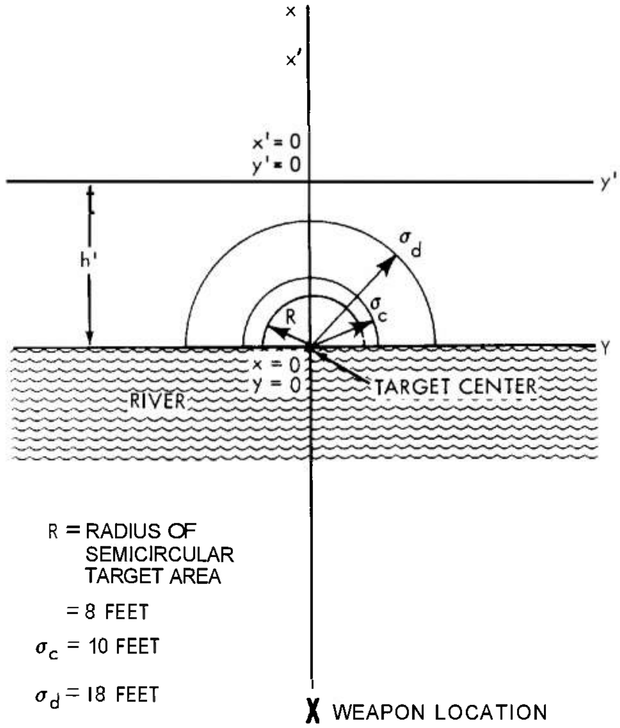


Figure 4-15. Plan view of weapon and target locations for an illustrative example.

1. The frontal area of the target (as presented to the mortar trajectory) can be represented by a semicircle having a radius R of 8 feet.
2. The vulnerable radius σ_c for the ammunition to be employed has been determined to be 10 feet.
3. All projectiles that fall short of the target will land in the river (depicted by cross-hatching in Fig. 4-15) and have zero effectiveness.
4. For the target range concerned, the mortar is known to have a uniform dispersion whose standard deviation σ_d is 18 feet.
5. The bias error h has been reduced to zero by effective spotting.

<u>Problem No. 1</u>:

Under the conditions noted, what is the single-shot hit probability?

<u>Solution to Problem No. 1</u>:

Since the hit probability is zero in the half-plane that includes the river, the angle in Eq. 4-65 is integrated from 0 to π instead of from 0 to 2π. The result of this integration is

$$(Pr)_{ssh} = \frac{1}{2}\left(1 - \exp\left[-\frac{R^2}{2\sigma_d^2}\right]\right) \tag{4-80}$$

Note that the right-hand side of Eq. 4-80 is one-half the right-hand side of Eq. 4-68. Inserting the known numerical values in Eq. 4-80 yields

$$(Pr)_{ssh} = \frac{1}{2}\left(1 - \exp\left[-\frac{(8)^2}{2(18)^2}\right]\right) = \frac{1}{2}\left(1 - e\,0.099\right) = 0.047.$$

Problem No. 2:

Observing that, on the average, half of the projectiles splash harmlessly into the river, the gun commander orders the weapon to be pointed so as to introduce an intentional bias directly back from the edge of the river, with successive values of bias of 0, 3, 6, 9, 12 and 18 feet. It is theoretically possible to compute the hit probability under these conditions but an analytic solution would be extremely complex – if not impossible. However, the kill probability can be determined in a straightforward manner and will serve to indicate the effectiveness of modifying the fire control elevation solution by changing the bias. The problem is then: What are the kill probabilities for the specified values of bias?

Solution to Problem No. 2:

In the derivation of Eq. 4-58, the transformation $x' = x - (h/m+1)$ was performed, where $m = \sigma_d^2/\sigma_c^2$. This is equivalent to translating the center of the x, y coordinate system a distance $h' = h/(m+1)$ away from the river edge along the x-axis, as shown in Fig. 4-15. Integration can now be carried out in the (x', y') coordinate system whose center is located at $x = h'$, $y = 0$. The lower limit of the x' integration should be now changed from $-\infty$ to $-h'$ since the kill probability has been assumed to be zero in the area from $x' = -\infty$ to $x' = -h'$, i.e., the region that includes the weapon and the river right up to the river bank at which the target is located.

If, in Eq. D4-3.11, the p(z) distribution is eliminated and k = 0, the reduced equation is

$$(Pr)_{ssk} = \frac{1}{2\pi\sigma_d^2} \exp\left[-\frac{h^2}{2(m+1)\sigma_c^2}\right] \int_{-\infty}^{\infty} dy' \int_{-h'}^{\infty} \exp\left[-\frac{m+1}{2m\sigma_c^2}\left(x'^2 + y'^2\right)\right] dx'. \tag{4-81}$$

The double integral of Eq. 4-81 is evaluated by dividing the exponential function into the product of two exponentials, thus separating the variables, i.e.,

$$\begin{aligned}
&\int_{-\infty}^{\infty} dy' \int_{-h'}^{\infty} \exp\left[-\frac{m+1}{2m\sigma_c^2}\left(x'^2 + y'^2\right)\right] dx' \\
&= \int_{-\infty}^{\infty} \exp\left[-\frac{m+1}{2m\sigma_c^2}\, y'^2\right] dy' \int_{-h'}^{\infty} \exp\left[-\frac{m+1}{2m\sigma_c^2}\, x'^2\right] dx' \qquad (4\text{-}82) \\
&= \int_{-\infty}^{\infty} \exp\left[-\frac{y'^2}{2\sigma_1^2}\right] dy' \int_{-h'}^{\infty} \exp\left[-\frac{x'^2}{2\sigma_1^2}\right] dx'
\end{aligned}$$

with the substitution of σ_1^2 for $(m/m+1)\,\sigma_c^2 = \sigma_c^2/(\sigma_d^2+\sigma_c^2)$.

The value of the first integral on the right-hand side of Eq. 4-82 can readily be determined to be $\sigma_1\sqrt{2\pi}$ by reference to mathematical tables.*

The integration of the second integral is illustrated by Fig. 4-16 which shows a plot of the exponential function $\exp[x'^2/2\sigma_1^2]$. It is desired to obtain the area under the curve from $x' = -h'$ to $x' = \infty$ since the probability associated with the cross-hatched area has been assumed to be zero for the particular problem under consideration. From symmetry, it is apparent that the area under the curve from $-h'$ to zero is one-half that from $-h'$ to $+h'$.

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* See, for example, definite integral No. 861.3 of Dwight.[23]

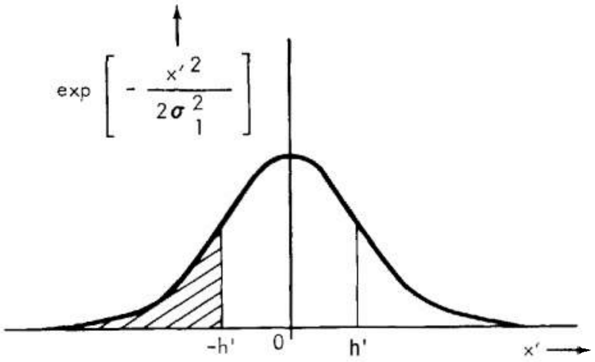


Figure 4-16. Plot of exp $[-x'^2/2\sigma_1^2]$ as a function of x'.

Therefore

$$\int_{-h'}^{\infty} \exp\left[-\frac{x'^2}{2\sigma_1^2}\right]dx' = \frac{1}{2}\int_{-h'}^{h'} \exp\left[-\frac{x'^2}{2\sigma_1^2}\right]dx' + \int_{0}^{\infty} \exp\left[-\frac{x'^2}{2\sigma_1^2}\right]dx'. \qquad (4\text{-}83)$$

Evaluation of the second term on the right-hand side of Eq. 4-83 by the use of the same definite integral employed for the first integral on the right-hand side of Eq. 4-82 shows that it has the value $\sigma_1\sqrt{2\pi}/2$. As has just been noted, the first term on the right-hand side of Eq. 4-83 represents the area under the curve of Fig. 4-16 that lies between $x' = -h'$ and $x' = 0$. This first term can be evaluated by making the arbitrary substitutions $t = x'/\sigma_1$ and $X = h'/\sigma_1$. Since $dx' = \sigma_1\, dt$ and since $t = \pm X$ when $x' = \pm h'$, the first term on the right-hand side of Eq. 4-83 can be rewritten as follows:

$$\frac{1}{2}\int_{-h'}^{h'} \exp\left[-\frac{x'^2}{2\sigma_1^2}\right]dx' = \frac{\sigma_1}{2}\int_{-X}^{X} \exp\left[-\frac{t^2}{2}\right]dt. \qquad (4\text{-}84)$$

The expression on the right-hand side of Eq. 4-84 can be recognized as a form of the error function erf $(X/\sqrt{2})$, which is given by Eq. 4-44. Accordingly

$$\frac{1}{2}\int_{-h'}^{h'} \exp\left[-\frac{x'^2}{2\sigma_1^2}\right]dx' = \frac{\sigma_1\sqrt{2\pi}}{2}\,\mathrm{erf}\left(\frac{X}{\sqrt{2}}\right) \qquad (4\text{-}85)$$

where

$$\mathrm{erf}\left(\frac{X}{\sqrt{2}}\right) = \frac{1}{\sqrt{2\pi}}\int_{-X}^{X} \exp\left[-\frac{t^2}{2}\right]dt. \qquad (4\text{-}86)$$

The error function erf $(X/\sqrt{2})$ can be found in such mathematical tables as References 23 and 26.

When the evaluations of the integrals thus obtained are substituted into Eq. 4-81 the result is

$$(Pr)_{ssk} = \frac{1}{2\pi\sigma_d^2} \exp\left[-\frac{h^2}{2(m+1)\sigma_c^2}\right](\sigma_1\sqrt{2\pi})\left(\frac{\sigma_1\sqrt{2\pi}}{2}\operatorname{erf}\left(\frac{X}{\sqrt{2}}\right)+\frac{\sigma_1\sqrt{2\pi}}{2}\right)$$

$$= \frac{\sigma_1^2}{2\sigma_d^2} \exp\left[-\frac{h^2}{2(m+1)\sigma_c^2}\right]\left\{1+\operatorname{erf}\left(\frac{X}{\sqrt{2}}\right)\right\}. \tag{4-87}$$

This equation can be evaluated in terms of h by determining values of σ_1 and X. The values for the lethal radius σ_c and the standard deviation of the dispersion σ_d have been assumed to be 10 feet and 18 feet, respectively. The substitution of these values in the previously established relationships for σ_1 and X shows that

$$\sigma_1^2 = \frac{\sigma_c^2\sigma_d^2}{\sigma_c^2+\sigma_d^2} = 76.5; \quad \sigma_1 = 8.75 \text{ feet} \tag{4-88}$$

and

$$X = \frac{h'}{\sigma_1} = \frac{}{\sigma_d^2} = 0.027\, h. \tag{4-89}$$

Accordingly, with these substitutions, Eq. 4-87 shows that

$$(Pr)_{ssk} = \frac{76.5}{2\times 324} \exp\left[-\frac{h^2}{2(3.24+1)100}\right]\left(1+\operatorname{erf}\frac{0.027\,h}{\sqrt{2}}\right) = 0.118 \exp\left[-0.00118h^2\right]\left(1+\operatorname{erf}\frac{0.027h}{\sqrt{2}}\right). \tag{4-90}$$

This equation can be evaluated for specific values of h. For example, with h = 10 feet, Eq. 4-90 becomes

$$(Pr)_{ssk} = 0.118e^{-0.118}\left(1+\operatorname{erf}\frac{0.27}{\sqrt{2}}\right). \tag{4-91}$$

Application of Table 1045 of Reference 23 shows that the error function erf $(0.27/\sqrt{2})$ is equal to 0.2128. Accordingly,

$$(Pr)_{ssk} = (0.118)\ (0.894)\ (1.213) = 0.128 \tag{4-92}$$

The single-shot kill probability $(Pr)_{ssk}$ has been plotted against bias h in Fig. 4-17. This figure illustrates the effect of an intentional bias in this particular problem. It can be seen that the introduction of bias gives a modest increase in kill probability, with the optimum bias at about 7 to 8 feet. As might be expected, the kill probability drops off rapidly

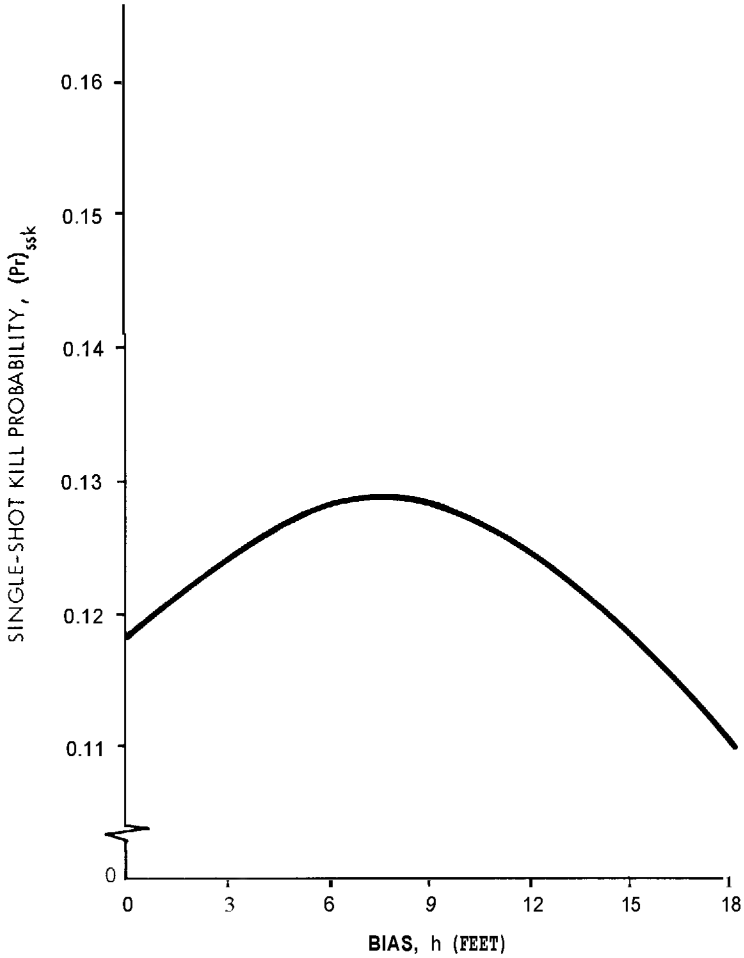


Figure 4-17. Single-shot kill probability as a function of bias for an illustrative example.

as the bias exceeds the lethal radius of the target.

4-4.4 ERROR ANALYSIS IN FIRE CONTROL SYSTEMS

4-4.4.1 Introduction

The material in par 4-4.3 describes a means of determining the allowable error in a weapon system, given a specified engagement hit probability and the number of shots in an engagement. The allowable error is usually expressed in terms of its systematic and random components, specified respectively by the variance of the bias σ_b^2 and the variance of the dispersion σ_d^2. As noted in the procedural summary given in par 4-4.1.3, the next step in the design of a fire control system from the accuracy standpoint is the determination of those errors of the weapon system that are inherently beyond the control of the fire control system designer. For example, in the input portion of the weapon system, the nature of the target's motion may cause errors that are beyond the control of the fire control system designer. Similarly, errors associated with the weapon and its projectile may cause errors at the output end of the weapon system that are beyond the control of the fire control system designer. A discussion of these input and output weapon-system errors is given in par 4-4.5.

Before proceeding to this topic, however, it is in order to consider the steps involved in an error analysis of a fire control system and to derive useful error-propagation equations.

For reference purposes in this error-analysis presentation, Fig. 4-18 provides a conceptual layout of a weapon system. This figure shows the major divisions of a weapon system as follows:

1. The input part of the weapon system, which comprises the acquisition and tracking system.
2. The fire control computing system.
3. The output part of the weapon system, which comprises the weapon and weapon-pointing system.

The flow of signals through a weapon system is the same as the flow of their associated errors. The diagram of Fig. 4-18 indicates only the error components, inasmuch as they are all that are of concern in the present discussion. Input errors enter the weapon system from the target and sometimes from other sources – e. g., from a gyroscopein the tracking system or from a command post. The final result of all errors associated with the weapon system is the error in the system output and is usually expressed in terms of the variance of the bias σ_b^2 and the variance of the dispersion σ_d^2.

Figure 4-18 shows the flow of errors between the three major subsystems and also indicates a possible arrangement of elements within the subsystems. Many variations of the arrangement of elements and the interconnections between them are, of course, possible. In order to determine the error in the output of a subsystem, the flow of errors through the group of elements and the modifications made on the errors by the elements during this flow must be determined. It is for this purpose that the error-propagation equations are employed.

A typical fire control system includes such elements as tracking servos, weapon-pointing servos, analog and digital computing devices, and data-transmission and conversion elements. Except for the digital computing elements, all of these devices may be classified as analog computing elements. Since the digital computing elements, if present, are coupled to the analog input and output elements by analog-digital converters, the combination of the

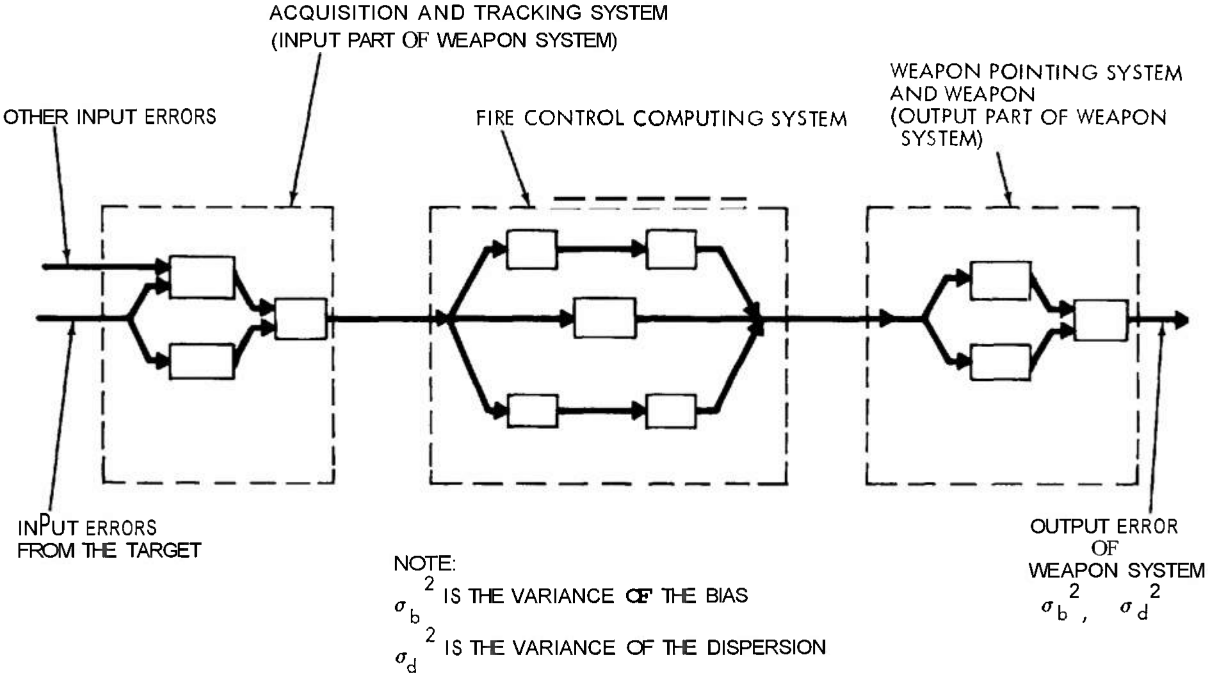


Figure 4-18. Representative error diagram associated with a weapon system.

digital computing elements and the analog-digital converters may be treated as a unit having analog inputs and outputs. Methods of error analysis that were originally devised for analog computers may, therefore, be applied to an entire fire control system.

The procedure for allocating allowable errors in a fire control system during the course of its design is as follows:

1. First, determine from a functional block diagram of the system* and from the sets of error-propagation equations that are developed in the following paragraphs the apportionment of the total allowable error among the major subsystems.

2. Next, extend this same procedure to determine the allowable error in each of the components that make up the subystems.

It should be emphasized that this phase of the actual design process is far more complex than the straightforward procedure that might be inferred from the simple statement of these two steps. In the first place, the equations of error propagation are much easier to employ in the analysis of a fire control system than they are in its synthesis – i.e., in the actual designing of the system from the beginning – based on performance requirements. Therefore, in synthesis, a common procedure is to initially assign errors (either arbitrarily or based on experience) to the various elements of the analog system under consideration. Next, a trial system error is determined by means of the appropriate error-propagation equations. In so doing, the places where the specifications should be relaxed or tightened will become evident. The process must then be repeated as many times as necessary to achieve a system of the required accuracy with realistic errors assigned to the individual elements of the system. Rearrangement of system elements may be required in some instances. For example, if the initial design of a computer called for the generation of an output quantity by the subtraction of one large input variable from another, it might be found that the error in the output quantity was large in comparison with the output quantity itself. If at all possible, the accuracy should be improved by rearrangement of the computer elements as required to avoid a subtraction involving two large quantities.

An analysis of error propagation in analog systems that are describable by equations other than differential equations can be carried out in a relatively straightforward fashion. Much greater complexity is introduced if differential equations must be employed to describe the system. Therefore, the simpler case is discussed first (see par 4-4.4.2), followed by the case involving differential equations (see par 4-4.4.3).

4-4.4.2 Analysis of Error Propagation in Systems Describable by Equations Other Than Differential Equations†

An analysis of the propagation of errors in assemblages of analog devices that can be described by algebraic, trigonometric, or empirical equations (but with differential equations excluded) can be made if it is assumed that the analog system can be separated into independent – i.e., noninteracting – operating elements, each of which has an identifiable error that is independent of that in the other elements. In actual systems, this independence is generally the case since – for convenience in design and maintenance – it is desirable to assemble such systems from standard modules, each of which has measurable characteristics that are largely independent of the preceding and following modules. Thus, components such as potentiometers and resolvers are either isolated by amplifiers, or designed with impedance levels such that cascaded components do not appreciably load one another.

The simplest type of independent element is one that has only one input and only one output. This element is represented by the functional diagram of Fig. 4-19(A). It is assumed

* See Chapter 3 for a discussion of functional block diagrams.

† The analysis presented in par 4-4.4.2 is adapted from that presented by Dr. J. G. Tappert in References 32 and 33.

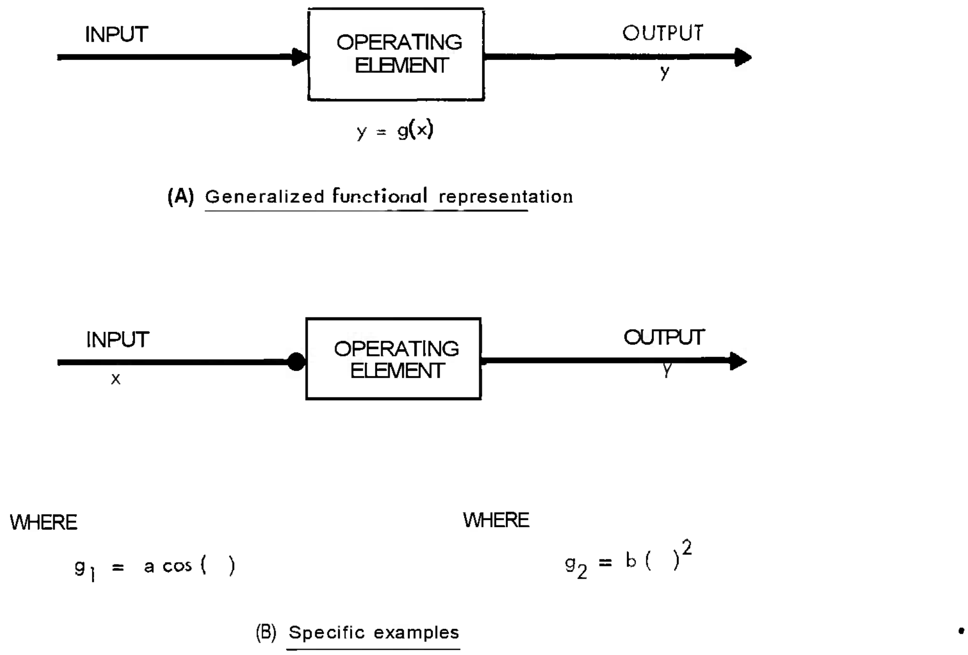


Figure 4-19. Functional representation of an element having a single input and a single output.

that the output y is a function of the input x but is independent of time or any other variable.* The functional dependence between the input and the output of the element can be expressed by the generalized equation

$$y = [PO]x \tag{4-93}$$

where

[PO] = performance operator of the element.

The performance operator is a broad concept that is employed to relate the input and output of an operating element.† Equation 4-93, which is referred to as the generalized performance equation of the operating element, states that the output y of the element results from the action of the performance operator of the element [PO] on the input x.

For the purposes of par 4-4.4.2, which is concerned with the analysis of error propagation in systems describable by other than differential equations, the generalized symbol for the performance operator [PO] will be replaced by the simpler notation g. With this substitution, Eq. 4-93 becomes

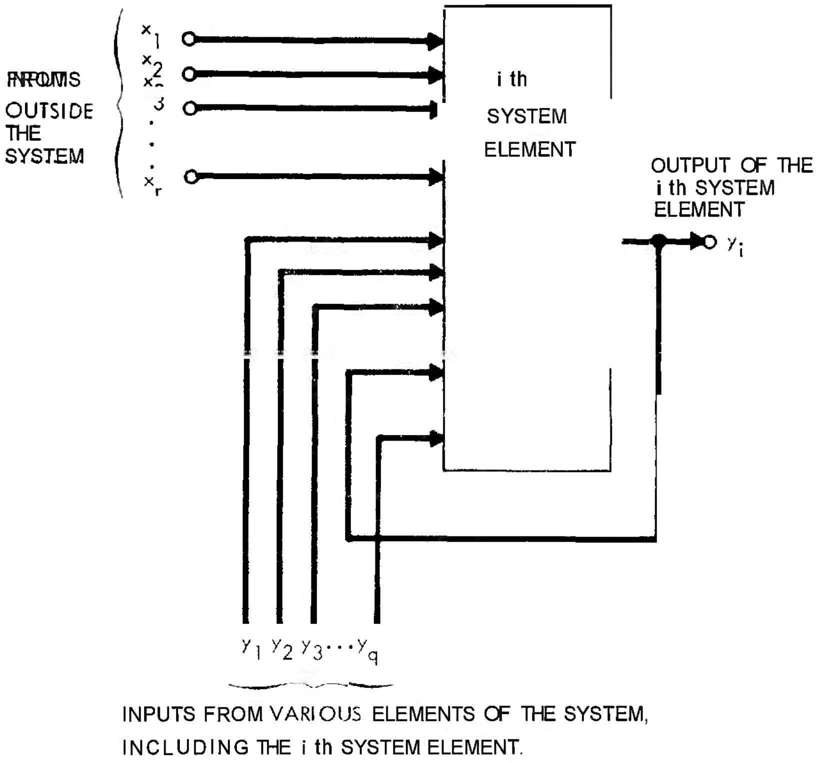
$$y = g(x) \tag{4-94}$$

where g represents the action of the operating element on the input x. The function g may represent any input-output relationship that does not involve differentials – e.g., $y = a \cos x$ and $y = bx^2$ – as depicted in Fig. 4-19(B).

The simplest single-input concept just described may be readily expanded to include the case of an operating element that receives multiple inputs and generates a single output (see Fig. 4-20). The i^{th} system element shown in Fig. 4-20 has r inputs from outside the sys-

* In par 44.4.3, the concept will be extended to functions that have a time response.

† For a complete discussion of the performance operator and its applications, see Reference 34.



$$y_i = g_i\,(x_1, \ldots, x_r;\ y_1, \ldots, y_i, \ldots, y_q)$$

WHERE

g_i = PERFORMANCE OPERATOR OF THE i th SYSTEM ELEMENT

Figure 4-20. Functional diagram of a typical system element.

tem, $x_1, x_2, \ldots, x_r$, and a single output y_i. (With this kind of breakdown, an actual system element with more than one output, such as a resolver, would be treated as two or more elements.) The system is considered to be made up of q such elements, with outputs $y_1, y_2, \ldots, y_q$, some of which are also the system outputs.

In the most general case, each system element will have r inputs from outside the system and q inputs from inside. The output of a typical element i of the system can then be expressed as

$$y_i = g_i\,(x_1, \cdots, x_r;\ y_1, \cdots, y_i, \cdots, y_q) \qquad (4\text{-}95)$$

where

$x_1, \ldots, x_r$ = r inputs to the i^{th} element from outside the system

$y_1, \ldots, y_i, \ldots, y_q$ = q inputs to the i^{th} element from the various elements of the system itself

g_i = performance operator of the i^{th} element.

Equation 4-95 gives the performance equation of the i^{th} element in general form, i.e., with g_i left unexpressed in specific terms. The performance operator g_i represents the action of the i^{th} element on the various inputs whereby the output of the element y_i is produced.

Simple explicit examples of typical multiple-input operating elements and their specific performance equations are represented in Fig. 4-21. Figure 4-21(A) represents a simple two-input element with a few of the many possible performance equations included. The case of an additive constant may be treated as a two-input element with one input held constant (see Fig. 4-21(B)). Figure 4-21(C) shows a case of output feedback. It should be noted that the performance equation given in this figure can be rewritten in the form

$$y = g_5(x,y) = \mu(x - y) = \frac{\mu}{\mu + 1}x \tag{4-96}$$

which is the familiar input-output relationship for a unity-feedback amplifier. The quantity μ is the amplification of the amplifier when no feedback is present.

A simple analog computer is shown in Fig. 4-22 as an example of the combination of typical elements into a small system. Application of Eq. 4-95 to the computer element

$y = ax_1 + bx_2 = g_1(x_1, x_2)$

WHERE

$g_1 = a(\ )_1 + b(\ )_2$

OR

$y = ax_1 x_2 = g_2(x_1, x_2)$

WHERE

$g_2 = a(\ )_1(\ )_2$

OR

$y = ax_1 \cos x_2 = g_3(x_1, x_2)$

WHERE

$g_3 = a(\ )_1 \cos(\ )_2$

(A) Element with two input variables

$y = a + bx^2 = g_4(x)$

WHERE

$g_4 = a + b(\ )^2$

(B) Element with one of its two inputs held constant

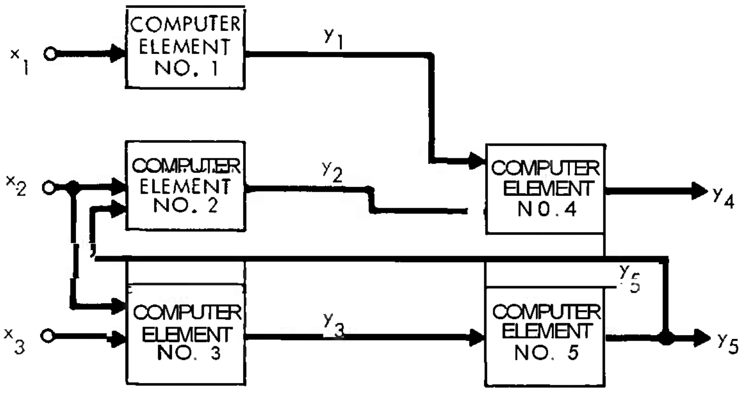
$y = \mu(x - y) = g_5(x, y)$

WHERE

$g_5 = \mu[(\ )_1 - (\ )_2] = \mu(\ )_1 - \mu(\ )_2$

(C) Element employing unity feedback

Figure 4-21. Examples of simple multiple-input system elements.



SPECIFIC EXPRESSIONS:

FOR COMPUTER ELEMENT NO. 1,
$y_1 = g_1(x_1)$

THEREFORE,
$y_1 - g_1(x_1) = f_1(x_1, y_1) = 0$

FOR COMPUTER ELEMENT NO. 2,
$y_2 = g_2(x_2, y_5)$

THEREFORE,
$y_2 - g_2(x_2, y_5) = f_2(x_2, y_2, y_5) = 0$

FOR COMPUTER ELEMENT NO. 3,
$y_3 = g_3(x_2, x_3)$

THEREFORE,
$y_3 - g_3(x_2, x_3) = f_3(x_2, x_3, y_3) = 0$

FOR COMPUTER ELEMENT NO. 4,
$y_4 = g_4(y_1, y_2)$

THEREFORE,
$y_4 - g_4(y_1, y_2) = f_4(y_1, y_2, y_4) = 0$

FOR COMPUTER ELEMENT NO. 5,
$y_5 = g_5(y_3)$

THEREFORE,
$y_5 - g_5(y_3) = f_5(y_3, y_5) = 0$

WHERE
g_1, g_2, g_3, g_4 AND g_5 = PERFORMANCE OPERATORS OF COMPUTER ELEMENTS NO. 1 THROUGH NO, 5, RESPECTIVELY

GENERAL EXPRESSION:
THIS EXAMPLE SHOWS THAT THE SPECIFIC EXPRESSIONS CAN BE EXPRESSED IN THE FOLLOWING COMPLETELY GENERALIZED FORM:

$$f_i(x_1, \ldots, x_r; y_1, \ldots, y_i, \ldots, y_q) = y_i - g_i(x_1, \ldots, x_r; y_1, \ldots, y_i, \ldots, y_q) = 0$$

Figure 4-22. Functional diagram of a typical analog computer with five elements and three inputs.

No. 2 in the figure yields the relationship

$$y_2 = g_2(x_2, y_5). \tag{4-97}$$

Suppose, for example, that computer element No. 2 is a resolver whose operation is represented by the equation

$$y_2 = x_2 \cos y_5 \tag{4-98}$$

Comparison of Eqs. 4-97 and 4-98 shows that

$$g_2(x_2, y_5) = x_2 \cos y_5 . \tag{4-99}$$

Equation 4-97 can also be written in the form

$$y_2 - g_2(x_2, y_5) = 0. \tag{4-97a}$$

A functional notation can then be introduced by defining

$$f_2(x_2, y_2, y_5) = y_2 - g_2(x_2, y_5) = 0. \tag{4-97b}$$

In this functional notation, the performance equation for the i^{th} system element that is given by Eq. 4-95 can be expressed in the completely generalized form

$$f_1(x_1, \cdots, x_r; y_1, \cdots, y_i, \cdots y_q) = y_i - g_i(x_1, \cdots x_r; y_1, \cdots, y_i, \cdots, y_q) = 0. \tag{4-100}$$

The specific performance equations for each of the five computer elements of Fig. 4-22 are given in this functional notation directly on that illustration.

The set of q simultaneous equations (one for eachcomputer element of the system) corresponding to Eq. 4-100 are the equations solved by an ideal system.

An actual system, which would usually be nonideal to some degree, is, in general, comprised of q nonideal elements. Even with perfect inputs, the output from each element will contain an error component due to the nonideal nature of the element. Therefore, the output of the i^{th} element will comprise thecorrect output y_i plus an output error component that is designated m_i (see Fig. 4-23),,

In order to determine the effect of the nonideal element in producing the error m_i, it is possible to postulate a generalized function

$$h_i(x_i, \cdots, x_r; y_1, \cdots y_i + m_i, \cdots, y_q) \tag{4-101}$$

for this element and establish its relationship to $f_i(x_1, \ldots, x_r; y_1, \ldots, y_q)$, the generalized function for the corresponding ideal element that appears in **Eq.** 4-100.* The resulting relationship will be a performance equation for the i^{th} nonideal element, stated in functional form. There will be q such performance equations for the complete system of q elements. Once these performance equations for the nonideal elements are determined, error equations that establish the output errors resulting from erroneous inputs to these nonideal elements can then be determined.

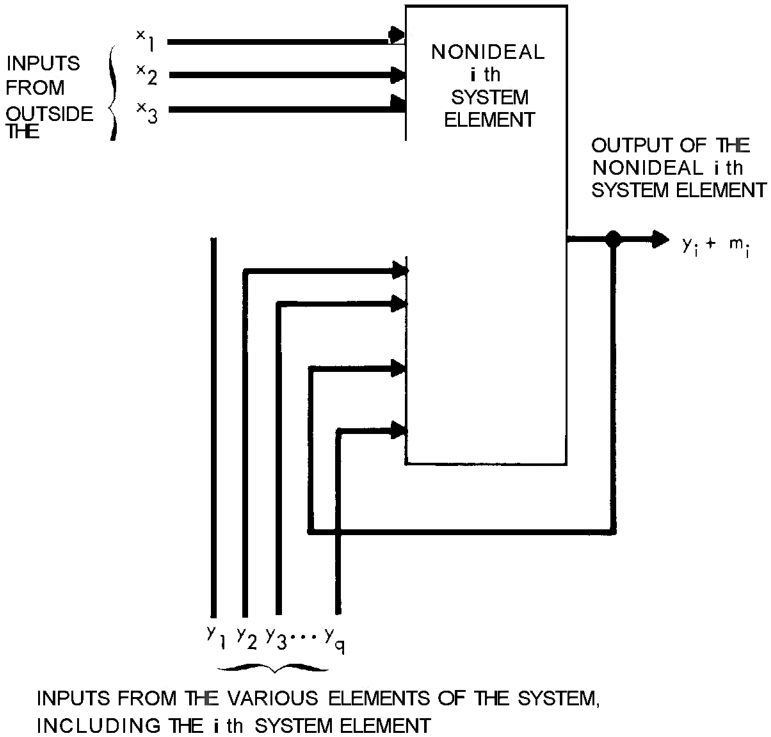
In order to establish the desired performance equations for the q nonideal elements, the error terms must be separated from the function h_i.† As a first step in this separation of error terms, the functions f_i and h_i are expanded by use of Taylor's series. This type of series provides a means for approximating the value of any differentiable function at a point (x + a) in terms of a power-series expansion of the function at point x, where a is a suitably chosen spacing along the x axis. The basic form for Taylor's series is**

$$f(x + a) = f(x) + \frac{a}{1!} f'(x) + \frac{a^2}{2!} f''(x) + \frac{a^3}{3!} f'''(x) + \cdots \tag{4-102}$$

* From here on, the simplified notation f_i in place of $f_i(x_1, \ldots, x_r; y_1, \ldots, y_q)$ and $h_i(x_1, \ldots, x_r; y_1, \ldots, y_i + m_i, \ldots, y_q)$ will be employed for the sake of convenience.

† The procedure followed in this error separation is a perturbation technique whose general features are similar to those used in calculating space-flight trajectories; see Reference 35 for example.

** See, for example, item No. 39 of Reference 23.



$$\left\{\begin{array}{l}\text{OUTPUT OF NONIDEAL} \\ \text{i th SYSTEM ELEMENT}\end{array}\right\} = y_i + m_i$$

$$= G_i(x_1, \ldots, x_r; y_1, \ldots, y_i + m_i, \ldots, y_q)$$

WHERE

G_i = PERFORMANCE OPERATOR OF THE NONIDEAL i th SYSTEM ELEMENT

y_i = CORRECT OUTPUT OF THE i th SYSTEM ELEMENT

m_i = ERROR IN THE OUTPUT OF THE i th SYSTEM ELEMENT BY VIRTUE OF ITS BEING NONIDEAL

IN TERMS OF THE GENERALIZED FUNCTION POSTULATED FOR A NONIDEAL SYSTEM,

$$y_i + m_i - G_i\,(x_1; \ldots, x_r; y_1, \ldots, y_i + m_i, y_q) = h_i\,(x_1, \ldots, x_r; y_1, \ldots, y_i + m_i, \ldots, y_q) = 0$$

Figure 4-23. Functional representation and associated relationships for a typical nonideal system element.

where $f'(x)$, $f''(x)$, etc. are, respectively, the first, second and higher derivatives of $f(x)$.

The expansion of f_i and h_i by use of Taylor's series yieldstwo sets of $q + r$ expansions, each of which will have the form given on the right-hand side of Eq. 4-102. For example, the expansion of $h_i(x_1)$ is performed by first choosing an initial value of x_1, designated by the symbol x_{1o}. It is evident that $h_i(x_1)$ is equal to $h_i(x_{1o} + x_1 - x_{1o})$. Since $x_1 - x_{1o}$ is the displacement from the initial to the desired value of x_1, it corresponds to the quantity a in Eq. 4-102. Accordingly, the series can be written by inspection as

$$h_i(x_1) = h_i(x_{1o}) + \frac{(x_1 - x_{1o})}{1!}\frac{\partial h_i(x_{1o})}{\partial x_1} + \frac{(x_1 - x_{1o})^2}{2!}\frac{\partial^2 h_i(x_{1o})}{\partial x_1^2} + \cdots \tag{4-103}$$

The multiplicity of independent variables in the function h_i requires the introduction of the partial derivatives in Eq. 4-103; x_{1o} is the point at which the partial derivatives are evaluated; x_1 is the point at which h is evaluated, and is separated from x_{1o} by spacing a.

The expansion of f_i, in accordance with the example of the preceding paragraph, is

$$\begin{aligned}
f_i = {} & f_i(x_{1o}) + \frac{(x_1 - x_{1o})}{1!}\frac{\partial f_i(x_{1o})}{\partial x_1} + \frac{(x_1 - x_{1o})^2}{2!}\frac{\partial^2 f_i(x_{1o})}{\partial x_1^2} + \cdots \\
& - - - - - - - - - - - - - - - \\
& + f_i(x_{ro}) + \frac{x_r - x_{ro}}{1!}\frac{\partial f_i(x_{ro})}{\partial x_r} + \frac{(x_r - x_{ro})^2}{2!}\frac{\partial^2 f_i(x_{ro})}{\partial x_r^2} + \cdots \\
& - - - - - - - - - - - - - - - \\
& + f_i(y_{io}) + \frac{(y_1 - y_{io})}{1!}\frac{\partial f_i(y_{io})}{\partial y_i} + \frac{(y_i - y_{io})^2}{2!}\frac{\partial^2 f_i(y_{io})}{\partial y_i^2} + \cdots \\
& - - - - - - - - - - - - - - - \\
& + f_i(y_{qo}) + \frac{y_q - y_{qo}}{1!}\frac{\partial f_i(y_{qo})}{\partial y_q} + \frac{(y_q - y_{qo})^2}{2!}\frac{\partial^2 f_i(y_{qo})}{\partial y_q^2} + \cdots
\end{aligned} \tag{4-104}$$

which contains a few typical terms. Similarly, the expansion of h_i is indicated for a few typical terms by the relationship

$$
\begin{aligned}
h_i = h_i(x_{1o}) &+ \frac{x_1 - x_{1o}}{1!}\,\frac{\partial h_i(x_{1o})}{\partial x_i} + \frac{(x_1 - x_{1o})^2}{2!}\,\frac{\partial^2 h_i(x_{1o})}{\partial x_1^2} + \cdots \\
&+ h_i(x_{ro}) + \frac{(x_r - x_{ro})}{1!}\,\frac{\partial h_i(x_{ro})}{\partial x_r} + \frac{(x_r - x_{ro})^2}{2!}\,\frac{\partial^2 h_i(x_{ro})}{a x_r^2} + \cdots \\
&+ h_i(y_{1o}) + \frac{(y_1 - y_{1o})}{1!}\,\frac{\partial h_i(y_{1o})}{\partial y_1} + \frac{(y_1 - y_{1o})^2}{2!}\,\frac{\partial^2 h_i(y_{1o})}{\partial y_1^2} + \cdots \\
&+ h_i(y_{io}) + \frac{(y_i + m_i + y_{io})}{1!}\,\frac{\partial h_i(y_{io})}{\partial y_i} + \frac{(y_i + m_i - y_{io})^2}{2!}\,\frac{\partial^2 h_i(y_{io})}{\partial y_i^2} + \cdots \\
&+ h_i(y_{qo}) + \frac{(y_q - y_{qo})}{1!}\,\frac{\partial h_i(y_{qo})}{\partial y_q} + \frac{(y_q - y_{qo})^2}{2!}\,\frac{\partial^2 h_i(y_{qo})}{\partial y_q^2} + \cdots
\end{aligned}
\tag{4-105}
$$

The terms on the fourth line of Eq. 4-105 – for the $y_i + m_i$ output – require some explanation. The value of the quantity a that has been selected for this expression is $y_i + m_i - y_{io}$, which is valid for cases in which m_i is small. Also, $d(y_i + m_i) \approx dy_i$ for $m_i << y_i$.

Since a can be made as small as desired and since m_i can be assumed to be small, all partial-derivative terms in Eq. 4-105 beyond the first can be neglected. The remaining terms can then be rearranged to give

$$
\begin{aligned}
h_i = h_i(x_{1o}) + (x_1 - x_{1o})\,\frac{\partial h_i(x_{1o})}{\partial x_1} &+ \cdots + h_i(x_{ro}) + (x_r - x_{ro})\,\frac{\partial h_i(x_{ro})}{\partial x_r} + \cdots \\
&+ h_i(y_{1o}) + (y_1 - y_{1o})\,\frac{\partial h_i(y_{1o})}{\partial y_1} + \cdots + h_i(y_{io}) + (y_i - y_{io})\,\frac{\partial h_i(y_{io})}{\partial y_i} + m_i\,\frac{\partial h_i(y_{io})}{\partial y_i} + \cdots \\
&+ h_i(y_{qo}) + (y_q - y_{qo})\,\frac{\partial h_i(y_{qo})}{}
\end{aligned}
\tag{4-106}
$$

The expansion of f_i may be similarly approximated by dropping from **Eq.** 4-104 all terms containing partial derivatives higher than the first. Equation 4-104 then reduces to

$$f_i = f_i(x_{1_o}) + (x_1 - x_{1_o}) \frac{\partial f_i(x_{1_o})}{\partial x_1} + \cdots + f_i(y_{i_o}) + (y_i - y_{i_o}) \frac{\partial f_i(y_{i_o})}{\partial y_i} + \cdots \tag{4-107}$$

If m_i is allowed to go to zero in Eq. 4-106, the remaining terms represent the performance equation for the ideal system. These remaining terms of Eq. 4-106 must, therefore, be identical with the corresponding terms of Eq. 4-107. One can, therefore, substitute for each h_i term in Eq. 4-106 the corresponding f_i term in Eq. 4-107; i. e., in terms of the simplified notation,

$$[h_i]_{m_i=0} = f_i \tag{4-108}$$

If m_i is not zero, the last term of the fourth line of Eq. 4-106 must be reintroduced, which yields

$$h_i = f_i + m_i \frac{\partial f_i(y_{i_o})}{\partial y_i} \tag{4-109}$$

This expression can be rewritten in the simplified form

$$\boxed{h_i = f_i + m_i \frac{\partial f_i}{\partial y_i}} \tag{4-110}$$

where h_i is simplified notation for $h_i(x_1, \ldots, x_r; y_1, \ldots, y_i + m_i, \ldots, y_q)$ and f_i is simplified notation for $f_i(x_1, \ldots, x_r; y_1, \ldots, y_q)$. It is to be understood that the partial derivative in Eq. 4-110 must be evaluated at some set of values of the input and outputs that satisfies the performance equation of the element concerned. Equation 4-110 is the performance equation, in functional notation, of the $\underline{i}^{th}$ element of a nonideal system.

Now consider the general case of a system that has errors in its inputs in addition to being made up of imperfect elements. In this case (see Fig. 4-24), consideration must be given not only to the error m_i that is produced by the $\underline{i}^{th}$ system element as a result of its being nonideal but also to (1) the errors in the inputs $x_1, \ldots, x_r$ that are applied to the $\underline{i}^{th}$ system element from outside the system and (2) the errors in the inputs $y_1, \ldots, y_q$ that are applied to the $\underline{i}^{th}$ system element from the various elements of the system. Assume the system-element inputs to be of the form $x_j + \epsilon_{x_j}$, where x_j is the correct value of the input and ϵ_{x_j} is the error in that input. Similarly, assume the system-element outputs to be of the form $y_i + \epsilon_{y_i}$, where y_i is the correct value of the output from the $\underline{i}^{th}$ system element and ϵ_{y_i} is the error in that output. The error ϵ_{y_i} includes the error m_i that is the result of the $\underline{i}^{th}$ system element being an imperfect element.

The performance equation of the $\underline{i}^{th}$ element of a nonideal system, which is given in functional notation by Eq. 4-110, can be adapted to the case in which nonideal inputs are applied to that element merely by replacing each input x_j by $x_j + \epsilon_{x_j}$ and each input y_i by $y_i + \epsilon_{y_i}$ in the function $h_i(x_1, \ldots, x_r; y_1, \ldots, y_i + m_i, \ldots, y_q)$. The m_i error term can be dropped in this process since, as just pointed out, it is included in the error ϵ_{y_i}. It is important to note that it is not necessary to define a new function to replace h_i for the $\underline{i}^{th}$ system element; rather, it is sufficient to introduce new independent variables that account for the error com-

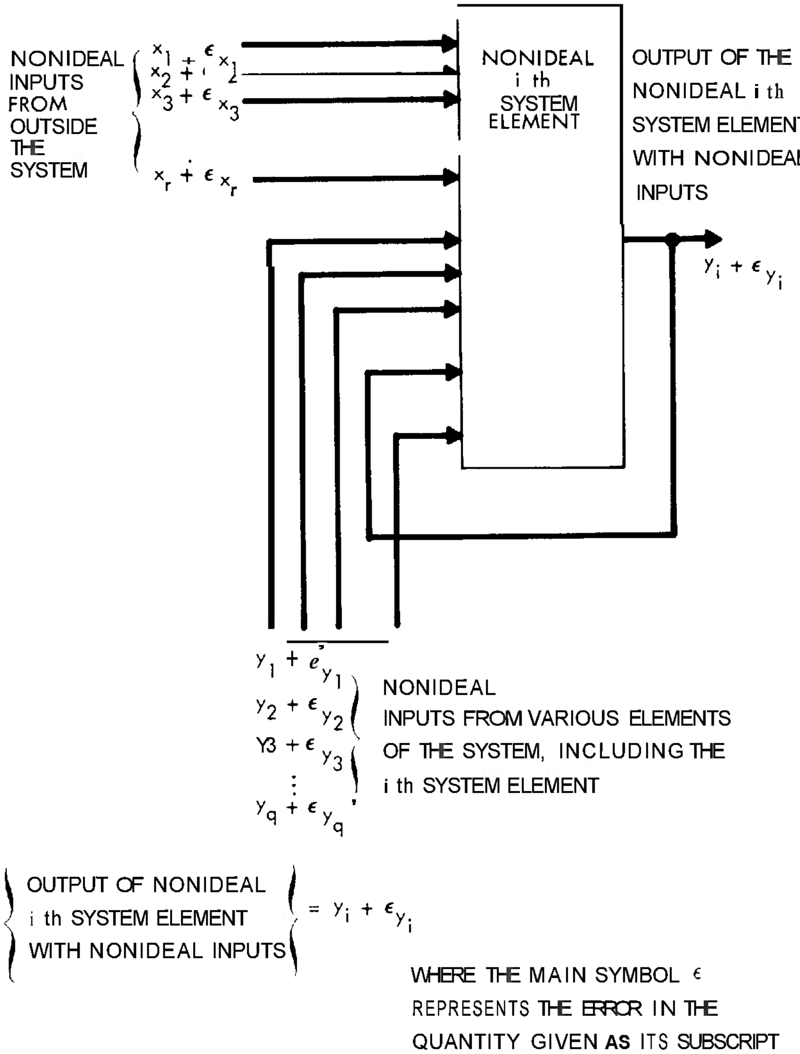


Figure 4-24. Functional diagram and associated relationships for a typical nonideal system element with nonideal inputs.

ponents in the input quantities. Equation 4-110 then becomes

$$h_i(x_1 + \epsilon_{x_1}, \cdots, x_r + \epsilon_{x_r}; y_1 + \epsilon_{y_1}, \cdots, y_i + \epsilon_{y_i}, \cdots, y_q + \epsilon_{y_q}) = f_i + m_i \frac{\partial f_i}{\partial y_i} . \tag{4-111}$$

The errorterms can be separated fromthe function on the left-hand side of Eq. 4-111 by expanding it, through use of Taylor's series, in the same manner as was employed to obtain Eq. 4-105. Again it will be assumed that all partial-derivative terms beyond the first can be neglected and that differentials of errors can be neglected, e.g., $d(y_i + \epsilon_{y_i}) \approx dy_i$. The

resulting expansion gives

$$h_i(x_1 + \epsilon_{x_1}, \cdots, x_r + \epsilon_{x_r}; y_1 + \epsilon_{y_1}, \cdots, y_i + \epsilon_{y_i}, \cdots, y_q + \epsilon_{y_q})$$

$$= h_i(x_{1o}) + (x_1 + \epsilon_{x_1} - x_{1o})\frac{\partial h_i(x_{1o})}{\partial x_1} + \cdots + h_i(y_{io}) + (y_i + \epsilon_{y_i} - y_{io})\frac{\partial h_i(y_{io})}{\partial y_i}$$

$$+ \cdots + h_i(y_{qo}) + (y_q + \epsilon_{y_q} - y_{qo})\frac{\partial h_i(y_{qo})}{\partial y_q}$$

$$h_i(x_{1o}) + (x_1 - x_{1o})\frac{\partial h_i(x_{1o})}{\partial x_1} \quad + h_i(y_{io}) + (y_i - y_{io})\frac{\partial h_i(y_{io})}{\partial y_i} \tag{4-112}$$

$$+ \cdots + h_i(y_{qo}) + (y_q - y_{qo})\frac{\partial h_i(y_{qo})}{\partial y_q} + \epsilon_{x_1}\frac{\partial h_i(x_{1o})}{\partial x_1}$$

$$\frac{\partial h_i(y_{io})}{\quad} + \cdots + \epsilon \quad \frac{\partial h_i(y_{qo})}{\quad}$$

It has already been shown in connection with Eq. 4-106 that the portion of the expansion of Eq. 4-112 that does not contain error terms is identical with f_i. Also, in connection with Eq. 4-108 it has beenshown that h_i can be replaced by f_i in the partial derivatives. Accordingly

$$h_i(x_1 + \epsilon_{x_1}, \cdots, x_r + \epsilon_{x_r}, y_1 + \epsilon_{y_1}, \cdots, y_i + \epsilon_{y_i}, \cdots, y_q + \epsilon_{y_q})$$

$$= f_i + \epsilon_{x_1}\frac{\partial f_i(x_{1o})}{\partial x_1} + \cdots + \epsilon_{y_i}\frac{\partial f_i(y_{io})}{\partial y_i} + \cdots + \epsilon_{y_q}\frac{\partial f_i(y_{qo})}{\partial y_q} \tag{4-113}$$

A comparison of Eqs. 4-111 and 4-113 shows that

$$\epsilon_{x_1}\frac{\partial f_i(x_{1o})}{\partial x_1} + \cdots + \epsilon_{y_i}\frac{\partial f_i(y_{io})}{\partial y_i} + \cdots + \epsilon_{y_q}\frac{\partial f_i(y_{qo})}{\partial y_q} = m_i \frac{\partial f_i}{\partial y_i} \tag{4-114}$$

This relationship can be rewritten in the more compact form

$$\boxed{\sum_{k=1} \frac{\partial f_i}{\partial y_k}\epsilon_{y_k} \qquad \sum_{n=1} \frac{\quad}{\partial x}\epsilon_{x_n} + m_i \frac{\quad}{\quad}} \tag{4-115}$$

where f_i is simplified notation for $f_i(x_1, \ldots, x_r; y_1, \ldots, y_q)$. This is the error equation for the $\underline{i}^{th}$ system element. The particular order of terms that appears in Eq. 4-115 was chosen in order to put the output errors ϵ_{y_k} on the dependent-variable side of the equation, inasmuch as they are the quantities of primary interest. As in the case for Eq. 4-110, it is to be understood that the partial derivatives in Eq. 4-115 must be evaluated at some set of values of the inputs and outputs that satisfies the performance equation of the element concerned.

In the same compact form given by Eq. 4-115 for the $\underline{i}^{th}$ system element, the final set of error equations for the entire system of q elements can be written in the form

$$
\begin{aligned}
\sum_{k=1}^{q} \frac{\partial f_1}{\partial y_k} \epsilon_{y_k} &= - \sum_{n=1}^{r} \frac{\partial f_1}{\partial x_n} \epsilon_{x_n} + m_1 \frac{\partial f_1}{\partial y_1} \\
&\cdots\cdots\cdots\cdots\cdots \\
\sum_{k=1}^{q} \frac{\partial f_i}{\partial y_k} \epsilon_{y_k} &= - \sum_{n=1}^{r} \frac{\partial f_i}{\partial x_n} \epsilon_{x_n} + m_i \frac{\partial f_i}{\partial y_i} \\
&\cdots\cdots\cdots\cdots\cdots \\
\sum_{k=1}^{q} \frac{\partial f_q}{\partial y_k} \epsilon_{y_k} &= - \sum_{n=1}^{r} \frac{\partial f_q}{\partial x_n} \epsilon_{x_n} + m_q \frac{\partial f_q}{\partial y_q}
\end{aligned}
\qquad (4\text{-}116)
$$

This set of equations is the result of the subscript i in Eq. 4-115 taking on all integral values from 1 to q. In Eq. 4-116, $x_1, x_2, x_3, \ldots, x_r$ represent all of the inputs to the system elements that come from outside the system.

Note that Eqs. 4-116 comprise a set of q equations, where q is the number of system elements. The derivation of Eqs. 4-116 depends only on the assumptions that the errors involved – i.e., the ϵ's and the m's – are small, and that the performance equations of the system elements are sufficiently linear in the vicinity of the point at which the partial derivatives are evaluated that the higher partial derivatives can be neglected. In practice, these conditions are usually fulfilled.

The errors involved can be classified as either dependent errors or independent errors. Each dependent error is a function of a system input or output, while each independent error is either constant or is a function of some other variable such as time or an interference type of input, e.g., a temperature change might modify the performance equation of a system element. Some of the independent errors are modified by the partial derivatives so that their effect on the output is thereby made to be dependent on an input variable.

In general, each error will be made up of a systematic component and a random component. Each of these two types of components can be either dependent or independent (as defined in the preceding paragraph), with the form of the dependency not necessarily the same for both the systematic and the random components of a given error. A given set of errors can, therefore, be separated into a group of systematic error components and a group of random error components. Each group can then be treated separately as discussed in subsequent paragraphs.

Inspection of Eqs. 4-116 shows that the evaluation of a particular output error – e.g., ϵ_{y_3} – is obtained by making a linear combination of the terms associated with each input error, each component error, and each of the remaining output errors. The contributions of the input errors ϵ_{x_n} and the component errors m_i are, therefore, independent of one another.

The solution of Eqs. 4-116 for the errors associated with the system outputs is a straightforward, although tedious, process. There are $q(r + q + 1)$ terms involved in all, but usually many of the partial derivatives are zero, which reduces considerably the total number of terms that must be taken into account. The procedure is as follows:

1. Evaluate the partial derivatives at chosen values of the x's and at the corresponding y values.
2. Express all dependent ϵ's and m's as explicit functions of the independent variables.
3. Compute each error term by multiplying the error by the corresponding partial derivative.
4. If the system is complex, tabulate the error terms in matrix form for ease of computation.
5. Either known values, or the variances, of the independent ϵ_x's will be assumed to be given. Solve the equations for specific values or variances, respectively, of the ϵ_y's that correspond to the system outputs.

For a simple example of how Eqs. 4-116 can be applied, see Example 4-6 in the Appendix to Chapter 4, which concerns the error associated with a simple amplifier. Because of the simplicity of this example, a direct solution that – in effect – reproduces the steps in the preceding derivations of Eqs. 4-116 is presented as Solution A. Solution B, on the other hand, is obtained by substitution into Eqs. 4-116 after computation of the partial derivatives. A more complex example, based on Tappert,[32] is given in Example 4-7 in the Appendix to Chapter 4. This example demonstrates the evaluation of the error propagation through a simple computer circuit that consists solely of an electromagnetic resolver and a summing network. The appropriate impedance-matching amplifiers are considered as part of the resolver. Figure 4-25 depicts the circuit under consideration. Figure 4-26 shows the associated error diagram.

Exhaustive experimental and analytical studies have been made on the errors (m_i) of most types of fire-control-system elements. These errors can be categorized as being either dependent or independent, in accordance with whether they are dependent on or independent of the inputs to the element. The errors can be further categorized as being either systematic or random. A systematic error is the deviation of the mean value of an actual variable from the true value of that variable, i.e., the value it should have ideally. A random error is the deviation of the actual value from the mean value, regardless of whether or not a systematic error is present. Because of their erratic nature, random errors are also referred to as uncertainties,, Random errors can be described only in terms of statistical parameters – usually the variance σ^2. In the instance of an electromagnetic resolver, for example, the significant errors are those listed in Table 4-2, which is based on Reference 36.

The paragraphs which follow give the detailed procedures for determining the output errors, ϵ_{y_i}, of a fire control system. For reference purposes, the general overall procedure can be outlined as follows:

1. Separate the systematic and random components of all input errors and system-element errors.
2. Find the dependent and independent components of the individual systematic errors, including both those associated with inputs and those associated with elements.
3. Repeat Step 2 for the random errors.
4. Determine the random error in each system output by combining the individual random errors of the system inputs and the system elements.
5. Repeat Step 4 for the systematic errors.
6. Find the total error in each system output from the results of Steps 4 and 5.

The detailed procedures follow:

1. Separate the systematic components and the random components of all input and element errors. For example, the input error for the n^{th} element can be expressed as

$$\epsilon_{x_n} = \epsilon_{x_n s} + \epsilon_{x_n r} \qquad (4\text{-}117)$$

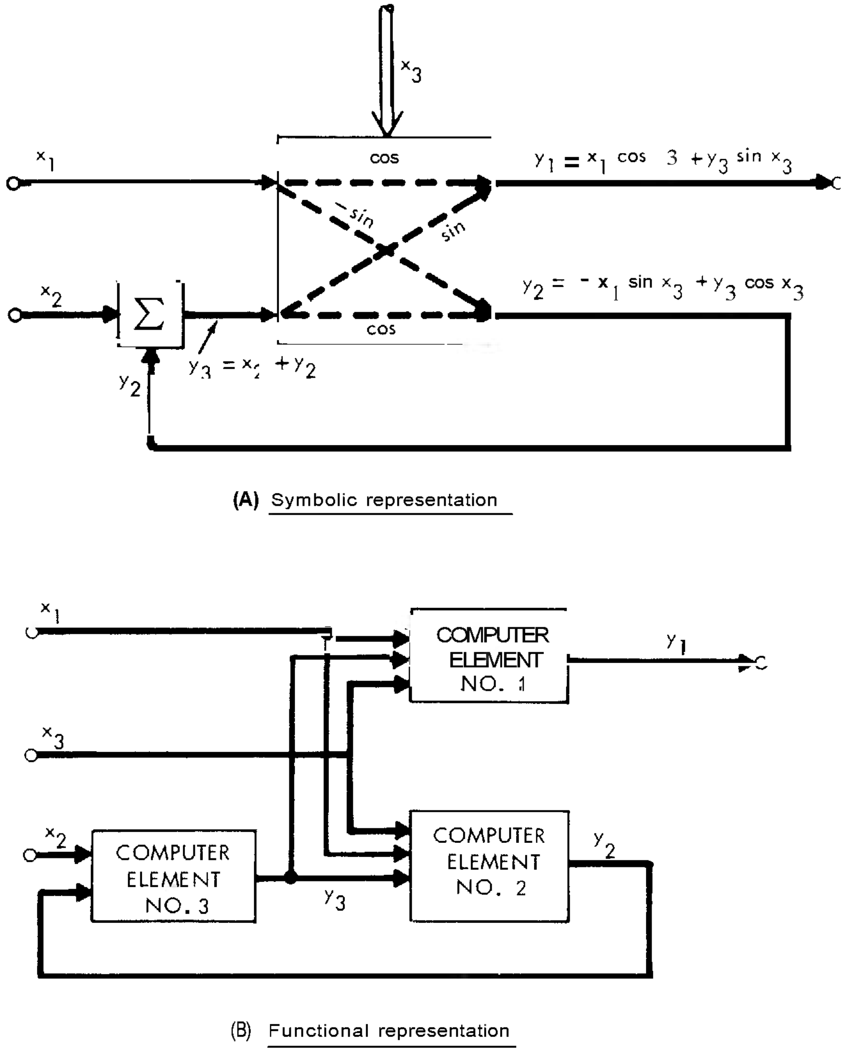


Figure 4-25. Representations of the simple computer circuit considered in Example 4-7.

where

$\epsilon_{x_n s}$ = systematic component of ϵ_{x_n}

$\epsilon_{x_n r}$ = random component of ϵ_{x_n}.

Note that the value of a random component would normally be specified by some statistical parameter such as the variance of the input error $\sigma^2_{\epsilon_{x_n r}}$, for example.

2. Find the dependent and independent components of the individual systematic errors, including both those associated with inputs and those associated with elements. For example, the systematic component of a particular input error, ϵ_{x_n}, is designated $\epsilon_{x_n s}$ (see Eq. 4-117) and might be made up of subcomponents such as are indicated in the following equation:

$$\epsilon_{x_n s} = \epsilon_{x_n so} + \epsilon_{x_n s}(x_1) + \epsilon_{x_n s}(x_2) + \cdots \epsilon_{x_n s}(y_1) + \cdots \qquad (4\text{-}118)$$

where

$\epsilon_{x_n so}$ = independent subcomponent of $\epsilon_{x_n s}$

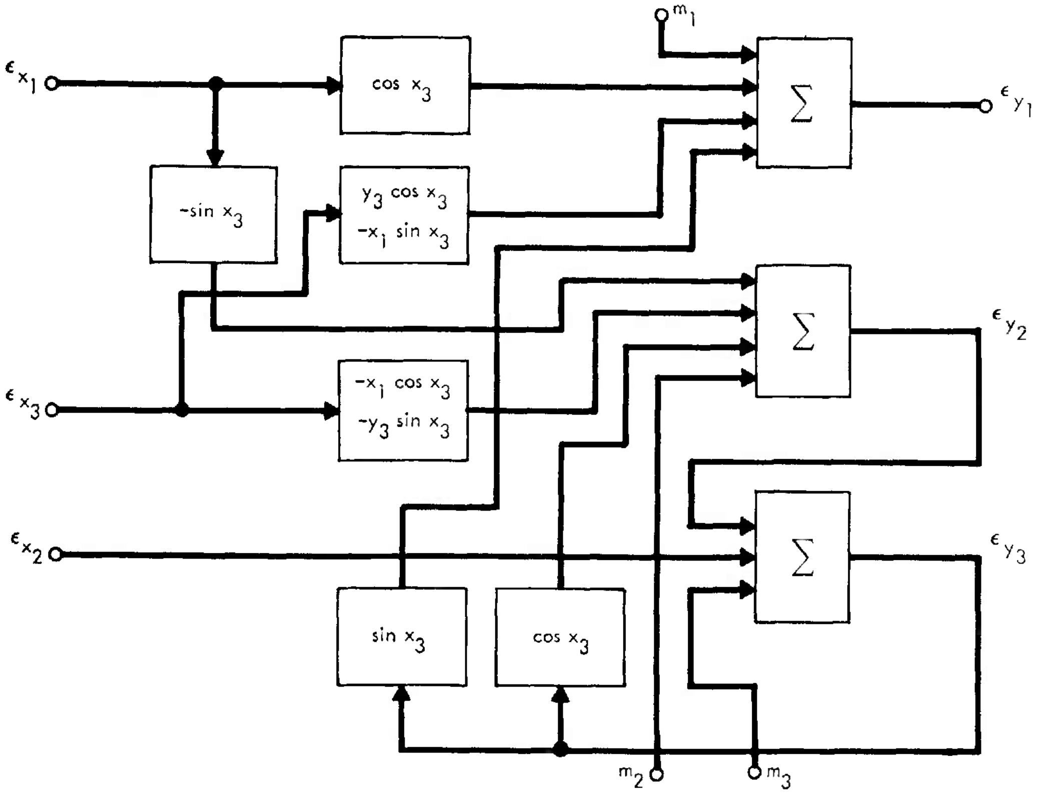


Figure 4-26. Error diagram for the computer circuit represented in Figure 4-25.

TABLE 4-2. SOURCES OF ERROR IN AN ELECTROMAGNETIC RESOLVER.

Independent Errors	
Systematic	**Random**
Transformation ratio inaccuracy	Electrical noise
Phase shift	Bearing play
Axis misalignment	Power-supply variations
Amplifier drift and offset	
Dependent Errors	
Shaft-Angle Dependent	**Input-Voltage Dependent**
Winding inaccuracy	Magnetic nonlinearity
Eccentricity	Amplifier nonlinearity
Phase shift	Null voltage
Null voltage	Phase shift

$\epsilon_{x_{ns}(\ )}$ = subcomponents of $\epsilon_{x_n s}$ that are dependent on the input and output variables, with the specific dependency indicated by the quantity within the parentheses.

Similarly, an error in the i^{th} element m_i with systematic component m_{is} might have subcomponents as in the following equation:

$$m_{is} = m_{iso} + m_{is}(x_1) + m_{is}(x_2) + \cdots + m_{is}(y_1) + \cdots \tag{4-119}$$

where

m_{iso} = independent subcomponent of m_{is}

$m_{is}(\ )$ = subcomponents of m_{is} that are dependent on the input and output variables, with the specific dependency indicated by the quantity within the parentheses.

In practice, whenever a dependent error is small and is not greatly amplified, a fixed maximum value can be employed in place of the complete functional dependence. The maximum error can be obtained by plotting each term in the error Eqs. 4-116 as a family of curves with the input variables as parameters (see Fig. 4-27, for example) and picking off the maximum value.

In many cases, only the maximum possible output error for any combination of the inputs may be desired. This "worst-case" error can be obtained by summing the maximum values obtainedfor each of the terms of the error equations. Usually, however, a statistical description of the output error is the most useful representation. If a statistical description of the input variables is known or can be assumed, the corresponding description – usually the mean and the variance – for the output systematic error can be computed.

3. Find the dependent and independent components of the individual random errors. The procedure here is similar to that used in connection with systematic errors.

4. Determine the random output errors from combinations of the individual random errors. Random errors that are independent of one another can be combined in the error Equations 4-116 through use of the relationship*

$$\sigma_Y^2 = \sum_{i=1}^{n} \sigma_i^2 \tag{4-120}$$

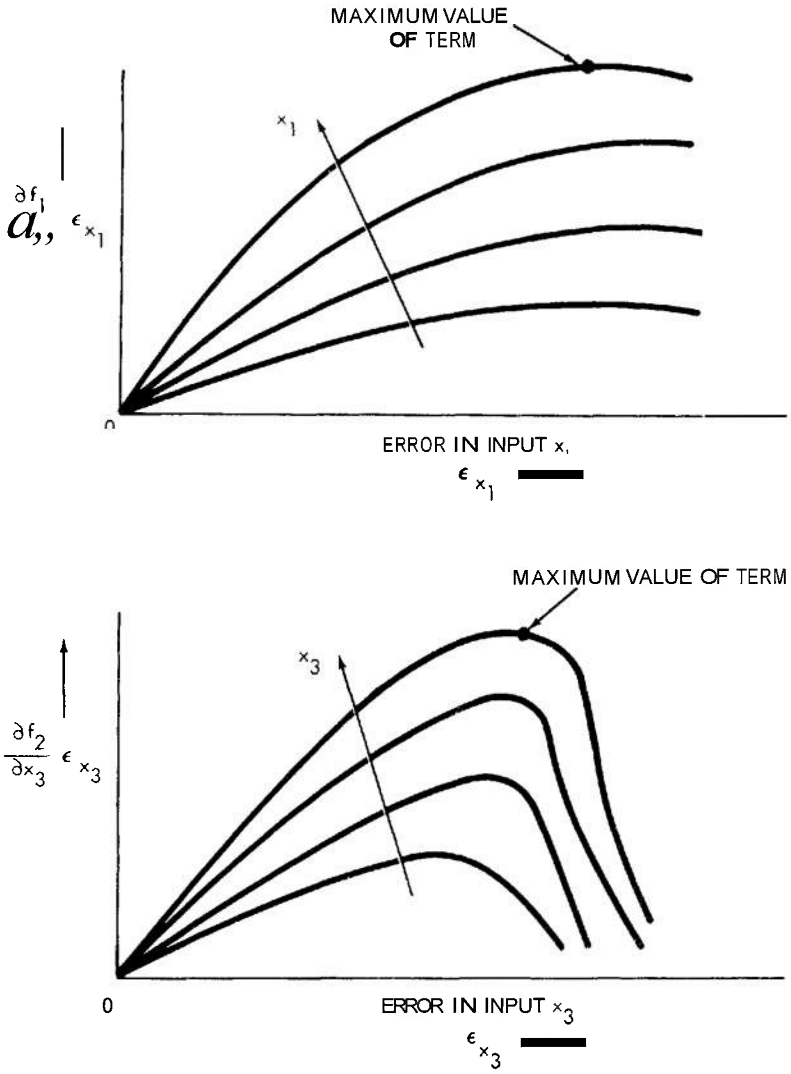
which pertains to a system element whose output y and inputs x_i are independent random variables with variances σ_Y^2 and σ_i^2, respectively, and are related by the general equation

$$y = \sum_{i=1}^{n} g_i(x_i). \tag{4-121}$$

A summation of the type represented by Eq. 4-120 is called a summation of squares. Since the quantities tabulated are often the standard deviations σ, the square root of each side of Eq. 4-120 is often employed. The result is known as a root-square summation.

Equation 4-120 states that the variance of the output of a system element σ_y^2 is equal to the sum of the variances σ_i^2 of the n inputs to that system element. In addition to being used to obtain the variance of the total error in the output of a system element from the variances of the individual random errors of the inputs, Eq. 4-120 can also be used to compute the variance of the output error from the variances of the input and element errors.

* The basic relationship for summing the variances of component errors is derived separately from the main text, in Derivation 4-6 of the Appendix to Chapter 4, in order to avoid interrupting the continuity of the text. As shown by Derivation 4-6, the derivation of Eq. 4-120 must be preceded by the derivation of a relationship for summing the means of the component errors.



NOTE:
THE CURVES ARE FOR FIXED VALUES OF x_1 AND x_3, RESPECTIVELY, THAT INCREASE IN THE DIRECTIONS INDICATED.

Figure 4-27. Typical families of curves for the terms of the error equations.

The random errors that depend on the same input variable x, and are therefore not independent of one another, must also be considered. These dependent errors may stem from errors in an input to an element, or from the nonideal behavior of the element itself. If the variances of a number of random errors are dependent on a single input variable, then the total variance for this group must first be determined. (The symbol used for this total variance is $\sigma_\epsilon^2(x)$.) For such a group, a single curve of the total variance can be plotted against the input variable concerned. (In general, there will be a particular value of the input at which the curve is a maximum.) If the statistical nature of the input variable x in question is known, then the total variance of the error group at any value of x can be weighted by the probability of that value of x, based on a mathematical development similar to that carried out in connection with the discussion of engagement hit probability *(see* par 4-4.3.4). The procedure involved for the case at hand is represented by Fig. 4-28, in which part (C) de-

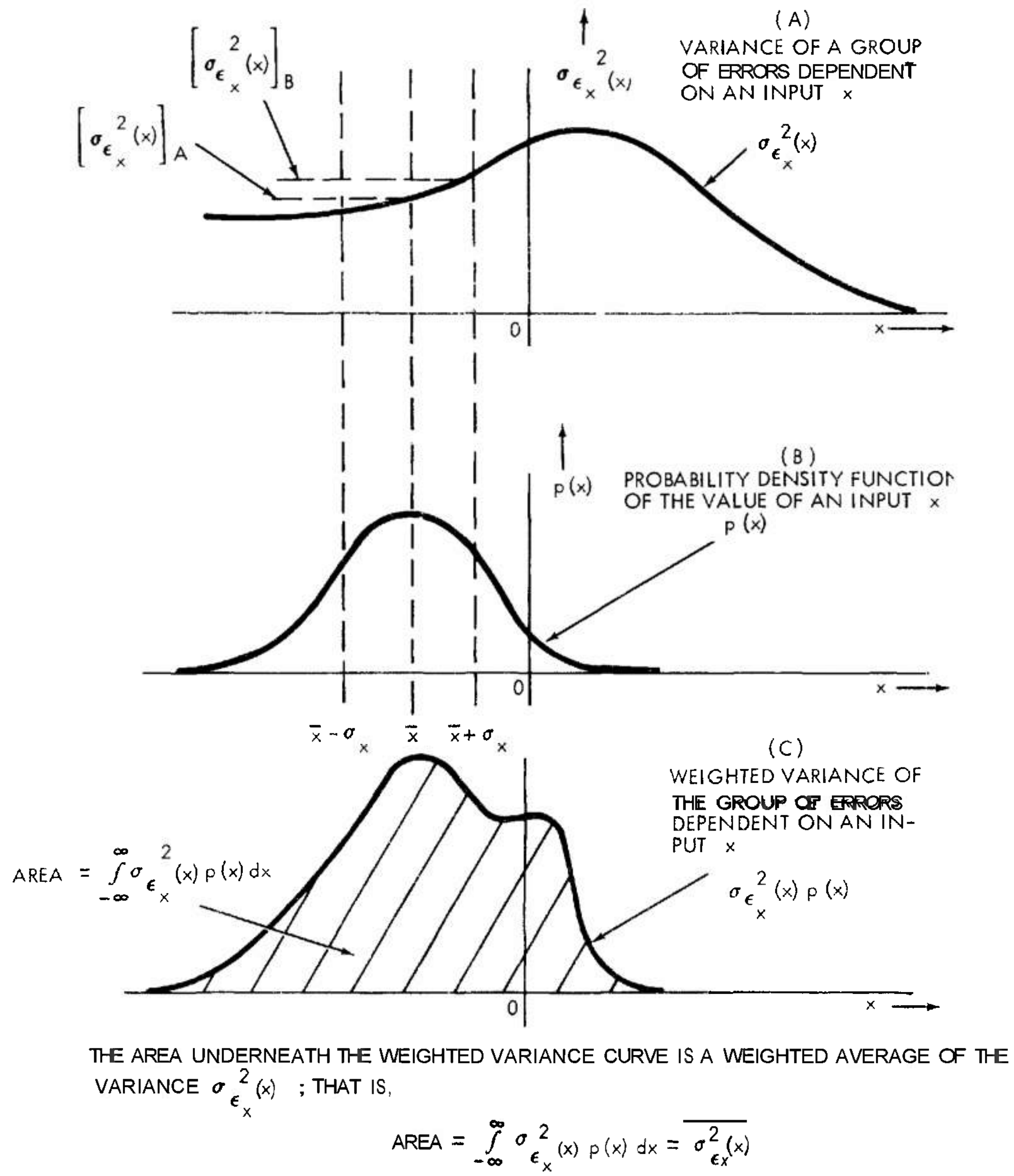


Figure 4-28. Development of the weighted average of a group of dependent random errors.

picts the weighted variance of the group of errors that are dependent on x. The area beneath the weighted curve is a weighted average value of the dependent group of errors.

Rarely, however, is such a detailed development required. Instead either of the two following methods can be employed to determine the most probable total error of a group of random errors that are dependent on the same variable: (1) the total error variance at the mean value of the input $\bar{x}$ or (2) the maximum error variance falling within the band from $\bar{x} - \sigma_x$ to $\bar{x} + \sigma_x$. Each is a convenient approximation of $\sigma_{\epsilon_x}^2(x)$ that will serve satisfactorily. These two approximations are designated respectively as $[\sigma_{\epsilon_x}^2(x)]_A$ and $[\sigma_{\epsilon_x}^2(x)]_B$ in Fig. 4-28(A).

The total variances that are determined for the dependent groups – i.e., $\sigma_\epsilon^2(x)$ values –

are independent of each other and of the remaining random errors. All these errors can, therefore, now be combined in accordance with the root-square summation given by Eq. 4-120 to give a variance σ_r^2 that is a measure of the total random error.

5. Determine the systematic output errors from combinations of the individual systematic errors. Dependent systematic errors are combined in a manner similar to that just employed for dependent random errors. The two approximate methods given for determining the most probable variance of a group of dependent random errors can also be applied to a group of dependent systematic errors that are all a function of the same variable. Thus, an approximate value of the most probable error can be obtained by determining the error corresponding to the mean value $\bar{x}$ of the input variable x. Alternatively, the band $\bar{x} - \sigma_x < x < \bar{x} + \sigma_x$ can be employed.

A third method is also available for dependent systematic errors. First, determine the maximum value of the total error of the group, either analytically or from a plot of total error versus x. (Note that this maximum value is not necessarily the sum of the maxima of the individual errors.) Second, assume that this maximum total dependent error is a fixed value, independent of x. This value of the error can be combined with other independent systematic errors as described in the next paragraph.

If a maximum possible value of the total systematic error were desired, the independent systematic errors could be summed linearly. However, since the probability that all the systematic errors will be simultaneously at a maximum is very small, the maximum error is a poor indication of the average performance of the system. A better indication is given by the most probable error. If the probability of the occurrence of the various independent systematic errors is known, the errors can be weighted in accordance with this probability to determine the most probable error. A conventional approximation to this procedure is to perform a root-square summation, for example,

$$\epsilon_{s(total)} = \left[\sum_{i=1}^{n} \epsilon_{si}^2 \right]^{1/2} . \qquad (4\text{-}122)$$

This approximation, in effect, assigns the same probability to each error if all are equal, but weights the larger errors more heavily if the errors are unequal. Since the maximum systematic errors of the dependent groups are each independent, the maximum total systematic error can be obtained by a root-square summation of these group errors, together with the maximum values of the independent errors.

The most-probable total systematic error can be obtained from the maximum total systematic error if it is assumed that the total systematic error has a Gaussian distribution.*: The maximum value is then equated to the 3σ value of the Gaussian distribution. (As shown by Fig. 4-10, only 0.3 percent of all possible values fall outside the 3σ limit.) The most-probable error is the 1σ value of the assumed Gaussian distribution and is, therefore, one-third of the maximum value.

6. Find the total output error from the combined systematic and random error. While separate values for the systematic and the random errors of an element or of a system are generally the most useful type of presentation, a single measure of the total error may be desired for certain purposes. One such measure is the total maximum error. This error can be computed by multiplying the standard deviation of the total random error by three, in order to obtain a maximum random error, and then obtaining the root-square sum of the maximum random and systematic errors.

Another single measure of the output error is the total most probable error, abbreviated

* This assumption is justified by the central-limit theorem (see par 4-4. 2.4).

as the total MPE. This error can be obtained from a root-square summation of the total random MPE and the total systematic MPE. In a particular system design, either the MPE or the maximum (or peak) error might be employed as a measure of performance.

As an example of the error-summation procedures described, see Example 4-8 in the Appendix to Chapter 4. This example, which is a continuation of the resolver-computer-circuit example of Example 4-7, illustrates the modification of errors that are dependent on the input variables to independent forms that can be combined with the independent errors.

4-4.4.3 Analysis of Error Propagation in Systems Describable by Differential Equations*

While the technique just described may be employed to determine the propagation of errors in certain portions of a fire control system – e.g., the coordinate-transformation and ballistic-correction elements – other parts of the system (such as those employed in the computation of lead angle) perform the solution of differential equations. The error analysis of a system that solves differential equations has major complications. In this case, the system equations include time derivatives of the inputs and outputs as well as the inputs and outputs themselves. This means that the set of q error equations represented by Eq. 4-116 now becomes a set of q differential equations of the form†

$$\sum_{k=1}^{q}\left(\frac{\partial f_1}{\partial y_k}\epsilon_{y_k}+\frac{\partial f_1}{\partial \dot{y}_k}\dot{\epsilon}_{y_k}+\frac{\partial f_1}{\partial \ddot{y}_k}\ddot{\epsilon}_{y_k}+\cdots\right)=-\sum_{n=1}^{r}\left(\frac{\partial f_1}{\partial x_n}\epsilon_{x_n}+\frac{\partial f_1}{\partial \dot{x}_n}\dot{\epsilon}_{n_n}+\frac{\partial f_1}{\partial \ddot{x}_n}\ddot{\epsilon}_{x_n}+\cdots\right)+m_1\frac{\partial f_1}{\partial y_1}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\sum_{k=1}^{q}\left(\frac{\partial f_i}{\partial y_k}\epsilon_{y_k}+\frac{\partial f_i}{\partial \dot{y}_k}\dot{\epsilon}_{y_k}+\frac{\partial f_i}{\partial \ddot{y}_k}\ddot{\epsilon}_{y_k}+\cdots\right)=-\sum_{n=1}^{r}\left(\frac{\partial f_i}{\partial x_n}\epsilon_{x_n}+\frac{\partial f_i}{\partial \dot{x}_n}\dot{\epsilon}_{x_n}+\frac{\partial f_i}{\partial \ddot{x}_n}\ddot{\epsilon}_{x_n}+\cdots\right)+m_i\frac{\partial f_i}{\partial y_i}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\sum_{k=1}^{q}\left(\frac{\partial f_q}{\partial y_k}\epsilon_{y_k}+\frac{\partial f_q}{\partial \dot{y}_k}\dot{\epsilon}_{y_k}+\frac{\partial f_q}{\partial \ddot{y}_k}\ddot{\epsilon}_{y_k}+\cdots\right)=-\sum_{n=1}^{r}\left(\frac{\partial f_q}{\partial x_n}\epsilon_{x_n}+\frac{\partial f_q}{\partial \dot{x}_n}\dot{\epsilon}_{x_n}+\frac{\partial f_q}{\partial \ddot{x}_n}\ddot{\epsilon}_{x_n}+\cdots\right)+m_q\frac{\partial f_q}{\partial y_q}$$

(4-123)

Unfortunately, no general method of obtaining RMS errors (or variances) of the outputs in terms of RMS errors of the inputs and the components is available. In fact, it is obvious that more than RMS values of input and component errors are required since the solution clearly depends on the time variation of these errors – specifically, their autocorrelations – as well as on their RMS values.

A solution to this problem requires the introduction of certain additional fundamental concepts of a random variable. Because of the complexity of these concepts, explanations of them are presented here in a nonrigorous fashion. This presentation is based on the two following approaches to the solution of error Eqs. 4-123:

1. The impulse-response approach in the time domain. (This approach will be described first.)

2. The transfer-function approach in the frequency domain.

Methods of error analysis employing the time domain (impulse-response approach) and methods employing the frequency domain (transfer-function approach) are described in the paragraphs which follow (see par 4-4.4.3.1 and par 4-4.4.3.2, respectively). Since use of the

*The analysis presented in par 4-4.4.3 is largely adapted from that presented by Dr. J. G. Tappert in References 32 and 33.

† See Derivation 4-8 for a simple example of how this set of equations may be employed.

time domain requires convolution integrals, whereas use of the frequency domain requires only simple multiplications, the solution of error-analysis problems in the frequency domain is often advantageous. However, for obtaining an understanding of a problem, the use of the time domain offers advantages. Means of transforming from one domain to the other by means of direct and inverse Fourier transforms are given in par 4-4.4.3.2.

For further details on derivations of these important concepts, the reader is referred to References 1, 16, and 25.

4-4.4.3.1 Use of the Impulse-Response Approach

As already noted in par 4-4.4.2, a fire control system can be considered to be made up of an assemblage of elements, such as the one depicted in Fig. 4-20. As indicated thereon, the output of each such element can be expressed as a function of the inputs by the performance equation

$$y_i = g_i(x_1, \cdots x_r; y_1, \cdots, y_i, \cdots, y_q) \qquad \text{[Eq. 4-95 repeated]}$$

With the impulse-response approach, the first step toward the solution of the error Eqs. 4-123 which are considered to have time-varying inputs, is to derive a particular form of the performance operator g_i that relates an output function of time to an input function of time. This form of the performance operator is known as the weighting function and is the response of the system element to a unit impulse function.

These newly introduced functions will first be defined and derived for a simple system element having one input and one output. This element is represented in Fig. 4-29. A unit impulse function can be defined in terms of a unit pulse function p(t) which is shown in Fig. 4-30(A). The unit pulse function has an amplitude of $1/\Delta t$ between $t = 0$ and $t = \Delta t$, and has zero amplitude everywhere else. As Δt approaches zero, the unit pulse function becomes a unit impulse function u(t) which has infinite amplitude at $t = 0$ and is zero everywhere else (see Fig. 4-30(B)). Note that the area under the unit pulse function is unity, and remains at this same value in the limit as Δt approaches zero. Therefore, the area under the unit impulse function is also unity.

When the unit pulse function is applied to the system element at time $t = 0$, there is a time response v(t) that, typically, might have a form similar to the solid curve shown in Fig. 4-31. This curve is identified by the symbol $v_o(t)$ to indicate that it is the response to a unit pulse function applied at time $t = 0$. If the pulse width is sufficiently narrow,* the pulse response will approach the impulse response. (For reasons discussed subsequently, the impulse response is also called the weighting function r(t).)

The dashed curve shown in Fig. 4-31 is the response to a pulse applied at time $t = t_k$. Accordingly, this response is identified by the symbol $v_k(t)$. For a linear system element† the response $v_o(t)$ and the response $v_k(t)$ have the same shape and are separated by a time interval t_k. In mathematical terms, the constancy of shape of the responses is expressed by the equation

$$v_k(t_1) = v_o(t_1 - t_k) \tag{4-124}$$

where

v_k = response to a unit pulse occurring at some arbitrary time $t = t_k$

* The pulse width may be considered to be sufficiently narrow if it is smaller than the smallest time constant of the system element concerned.

† A linear element is one that obeys the principle of superposition. This principle states that the sum of the responses to each of a set of inputs is the same as the response to the sum of this set of inputs. An equivalent definition is that a linear element is one that is governed by a linear differential equation. For nonlinear elements, special techniques of linearization must be employed; for example, see par 3.1 in Chapter 2 of Reference 1.

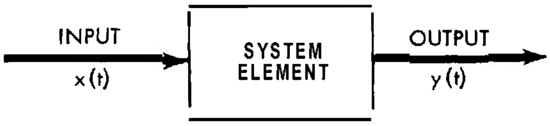


Figure 4-29. Functional representation of a system element with a single time-varying input and a single time-varying output.

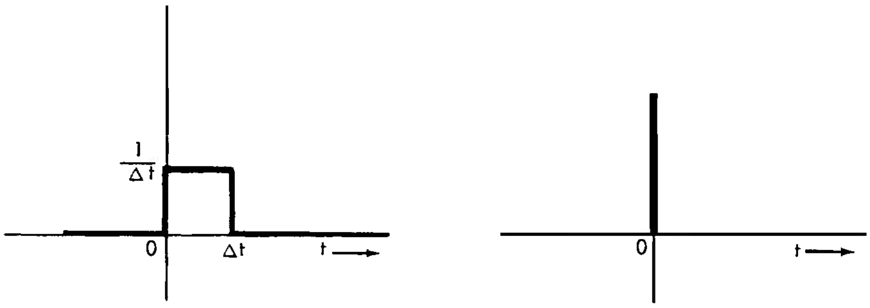


Figure 4-30. Graphical representation of the unit pulse function and the unit impulse function.

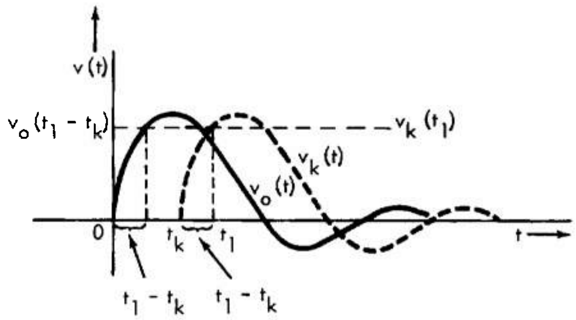


Figure 4-31. Typical pulse responses v(t).

v_o = response to a unit pulse occurring at time t = 0

t_1 = arbitrary point in time.

As shown in Fig. 4-32, a random input function x(t) can be approximated by a sum of pulse functions. A unit pulse function occurring at time $t = t_k$ is denoted $p(t_k)$ and is illustrated in Fig. 4-32(A). When the unit pulse function $p(t_k)$ is multiplied by Δt, a pulse of unit amplitude at time t_k is obtained, as shown in Fig. 4-32(B). The amplitude of x(t) is $x(t_k)$ at $t = t_k$; accordingly, a pulse function of width Δt and amplitude $x(t_k)$ will approximate the plot for x(t) at that point. This pulse function is obtained by multiplying the unit-amplitude pulse $p(t_k)\,\Delta t$ by the amplitude $x(t_k)$ to give $x(t_k)\,p(t_k)\,\Delta t$, as shown in Fig. 4-32(C). The function x(t) can thus be approximated by a "staircase" of such pulse functions, as shown in Fig. 4-32(D). The approximation is given in mathematical terms by the equation

$$x(t) \cong \sum_{k=-\infty}^{\infty} x(t_k)\, p(t_k)\, \Delta t \tag{4-125}$$

where k takes on the discrete values $n\Delta t$, with n representing all integral values from $-\infty$ to ∞.

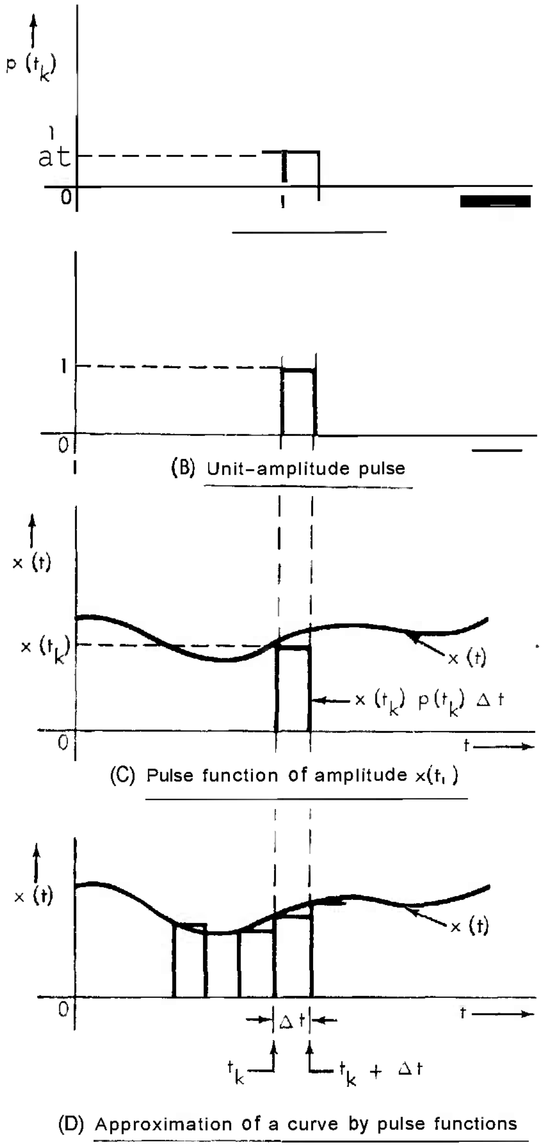


Figure 4-32. Approximation of a random input function by a series of pulse functions.

The response of a system element to a unit pulse function – i. e., a unit-area pulse – at time $t = t_k$ has been defined as $v_k(t)$ (see Fig. 4-31). Therefore, for a linear system element (see preceding footnote, the response to a unit-amplitude pulse is Δt times as great or $v_k(t)$ Δt, and the response to a pulse of amplitude $x(t_k)$ is $v_k(t)\, x(t_k)\, \Delta t$. If the output of the system element is denoted $y(t)$, then at time t_1 the output resulting from a single pulse of amplitude $x(t_k)$ at time t_k is

$$y(t_1) = v_k(t_1)\, x(t_k)\, \Delta t. \tag{4-126}$$

The substitution from Eq. 4-124 into Eq. 4-126 shows that

$$y(t_1) = v_o(t_1 - t_k)\, x(t_k)\, \Delta t. \tag{4-127}$$

Therefore, the response of the systemelement at time t_1 to the summation of pulses represented by Eq. 4-125 is given by the relationship

$$y(t_1) = \sum_{k=-\infty}^{\infty} v_o(t_1 - t_k)\, x(t_k)\, \Delta t. \tag{4-128}$$

As Δt approaches zero, the resultingfunction becomes continuous and is given mathematically by the equation

$$y(t_1) = \int_{-\infty}^{\infty} r_o(t_1 - t_k)\, x(t_k)\, dt_k \tag{4-129}$$

where

$r_o(t_1 - t_k)$ = weighting function, or response at time $t_1 - t_k$ of the system element to a unit impulse function applied at time $t = 0$.

The substitution of a new time variable τ for $t_1 = t_k$ in Eq. 4-129 yields the relationship

$$y(t_1) = -\int_{\infty}^{-\infty} r_o(\tau)\, x(t_1 - \tau)\, d\tau = \int_{-\infty}^{\infty} r_o(\tau)\, x(t_1 - \tau)\, d\tau \tag{4-130}$$

since $t_k = t_1 - \tau$, $dt_k = -d\tau$ for a constant value of t_1, and $\tau = -\infty$ when $t_k = \infty$. The integral on the right-hand side of this equation is a standard form that is known as the convolution integral or superposition integral.* Inasmuch as t_1 is any arbitrary point in time, it can be replaced by the general symbol for time t. In addition, for simplification, the subscript o will be dropped from $r_o(\tau)$ since the impulse function is conventionally applied at time $t = 0$ and this fact need not, therefore, be specifically indicated. These steps, when applied to Eq. 4-130, result in the most general form for the convolution integral, i. e.,

$$y(t) = \int_{-\infty}^{\infty} r(\tau)\, x(t - \tau)\, d\tau. \tag{4-131}$$

Inasmuch as t_k is also an arbitrary point in time, Eq. 4-129 can be generalized by replacing t_1 with t and t_k with τ' to give an alternative form of the convolution integral, i. e.,

$$y(t) = \int_{-\infty}^{\infty} r(t - \tau')\, x(\tau')\, d\tau'. \tag{4-132}$$

The convolution process represented by the right-hand side of Eq. 4-131 can be visualized with the aid of Fig. 4-33. First, the concept of $x(t - \tau)$ is shown. In Fig. 4-33, the upper curve represents $x(\tau)$, i. *e.*, $x(t - t_k)$ plotted against τ with an arbitrary time t indicated on the τ axis. If τ is zero, $x(t - \tau)$ is, of course, identical with $x(t)$. As τ increases from zero, the function $x(t - \tau)$ can be found from Fig. 4-33 by sweeping to the left an amount τ from the initial value t. In the lower curve of Fig. 4-33, as τ increases from zero, $r(\tau)$ will be swept out by a line moving to the right. Thus, the convolution process involves the integration of two time functions, one of which, $r(\tau)$, is being swept out in the direction of increasing τ, and the other, $x(t - \tau)$, is being swept out in the direction of decreasing τ.

* *See* pages 120-123 of Reference 1.

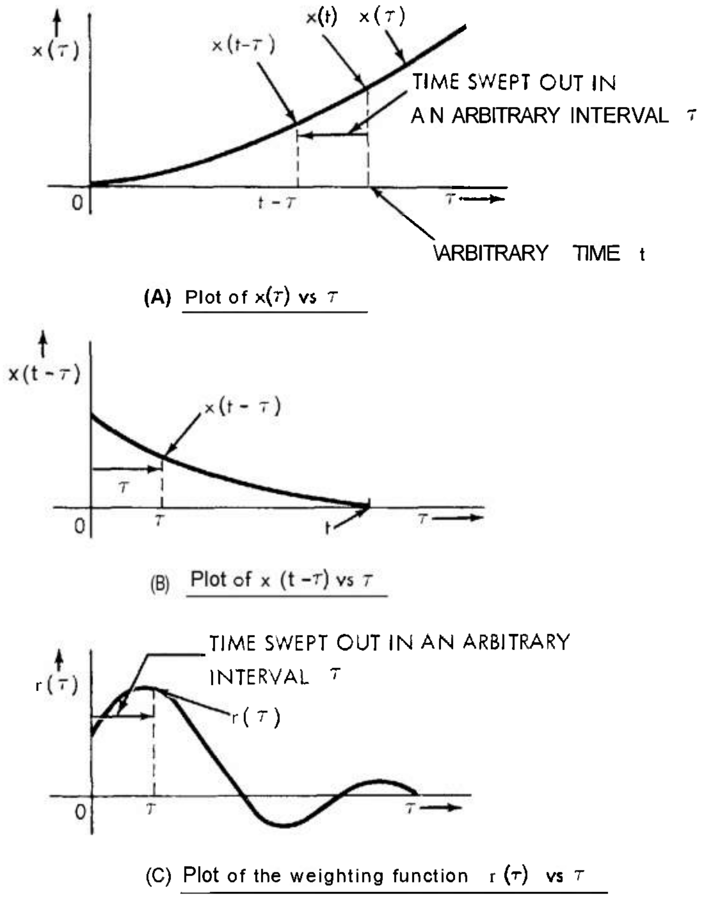


Figure 4-33. Pictorial representation of the convolution process.

The convolution integral can be conceived as the weighted sum of a large number of values of the input variable. In the second form of the convolution integral, which is given by Eq. 4-132, each value of the input variable $x(\tau')$ is weighted by the impulse response $r(t - \tau')$ at time $t - \tau'$, and the output $y(t)$ is then obtained as the infinite sum of these weighted values of the inputs. This is the origin of the alternate term, weighting function, for the impulse response.

By convention, in a real system, the input to a system element $x(\tau)$ and the impulse response of that system element $r(\tau)$ do not exist for negative time, i. e., for $\tau < 0$. In this case, Eq. 4-131 takes the form

$$y(t) = \int_0^t r(\tau)\, x(t - \tau)\, d\tau \tag{4-133}$$

where

$y(t)$ = output of the system element at time t.

The justification for Eq. 4-133 can be found by inspection of Fig. 4-33. If $r(\tau)$ is zero for $\tau < 0$, the integral on the right-hand side of Eq. 4-131 is also zero for $\tau < 0$. In addition, since $x(\tau)$ is zero for $\tau < 0$, $x(t - \tau)$ must be zero for $\tau > t$ and the integral must also be zero for $\tau > t$.

By means of the convolution integral on the right-hand side of Eq. 4-133, the output y(t) of a system element has thus been obtained by convolving the impulse response of the element $r(\tau)$ with the input $x(t - \tau)$. Since the impulse response to a linear element is a fixed property of that element, Eq. 4-133 can be considered to be a particular form of the performance Equation 4-95 for the case of a linear system element with a time-varying input.

Based on Eq. 4-132, y(t) can also be expressed in the alternative form

$$y(t) = \int_0^t r(t - \tau)\, x(\tau)\, d\tau. \qquad (4\text{-}134)$$

The corresponding error equation that relates an error in the input $\epsilon_x(\tau)$ to the resulting error in the output $\epsilon_y(t)$ can be written from Eq. 4-134 by inspection, utilizing the fact that, for a linear system element, an input error will be acted upon in the same way as an input. The result is

$$\epsilon_y(t) = \int_0^t r(t - \tau)\, \epsilon_x(\tau)\, d\tau \qquad (4\text{-}135)$$

where

$\epsilon_x(\tau)$ = error in $x(\tau)$, the single input to the system element under consideration

$\epsilon_y(t)$ = corresponding error in the output y(t).

As indicated by Eq. 4-135, the input error at time τ, weighted by the impulse response at time $t-\tau$, gives an incremental contribution to the output error. The total error $\epsilon_y(t)$ is then obtained as the summation of these individual incremental contributions.

For those situations in which the functions in Eq. 4-135 are not readily integrable, an approximate method can be employed. In this circumstance, the error in the input to the system element can be approximated by a succession of discrete impulse functions; i.e.,

$$\epsilon_x(\tau) = \epsilon_{x_1} + \epsilon_{x_2} + \cdots \epsilon_{x_n} \qquad (4\text{-}136)$$

where

$\epsilon_{x_1}, \epsilon_{x_2}, \ldots, \epsilon_{x_n}$ are the magnitudes of the discrete impulses at times $\tau_1, \tau_2, \ldots, \tau_n$, and are equal to the error in the input to the system element at the respective times, i.e.,

$$\epsilon_{x_1} = \epsilon_x(\tau_1);\ \epsilon_{x_2} = \epsilon_x(\tau_2);\ \epsilon_{x_3} = \epsilon_x(\tau_3);\ \text{etc.} \qquad (4\text{-}137)$$

Equation 4-127 shows that after a time interval t, the output error contribution from an input pulse ϵ_{x_1}, applied at time τ_1, is

$$\epsilon_{y_1}(t) = v_o(t - \tau_1)\, \epsilon_{x_1} \Delta t. \qquad (4\text{-}138)$$

Therefore, the total output error $\epsilon_y(t)$ is the sum of the contributions from the total of n separate impulses employed, i.e.,

$$\epsilon_y(t) = \epsilon_{y_1}(t) + \epsilon_{y_2}(t) + \cdots + \epsilon_{y_n}(t). \qquad (4\text{-}139)$$

Equation 4-135 can be generalized to apply to a multi-element system by means of the following procedure:

1. In Eq. 4-135, an expression for a single system element is available that relates a single time-varying input error to a single time-varying output error. For a system element that has multiple inputs, the output error has the form of a summation of expressions like that of Eq. 4-135, i.e., for p inputs,

$$\epsilon_y(t) = \sum_{n=1}^{p} \int_{o}^{t} r_n(t-\tau)\,\epsilon_{x_n}(\tau)\,d\tau \tag{4-140}$$

where

$\epsilon_{x_n}(\tau)$ = error in $x_n(\tau)$, the nth input of the system element

$r_n(t-\tau)$ = response of the element to an impulse function applied in place of an input x_n

$\epsilon_y(t)$ = error in the output of the system element and n takes on all integral values from 1 to p.

2. In a multi-element element system each element has as inputs some of the outputs of other system elements, as already discussed earlier in connection with Fig. 4-20. Adding terms of these outputs to Eq. 4-140 yields, for the error in the output of the ith of q system elements

$$\epsilon_{y_i}(t) = \sum_{n=1}^{p} \int_{o}^{t} r_{i_n}(t-\tau)\,\epsilon_{x_n}(\tau)\,d\tau + \sum_{k=1}^{q} \int_{o}^{t} r_{i_k}(t-\tau)\,e_{y_k}(\tau)\,d\tau \tag{4-141}$$

where

$r_{i_n}(t-\tau)$ = response of the ith element to an impulse applied in place of an input x_n

$r_{i_k}(t-\tau)$ = response of the ith element to an impulse applied in place of an input y_k

ϵ_{y_k} = error in an input y_k

and k takes on all integral values from 1 to q. The remaining quantities are as defined previously. Equation 4-141 represents the ith equation of a set of q equations that define the output errors of a system containing q elements. Equation 4-141 is a particular solution of Eqs. 4-123 that applies to a system that has only linear elements with time-varying inputs and no element errors m_i.

3. If the system under consideration has nonzero element errors, the final set of q error equations can be represented by the equation for the ith element, which is

$$\sum_{n=1} \int_{o} r_{i_n}(t-\tau)\,\epsilon_{x_n}(\tau)\,d\tau + \sum_{k=1} \int r_{i_k}(t-\tau)\,\epsilon_{y_k}(\tau)\,d\tau + m_i(t) \tag{4-142}$$

where

$m_i(t)$ = error in the ith element.

For further development of expressions in the form of Eq. 4-142 and examples of how they can be employed in error analyses of systems having time-varying inputs, see pages 52-62 of Reference 37, pages 136-163 of Reference 38, and Reference 39.

4-4.4.3.2 Use of the Transfer-Function Approach

As noted in par 4-4.4.3, an alternative approach to carrying out an error analysis for systems whose performance is represented by differential equations is the use of functions of frequency in place of functions of time. As is shown in the paragraphs which follow, an error analysis in the frequency domain has the advantage of requiring only the multiplication of functions of frequency, in contrast with the complex operation of convolution required for time functions. Also, the frequency response of a system element — i.e., the amplitude and phase angle of the output as compared, respectively, with the amplitude and phase angle of a sinusoidal input function — is quite commonly available for the usual computer elements, servo elements, etc. of fire control systems. This frequency-domain technique of error analysis has been extensively applied at Frankford Arsenal (see, for example, Reference 33) and will be described herein in terms that are most useful for fire-control calculations. (The remaining material of par 4-4.4.4 is based directly on the analyses developed by Dr. J. G.

Tappert and presented in References 32 and 33.)

With reference to the simple single-input, single-output system element depicted in Fig. 4-29, consider the problem of determining a statistical measure of the output error ϵ_y in terms of the known measure of the independent input errors ϵ_x. First, define the transfer function:*, $R(j\omega)$, as the complex ratio, at a particular angular frequency, ω, of the Fourier transform of the output to the Fourier transform of the input; i.e.,

$$R(j\omega) = \frac{Y(j\omega)}{X(j\omega)} \tag{4-143}$$

where

$\omega = 2\pi f$

f = frequency of the sinusoidal input function

and $X(j\omega)$ and $Y(j\omega)$ are, respectively, the Fourier transforms of the input x (t) and the output y (t). The Fourier transform of the input x(t) is defined by the relationship†

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \tag{4-144}$$

Similarly, the Fourier transform of the output, y(t), is defined by the relationship

$$Y(j\omega) = \int_{-\infty}^{\infty} y(t)\, e^{-j\omega t}\, dt \tag{4-145}$$

In addition, postulate a linear element, which means that $R(j\omega)$ is not a function of the input. For the frequency-domain approach, the functional representation of a system element that is given in Fig. 4-29 becomes modified to the form shown in Fig. 4-34.

Since Eq. 4-163 in the frequency domain is analogous to Eq. 4-131 in the time domain it is desirable to develop a relationship between the transfer function $R(j\omega)$ and the impulse response $r(\tau)$. In order to derive this relationship, the time-domain input-output relationship given by Eq. 4-131 will be substituted into the definition of $Y(j\omega)$ given by Eq. 4-145. This substitution yields

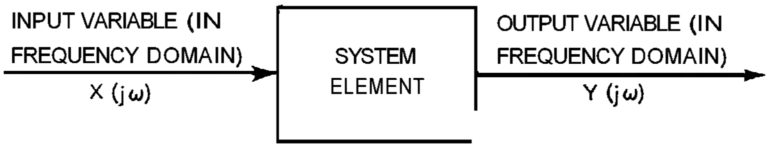
$$Y(j\omega) = \int \qquad \int^{\infty} r(\tau)\; x(t-\tau)\, d\tau \Big\} dt \tag{4-146}$$

Next, a change of the variable of integration will be made. Let $\nu = t-\tau$; then

$$Y(j\omega) = \int_{-\infty}^{\infty} e^{-j\omega(\tau+\nu)}\, d\nu \int_{-\infty}^{\infty} r(\tau)\, x(\nu)\, d\tau \tag{4-147}$$

* This transfer function is identical with the transfer function $G(j\omega)$ that is used in conventional servo theory; see Reference 40.

† The Fourier transform is a conventional mathematical tool that is covered thoroughly in such basic mathematical texts as Reference 31 and hence will be employed here without further background discussion. The notation $X(j\omega)$ will be employed throughout this chapter for the purpose of indicating the relationship between the Fourier and bilateral Laplace transforms. (Substitution of the Laplace operator, $s = \sigma + j\omega$, in place of $j\omega$ in the Fourier transform yields the corresponding bilateral (Laplace transform.) It should be noted, however, that the Fourier transform is written as $X(\omega)$ in many standard texts. For a more complete discussion of the Fourier and Laplace transforms and their applications, see par 2-2.3 through par 2-2.6 of Section 3 of the Fire Control Series.



$$Y(j\omega) = R(j\omega)\ X(j\omega)$$

WHERE

$X(j\omega)$ = FOURIER TRANSFORM OF $x(t)$

$Y(j\omega)$ = FOURIER TRANSFORM OF $y(t)$

$R(j\omega)$ = TRANSFER FUNCTION OF THE SYSTEM ELEMENT

THE TRANSFORMATION EQUATIONS ARE AS FOLLOWS:

FOURIER TRANSFORM	INVERSE FOURIER TRANSFORM
$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$	$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$
$Y(j\omega) = \int_{-\infty}^{\infty} y(t)\, e^{-j\omega t}\, dt$	$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(j\omega)\, e^{j\omega t}\, d\omega$
$R(j\omega) = \int_{-\infty}^{\infty} r(t)\, e^{-j\omega t}\, dt$	$r(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R(j\omega)\, e^{j\omega t}\, d\omega$

Figure **4-34.** Functional representation of a system element in the frequency domain.

The variables τ and ν may now be separated, yielding

$$Y(j\omega) = \int_{-\infty}^{\infty} x(\nu)\, e^{-j\omega\nu}\, d\nu \int_{-\infty}^{\infty} r(\tau)\, e^{-j\omega\tau}\, d\tau \qquad \textbf{(4-148)}$$

With the variables separated, the double integral may be integrated as the product of two single integrals. From Eq. **4-144,** the first integral is the Fourier transform of $x(\nu)$, that is, $X(j\omega)$. Thus,

$$Y(j\omega) = X(j\omega) \int_{-\infty}^{\infty} r(\tau)\, e^{-j\omega\tau}\, d\tau \qquad \textbf{(4-149)}$$

Application of **Eq. 4-143** shows that

$$R(j\omega) = \int_{-\infty}^{\infty} r(\tau)\, e^{-j\omega\tau}\, d\tau \qquad \textbf{(4-150)}$$

Thus, the transfer function $R(j\omega)$ is the Fourier transform of $r(\tau)$, the impulse response of the system element.

In order to utilize Eq. 4-150 in connection with random error variables, it will be necessary to describe the input and output errors by means of two new statistical parameters: the autocorrelation function and the power spectral density function*. It is necessary to introduce these parameters because a random variable is not itself Fourier transformable.

The autocorrelation function is defined as the time average of the product of two values of the random variable that are measured at times separated by arbitrary time interval τ. The autocorrelation function is thus a measure of the randomness of the variable. It varies from zero, for a completely random function, to unity, for a function that has no random component, such as a pure sine wave. The defining equation for the autocorrelation function of the input variable x(t) is

$$\phi_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)\, x(t+\tau)\, dt \tag{4-151}$$

where

$\phi_{xx}(\tau)$ = autocorrelation function of the random variable x(t) (The double-x subscript represents the fact that the autocorrelation function involves the product of two values of the same random variable x(t).)

The power spectral density function is defined as the Fourier transform of the autocorrelation function. This particular Fourier transform is defined by the equation

$$\Phi_{xx}(j\omega) = \int_{-\infty}^{\infty} \phi_{xx}(\tau)\, e^{-j\omega\tau}\, d\tau \tag{4-152}$$

where

$\Phi_{xx}(j\omega)$ = power spectral density function of the random variable x(t). The power spectral density function is thus a measure of the randomness of the variable x in the frequency domain. It is instructive to note that the power spectral density function at any angular frequency ω is the contribution to the total average power that is made by a frequency element $d\omega$ from the complete frequency spectrum. The defining integral extends over both positive and negative frequencies. Negative frequencies, although they have no physical existence, are employed here since they permit the simple, compact exponential form of the Fourier integral. It is, of course, essential that negative frequencies be correctly interpreted in any physical problem. (See, for example, the relationships given for the mean square noise voltage at the end of the illustrative example presented in par 4-4.4.4.)

The power spectral density function has considerable usefulness in the design of fire control systems. In this connection, it is desirable to determine its relationship to the statistical moments $m^{(1)}$ and $m^{(2)}$, which are defined in Eqs. 4-29 and 4-30. With discussion restricted to stationary random processes (which are of primary concern in fire control systems), it is permissible to take time averages that are equivalent to the ensemble averages given by Eqs. 4-29 and 4-30. The first moment is then the time average of the random variable which is designated by the symbol $\widetilde{x(t)}$. The second moment is the time average of the variable squared, which is designated by the symbol $\widetilde{x^2(t)}$. Therefore, in accordance with Eqs. 4-26 and 4-27, respectively, and with t = -T and Δt = 2T,

$$m^{(1)} = m = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)\, dt = \widetilde{x(t)} \tag{4-153}$$

*These two parameters are frequently referred to simply as the autocorrelation and the power spectral density, respectively.

and

$$m^{(2)} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\,dt = \widetilde{x^2(t)} \tag{4-154}$$

where $\pm T$ are the arbitrary limits of the time-interval. Equations 4-153 and 4-154 are thus alternative expressions for the time average and mean-square time average (Ref. Eqs. 4-26 and 4-27). It should be noted that both $\widetilde{x(t)}$ and $x^2(t)$ are numbers and not time functions.

In order to obtain an expression for the first moment in the frequency domain, the Fourier transform of $x(t)$ is first expressed in the form

$$X(j\omega) = \lim_{T\to\infty} \int_{-T}^{T} x(t)\, e^{-j\omega t}\, dt \tag{4-155}$$

where $X(j\omega)$ is the Fourier transform of $x(t)$. The value of $X(j\omega)$ at zero frequency is evidently

$$X(j0) = \lim_{T\to\infty} \int_{-T}^{T} x(t)\, dt \tag{4-156}$$

Where $X(j0)$ is the value of $X(j\omega)$ at $\omega = 0$. A comparison of Eqs. 4-156 and 4-153 shows that

$$\widetilde{x(t)} = \lim_{T\to\infty} \left\{ \frac{X(j0)}{2T} \right\}. \tag{4-157}$$

The process used in arriving at Eq. 4-157 can be summarized as follows: First, the Fourier transform of a stationary random variable $x(t)$ is integrated over an interval, $2T$, that is of sufficient duration that an increase in this interval does not change the average and next, the result is normalized by this same interval. The zero-frequency component of this normalized Fourier transform is then the mean value of the variable. The symbol $\overline{X(j0)}$ will be employed to denote this mean value.

In order to determine an expression for the second moment in the frequency domain, set $\tau = 0$ in Eq. 4-151. This procedure shows that

$$\phi_{xx}(0) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\, dt. \tag{4-158}$$

Since the right-hand side of Eq. 4-158 is identical with the right-hand side of Eq. 4-154, it is evident that

$$\widetilde{x^2(t)} = \phi_{xx}(0). \tag{4-159}$$

The right-hand side of this equation can be evaluated by taking the inverse to the transformation of Eq. 4-152. This procedure gives

$$\phi_{xx}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_{xx}(j\omega)\, e^{j\omega\tau}\, d\omega \tag{4-160}$$

Setting $\tau = 0$ in order to obtain $\phi_{xx}(0)$ show that

$$\widetilde{x^2}(t) = \phi_{xx}(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_{xx}(j\omega)\, d\omega \tag{4-161}$$

which is the desired relationship between the power spectral density $\Phi_{xx}(j\omega)$ and the second moment $\widetilde{x^2}(t)$. Inasmuch as the power spectra? density at any angular frequency, ω, is the contribution to the total average power made by a frequency element $d\omega$ Eq. 4-161 shows that the second moment is proportional to the total average power in the random variable.

In order to determine the power in a frequency increment dw,* consider first the power in the frequency interval $-\infty < jw < j\omega$ as shown in Fig. 4-35. The average power in this interval can be defined by an expression analogous to that of Eq. 4-141; i.e.,

$$\widetilde{x^2}(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{\omega} \Phi_{xx}(jw)\, dw \qquad -\infty < jw < j\omega. \tag{4-162}$$

This average power is a function of ω_a, in contrast to the total average power (see Eq. 4-161), which is a constant. It may, therefore, be differentiated with respect to ω, giving

$$\frac{d\widetilde{x^2}(j\omega)}{d\omega} = \frac{1}{2\pi}\Phi_{xx}(j\omega) \tag{4-163}$$

'The combination of Eqs. 4-37, 4-153, and 4-154 yields the relationship

$$\widetilde{x^2} = \sigma^2 + \tilde{x}^2 \tag{4-164}$$

where σ^2 is the variance. If $\tilde{x}$, and therefore $\tilde{x}^2$, are invariant with frequency — which, for example, is the case for a sinusoid† — it is evident that

$$\widetilde{x^2}(j\omega) = \sigma^2(j\omega) + \text{constant}. \tag{4-165}$$

Differentiation of this relationship and substitution from Eq. 4-163 shows that

$$\frac{d\sigma^2(j\omega)}{d\omega} = \frac{1}{2\pi}\Phi_{xx}(j\omega). \tag{4-166}$$

* Here, w is an arbitrary (dummy) frequency variable of integration, introduced to avoid confusion with ω in the ensuing development.

† The errors in the Vigilante Weapon System Example given in par 4-6 are assumed to be sinusoid.

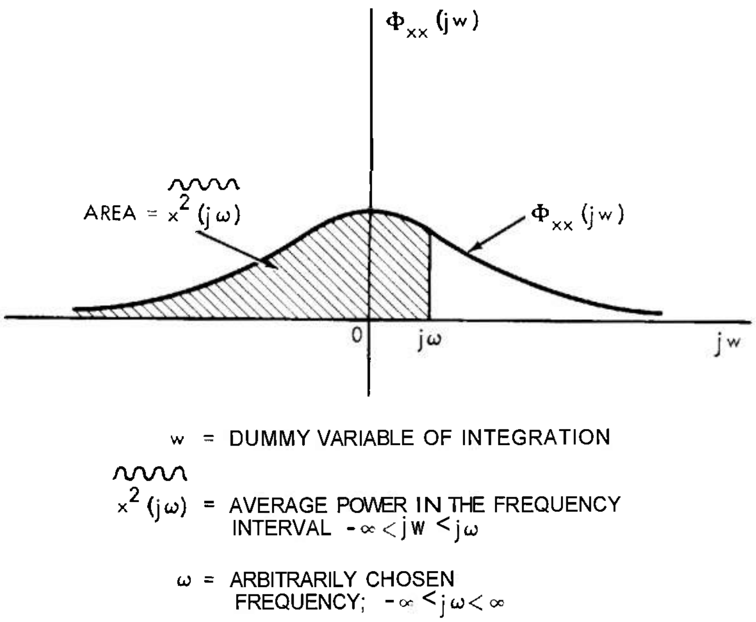


Figure 4-35. The power in a frequency interval.

Equation 4-166 demonstrates that the power spectral density at any frequency ω is proportional to the contribution to the total variance made by an incremental frequency do.

Important relationships between the autocorrelation functions and the power spectral densities of the input and output to the system element of Fig. 4-34 can now be derived. Equation 4-151 gives the autocorrelation function of the input variable x(t). The autocorrelation of the output variable y(t) is similar, i.e.,

$$\phi_{yy}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} y(t)y(t+\tau)dt. \tag{4-167}$$

The response relationship given by Eq. 4-131 can be expressed in the alternative form

$$y(t) = \int_{-\infty}^{\infty} r(\mu)x(t-\mu)\,d\mu \tag{4-168}$$

where μ is an arbitrary time variable of integration that is used instead of τ in order to avoid confusion with Eq. 4-167, in which τ represents an arbitrary time interval between two values of the random variable. Similarly, substituting ν for τ and $(t+\tau)$ for t_1 in Eq. 4-131 gives a second alternative form of the convolution of the impulse response with the

input response, i.e.,

$$y(t+\tau) = \int_{-\infty}^{\infty} r(\nu)x(t+\tau-\nu)d\nu \tag{4-169}$$

where ν is yet another time variable of integration. A substitution from Eqs. 4-168 and 4-169 into Eq. 4-167 yields the relationship

$$\phi_{yy}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} dt \left\{ \int_{-\infty}^{\infty} r(\mu)x(t-\mu)\,d\mu \right\} \left\{ \int_{-\infty}^{\infty} r(\nu)x(t+\tau-\nu)\,d\nu \right\}. \tag{4-170}$$

The familiar technique of reversing the order of integration yields

$$\phi_{yy}(\tau) = \int_{-\infty}^{\infty} r(\mu)d\mu \int_{-\infty}^{\infty} r(\nu)d\nu \left\{ \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t-\mu)\,x(t+\tau-\nu)dt \right\} \tag{4-171}$$

The time-average integral may be evaluated by changing the variables. Let $t_1 = t - \mu$, and $\rho = \tau + \mu - \nu$. Then

$$\begin{aligned} \lim_{T\to\infty} \frac{1}{2T} \int_{-T} x(t-\mu)\,x(t+\tau-\nu)\,dt &= \lim_{T\to\infty} \int_{-T-\mu}^{T-\mu} x(t_1)\,x(t_1+\tau+\mu-\nu)dt_1 \\ &\quad \lim_{T\to\infty} \frac{1}{2T} \int_{-T-\mu} x(t_1)\,x(t_1+\rho)\,dt_1 \\ &= \phi_{xx}(\rho) \end{aligned} \tag{4-172}$$

as evidenced by a substitution for the variables in Eq. 4-151. The quantity $\phi_{xx}(\rho)$ is the autocorrelation function of the input x(t) when the time interval between samples of the random variables is ρ. Therefore, substituting from Eq. 4-172 into Eq. 4-171 shows that

$$\phi_{yy}(\tau) = \int_{-\infty}^{\infty} r(\mu)d\mu \int_{-\infty}^{\infty} \phi_{xx}(\rho)r(\nu)d\nu \tag{4-173}$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} r(\mu)\,r(\nu)\,\phi_{xx}(\rho)\,d\mu\,d\nu \tag{4-174}$$

It can be shown that Eq. 4-173 is equivalent to the statement that the autocorrelation of the output of a system element is equal to the convolution of the autocorrelation of the impulse response with the autocorrelation of the input variable. This is accomplished by changing the variables of integration as shown in Derivation 4-3 of the Appendix to Chapter 4.

Equation 4-173 can now be transformed to the frequency domain in the manner which follows. The Fourier transforms of the autocorrelation functions have the form of Eq. 4-152, with the following indicated changes of the time variable:

$$\Phi_{xx}(j\omega) = \int_{-\infty}^{\infty} \phi_{xx}(\rho)\, e^{-j\omega\rho}\, d\rho \tag{4-175}$$

$$\Phi_{yy}(j\omega) = \int_{-\infty}^{\infty} \phi_{yy}(\tau)\, e^{-j\omega\tau}\, d\tau \tag{4-176}$$

Substitution from Eq. 4-174 into Eq. 4-176 gives

$$\begin{aligned} \Phi_{yy}(j\omega) &= \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} r(\mu)\, r(\nu)\, \phi_{xx}(\rho)\, d\mu\, d\nu \right\} e^{-j\omega\tau}\, d\tau \\ &= \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} r(\mu)\, e^{j\omega\mu}\, r(\nu)\, e^{-j\omega\nu}\, \phi_{xx}(\rho)\, d\mu\, d\nu \right\} e^{-j\omega(\tau+\mu-\nu)}\, d\tau. \end{aligned} \tag{4-177}$$

Reversing the order of integration shows that

$$\Phi_{yy}(j\omega) = \int_{-\infty} \int_{-\infty} r(\mu)\, e^{j\omega\mu}\, r(\nu)\, e^{-j} \left\{ \int_{-\infty} \phi_{xx}(\rho)\, e^{-j\omega(\tau+\mu-\nu} \right\} d\mu\, d\nu. \tag{4-178}$$

Since $p = \tau + \mu - \nu$ and therefore $d\tau = dp$, the innermost integral of Eq. 4-178 is identical with the right-hand side of Eq. 4-175. Accordingly

$$\Phi_{yy}(j\omega) = \Phi_{xx}(j\omega) \int_{-\infty}^{\infty} r(\mu)\, e^{j\omega\mu}\, d\mu \int_{-\infty}^{\infty} r(\nu)\, e^{-j\omega\nu}\, d\nu \tag{4-179}$$

The second integral on the right-hand side of this equation has the form of the integral in Eq. 4-150, as can be seen by rewriting that equation with ν in place of τ, i.e.,

$$R(j\omega) = \int_{-\infty}^{\infty} r(\nu)\, e^{-j\omega\nu}\, d\nu \tag{4-180}$$

The first integral can also be seen to have this form if in Eq. 4-150, $-\omega$ is substituted for ω and μ for τ, i.e.,

$$R(-j\omega) = \int_{-\infty}^{\infty} r(\mu)\, e^{j\omega\mu}\, d\mu. \tag{4-181}$$

This means that

$$\Phi_{YY}(j\omega) = \Phi_{xx}(j\omega)\, R(-j\omega)\, R(j\omega) \tag{4-182}$$

The power spectral density and the transfer function are even functions, i.e., they are symmetrical about $\omega = 0$.* Therefore,

$$R(-j\omega)\, R(j\omega) = |R(j\omega)|^2 \tag{4-183}$$

and

$$\Phi_{YY}(j\omega) = |R(j\omega)|^2\, \Phi_{xx}(j\omega). \tag{4-184}$$

If the input signal includes a random error ϵ_x with a variance that is a function of frequency $\sigma^2_{\epsilon_x}(j\omega)$, the power spectral density of the input error $\Phi_{\epsilon_{xx}}(j\omega)$ is given by analogy with Eq. 4-166 as

$$\Phi_{\epsilon_{xx}}(j\omega) = 2\pi \frac{d}{d\omega}\left\{\sigma^2_{\epsilon_x}(j\omega)\right\} \tag{4-185}$$

Similarly, the power spectral density of the output error $\Phi_{\epsilon_{yy}}(j\omega)$ is

$$\Phi_{\epsilon_{yy}}(j\omega) = 2\pi \frac{d}{d\omega}\left\{\sigma^2_{\epsilon_y}(j\omega)\right\} \tag{4-186}$$

where $\sigma^2_{\epsilon_y}(j\omega)$ is the variance of the output error ϵ_y. From Eq. 4-184, the power spectral density of the output error can also be seen to be given by the relationship

$$\Phi_{\epsilon_{yy}}(j\omega) = |R(j\omega)|^2\, \Phi_{\epsilon_{xx}}(j\omega) \tag{4-187}$$

Substitution from Eqs. 4-185 and 4-186 into Eq. 4-187 show that

$$2\pi \frac{d}{d\omega}\left\{\sigma^2_{\epsilon_y}(j\omega)\right\} = 2\pi\, |R(j\omega)|^2\, \frac{d}{d\omega}\left\{\sigma^2_{\epsilon_x}(j\omega)\right\}. \tag{4-188}$$

The total variance $\sigma^2_{\epsilon_y}$ of the output error is defined as the integral of the components at all physically possible frequencies, i.e.,

$$\sigma^2_{\epsilon_y} = \int_0^{\infty} d\left\{\sigma^2_{\epsilon_y}(j\omega)\right\} \tag{4-189}$$

* See Chapter 7 of Reference 25.

where $\sigma^2_{\epsilon_y}$ is independent of frequency. Therefore, integration of Eq. 4-188 between the limits of zero and infinity to give

$$\int_0^\infty \frac{d\left\{\sigma^2_{\epsilon_y}(j\omega)\right\}}{d\omega} d\omega = \int_0^\infty |R(j\omega)|^2 \frac{d\,\sigma^2_{\epsilon_x}(j\omega)}{d\omega} d\omega. \tag{4-190}$$

and substitution therein from Eq. 4-189 shows that

$$\sigma^2_{\epsilon_y} = \int_0^\infty |R(j\omega)|^2 \frac{d\,\sigma^2_{\epsilon_x}(j\omega)}{d\omega} d\omega \tag{4-191}$$

In order to limit the range of the variables, it is often convenient to substitute log ω for ω, where log ω is the logarithm of ω to any convenient base. When this is done, Eq. 4-191 can be rewritten in the form

$$\sigma^2_{\epsilon_y} = \int_{-\infty}^\infty |R(j\omega)|^2 \frac{d\,\sigma^2_{\epsilon_x}(j\omega)}{d(\log\omega)} d(\log\omega). \tag{4-192}$$

The forms given by Eqs. 4-191 and 4-192 can be used alternatively, but Eq. 4-192 has the advantage of employing an increment, d(log ω), whose width is a constant percentage of the frequency. The use of log ω, rather than ω, has the advantage in practical applications that the amplitude of the noise per octave usually varies less widely with log ω than noise per unit change in frequency varies with frequency. Further, the use of log ω permits the use of finite integration limits that are not unduly extensive. These considerations are important inasmuch as numerical integration normally has to be employed. In the development which follows d(log ω) is employed, but it should be understood that do can be used interchangeably with d(log ω).

Both Eq. 4-191 and Eq. 4-192 give the variance of a single output error due to a single input error, in terms of the power spectral density of the input error $\Phi_{\epsilon_{xx}}(j\omega)$ (see Eq. 4-185 and in terms of a transfer, or amplification, function of frequency R $(j\omega)$. The mathematical development can be expanded to include multiple-input, multiple-element systems by following a procedure that is directly analogous to that described for the time domain in Eqs. 4-141, 4-142, and 4-143. In the frequency domain, the output error for an element having p inputs is obtained by adapting Eq. 4-192 to give the expression, analogous to Eq. 4-140 in the time domain,

$$\sigma^2_{\epsilon_y} = \sum_{n=1}^{p} \int_{-\infty}^\infty |R_n(j\omega)|^2 \frac{d\,\sigma^2_{\epsilon_{x_n}}(j\omega)}{d(\log\omega)} d(\log\omega) \tag{4-193}$$

where

$\sigma^2_{\epsilon_Y}$ = total variance of the error ϵ_Y that is associated with the output y

$R_n(j\omega)$ = transfer function that is associated with the element for which the output is y, and is measured between the input x_n and the output y

$\sigma^2_{\epsilon_{x_n}}(j\omega)$ = variance of the error ϵ_{x_n} that is associated with the input x_n

$d(\log \omega)$ = increment of frequency

In a system having q elements but no element errors m_i, the output error of the ith element is given by the expression, analogous to Eq. 4-141 in the time domain,

$$\sigma^2_{\epsilon_{y_i}} = \sum_{n=1}^{p} \int_{-\infty}^{\infty} |R_{i_n}(j\omega)|^2 \frac{d\sigma^2_{\epsilon_{x_n}}(j\omega)}{d(\log \omega)} d(\log \omega) + \sum_{k=1}^{q} \int_{-\infty}^{\infty} |R_{i_k}(j\omega)|^2 \frac{d\sigma^2_{\epsilon_{y_k}}(j\omega)}{d(\log \omega)} d(\log \omega) \tag{4-194}$$

where

$\sigma^2_{\epsilon_{y_i}}$ = total variance of the error ϵ_{y_i} that is associated with the output y_i

$R_{i_n}(j\omega)$ = transfer function that is associated with the ith element, and is measured between the input x_n and the output y_i

$\sigma^2_{\epsilon_{y_k}}(j\omega)$ = variance of the error ϵ_{y_k} that is associated with an output y_k that is also an input to the ith element.

The final set of error equations in the frequency domain, for a system having **q** elements and nonzero element errors mi, is represented by the following equation for the ith element, analogous to Eq. 4-142 in the time domain:

$$\sum^{p} \int_{-\infty}^{\infty} |R_{i_n}(j\omega)|^2 \frac{d\sigma^2_{\epsilon_{x_n}}(j\omega)}{d(\log \omega)} d(\log a) + \sum_{k=1}^{q} \int_{-\infty}^{\infty} |R_{i_k}(j\omega)|^2 \frac{d\sigma^2_{\epsilon_{y_k}}(j\omega)}{d(\log \omega)} d(\log w) + m_i(j\omega) \tag{4-195}$$

where

$m_i(j\omega)$ = error of the ith element due to the element being nonideal.

The integrals in Eqs. 4-193, 4-194, and 4-195 can be evaluated by numerical methods. The R (jo) functions (that is, the transfer functions of the system elements) are, in general, known:!: frequency responses of the system elements. The power spectral density of an input error can normally be expected to be the same as the power spectral density of the corresponding input; the latter is either known or can be assumed. Thus, a means is available for determining the variance of the output error of a system that is describable by differential equations and is subjected to random time-varying input errors. 'This technique, in combination with the techniques developed in par 4-4.4.2 for systems describable by other than differential equations, provides the fire control system designer with the means for analyzing the errors of most of the systems with which he is likely to be confronted. The laboriousness of the calculations is extremely great for a complex system, as is evidenced by the Vigilante Antiaircraft Weapon System example of par 4-6. Therefore, it is

* If the R $(j\omega)$ functions are not known, but the equation that is solved by the system is known, it is possible to solve for these functions by using the relationship given by Eq. 4-195, with assumed sinusoidal sources of error introduced into all inputs and elements. An analogous experimental technique that can also be employed consists of applying sinusoidal signals as inputs to the system and measuring the corresponding outputs. Then Eq. 4-195 can be employed to compute the R $(j\omega)$ functions. It should be noted that these techniques are not particularly useful except for simple system configurations.

common practice to approximate the smaller errors and reserve detailed treatment for the larger errors.* Such approximations do not appreciably reduce the accuracy of the analysis, however. This is because one of the characteristics of a root-square summation, such as is employed in the error-summation relationships, is that the larger errors tend to be emphasized over the smaller ones.

Immediately following (see par 4-4.4.4), as an illustrative example of error analysis for a system describable by differential equations, is the error analysis of a tracking antenna. All of the techniques derived in the preceding discussion of error analysis (par 4-4.4.1 through par. 4-4.4.3) are employed in the solution of this problem but a number of simplifying approximations are made in order to reduce the labor of the analysis.

A detailed example of the analysis of a complete fire control system is presented in par 4-6. This fire control system concerned is that employed in the Vigilante Antiaircraft Weapon System. For this fire control system, highly sophisticated elaborations of the basic approach presented in par 4-4.4.3 are required in order to solve an involved problem.

4-4.4.4 Illustrative Example of an Error Analysis for a System Describable by Differential Equations.

In this illustrative example, the errors in a Cassegrainian 30-foot-reflector mobile satellite-tracking antenna system (see Fig. 4-36) are analyzed.† The range of azimuth motion for the antenna is ±300 deg; the maximum rate of travel is 10 deg/sec; and the maximum acceleration is 6 deg/sec^2. The range of elevation motion is from -2 deg to +92 deg; the maximum rate of travel is 5 deg/sec; and the maximum acceleration is 3 deg/sec^2. The antenna system is designed to track the beacon in a satellite automatically, using a four-horn monopulse receiver, in winds of up to 35-mph velocity.

The necessity in satellite communications for rapid acquisition of the satellite and reliable tracking with very-narrow beam widths means that the errors of the antenna system must be tightly controlled. The procedure followed in the design of the antenna system was to first assign reasonable error limits to the various subsystems and then carry through a preliminary design, with the objective of staying within the assigned error limits. A preliminary error analysis was then made to ascertain that the specified limits for the system could be met. Finally, the detailed design of the antenna subsystem was made, together with a detailed analysis of the errors.

The errors of the antenna system under consideration can be classified in accordance with their source as follows:

(a) Tracking-independent errors
(b) Tracking-dependent errors
(c) Radio-signal propagation errors
(d) Instrumentation errors.

As shown in Table 4-3, the various individual errors concerned can be tabulated in these groups and can also be classified as calibration errors (i.e., systematic errors that can be removed by calibration), bias errors (systematic errors that cannot be removed by calibration), and noise errors (random errors).

Of the four error-source classifications listed, the propagation errors and the instrumentation errors are not associated with the antenna proper, and hence will not be considered further in this example. In addition, the calibration errors will be assumed to have been removed by appropriate calibration of the antenna system. The noise and bias components of the tracking-dependent and tracking-independent errors were obtained by methods

*Further discussion of this type of approximation is given in Section 11-4 of Reference 7.

†This example is adapted from Reference 41, which gives a complete error analysis of the antenna system.

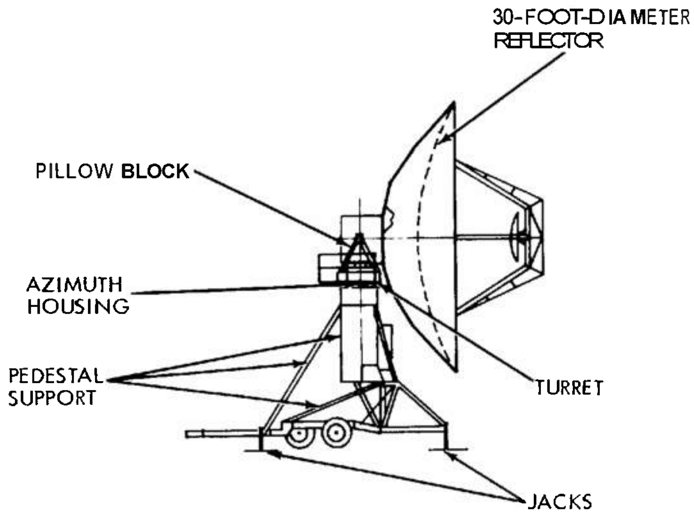


Figure 4-36. 30-foot-reflector mobile satellite-tracking antenna system.

TABLE 4-3. CLASSIFICATION OF TRACKING SYSTEM ERRORS.

Source of Error	Type of Error		
	Calibration	Bias	Noise
Tracking Independent (Static & Dynamic)	Gravity Alignment Readout Post-Comparator phase shift Pre-comparator unbalance Reference alignment Leveling (Above items removable at boresight)	Wind (Steady-state) Gravity Solar heating Leveling Lean in backlash Orthogonality of axes Readout Pre-Comparator phase shift Servo unbalance Servo dead zone	Wind gusts Receiver noise Gear tooth inaccuracies Variable phase shift in r-f receiver Data gear non-linearity and backlash Readout (Operator reading error)
Tracking Dependent (Dynamic)		Dynamic lag Multipath signal and other interfering errors	Glint Scintillation Vehicle tumbling Compliance (acceleration Hunting
Propagation	Predictable atmospheric behavior	Average atmospheric behavior	Atmospheric turbulence
Instrumentation	Inherent accuracy of optical equipment	Optical parallax Inherent accuracy of optical equipment	Reference instability Reading error

to be described in the paragraphs which follow. These error components are tabulated in Table 4-4 for the automatic-track-with-data-readout mode of operation.

An understanding of how the errors from the various components of the antenna system combine is aided by an error-flow diagram. Such a diagram is shown in Fig. 4-37 for the automatic-track-with-data-readout mode of operation. While it is not evident from this diagram, the azimuth and elevation components of the errors are independent of one another if the axes are orthogonal. Inasmuch as the deviation from orthogonality is quite small, the axes will be considered to be orthogonal in this analysis. As noted in the discussion which follows, however, certain azimuth errors are functions of the elevation angle.

An an example of the error-summation procedure, the azimuth bias errors caused by a steady wind will be considered. Most of the wind load on the antenna system is applied to the 30-foot reflector (see Fig. 4-36), and transmitted thence through the structure of the antenna system. The compliance* of the structural components can be calculated;† however, only the compliance of those components shown outside the azimuth servo loop in Fig. 4-37 (the azimuth housing, the pedestal support, and the jacks and anchors) need be considered. (The compliance of the ground can be assumed to be negligible.) Azimuth bias errors due to the compliance of the components noted are tabulated in the left-hand data column of Table 4-4 for the case of zero-degree antenna elevation (the worst case, since the wind direction is approximately parallel to the ground), and in the second column to the right for the case of 80-degrees elevation (the maximum elevation encountered in normal operation). The error caused by a steady wind load on the components within the azimuth servo loop is given by the loop gain in response to a torque input; this gain is called the torque-error constant.** The error of the servo loop is listed as the servo torque error in Table 4-4.

The component errors due to the wind load were all computed for the same amplitude and direction of the incident wind. (The azimuth angle chosen was that which produces the worst case of wind loading.) Therefore, the errors are correlated and are added linearly, as indicated in Table 4-4, to give the total compliance error. Separate computations are made for the cases of zero-degree and 80-degrees elevation.

The servo error caused by a time-varying input (this error is termed the dynamic lag error) was also computed for the combined worst-case conditions for both velocity and acceleration. These values and their linear sum are also entered in Table 4-4.

The remaining azimuth bias components are the servo static error, the receiver null-depth error, the power-train gearing error, and the data servo error. These errors are not correlated with each other or with either the dynamic lag error or the total compliance error.

The total probable peak (3σ) bias error can be obtained by a root-square summation, as described in par 4-4.4.3 and shown in Table 4-4. Similar summations are shown for the 1σ noise errors.

In order to obtain a figure for the total probable peak error, the total peak bias error was combined with three times the 1σ noise error in a root-square summation. The resultant total probable peak errors are shown at the bottom of Table 4-4.

As an example of the error calculation associated with a specific error component, the dynamic lag error (the servo error caused by a time-varying input) is examined in more detail in the paragraphs which follow.

Inasmuch as most of the errors of the azimuth and elevation antenna servos depend upon the associated open-loop transfer function — i.e., the frequency response — a deriva-

* Compliance is defined as the amount of angular displacement per unit of applied torque.

† The nature of these calculations is discussed in pages 7 and 8 of Reference 41.

** See page 10 of Reference 41.

TABLE 4-4. ERROR TABULATION FOR THE AUTOMATIC-TRACK-WITH-DATA-READOUT MODE OF OPERATION.

Error Component	Source of Error	Errors in Microradians					
		Azimuth Axis 0° Elev		Azimuth Axis 80° Elev		Elevation Axis	
		BIAS	NOISE	BIAS	NOISE	BIAS	NOISE
Servo Torque Error	Wind						
	Gravity					3.9	
Pillow Block Compliance Error	Wind					12.0	3.0
	Solar Heating					134.0	
Turret Compliance Error	Wind					40.0	10.0
	Gravity					45.0	
	Acceleration					4.2	
Azimuth Housing Compliance Error	Wind	5.4	1.4	1.2	0.3	17.0	4.2
	Gravity					17.0	
	Acceleration	0.4				0.8	
Pedestal Support Compliance Error	Wind	135.0	34.0	31.1	7.8	81.0	20.0
	Gravity	16.0		3.2		40.0	
	Solar Heating					113.0	
Jacks and Anchors Compliance Error	Wind	377.0	94.0	86.7	21.6	8.8	2.2
	Gravity					2.7	
Total Deflection	Wind	523.5	136.9	120.4	31.4	161.8	43.1
	Gravity	16.0		3.2		108.6	
	Solar Heating					247.0	
	Acceleration	0.4				5.0	
Servo Static Error	Unbalance	2.9		2.9		2.9	
	Dead Zone	5.7		5.7		5.7	
Hydraulic Motor Noise Error			55.0		55.0		27.0
Receiver Noise and Null Depth Error		8.8	36.0	50.7	204.0	8.8	36.0
Power-Train Gearing Error		166.0	23.0	166.0	23.0		23.0
Dynamic Lag Error	Velocity	52.0		257.0		52.0	
	Acceleration	2.0		92.0		2.0	
	Total	54.0		349.0		54.0	
Data-Train Gearing Error			10.0		10.0		10.0
Synchro Error			81.0		81.0		81.0

Error Component	Source of Error	Errors in Microradians: Azimuth Axis 0° Elev. BIAS	Azimuth Axis 0° Elev. NOISE	Azimuth Axis 80° Elev. BIAS	Azimuth Axis 80° Elev. NOISE	Elevation Axis BIAS	Elevation Axis NOISE
Data Servo Error		110.0		277.0		110.0	
Dial-Readout Error			87.0		87.0		87.0
Root-Square Total Error	Probable Peak (3 σ) Bias	563.0		493.2		337.8	
	1 σ Noise		194.5		249.1		136.5
Total Probable Peak (3 σ) Error		810.8		895.4		530.8	

tion of this function for an antenna servo is included here. Each servo comprises three loops, as shown in Fig. 4-38, and employs a valve-controlled hydraulic servomotor for the power drive.

The operation of the hydraulic servovalve* shown in Fig. 4-38 may be approximately described by the linear relationship

$$q_v = k_1 E_1 - k_2 p_m \tag{4-196}$$

where

q_v = output flow from the valve, measured in units of volume per unit of time
E_1 = input voltage applied to the electromagnetic torque motor in the valve
p_m = pressuredrop across the hydraulic motor, which constitutes the load on the valve
k_1 = voltage-flow constant for the valve
k_2 = pressure-flow constant for the valve.

Equation 4-196 is valid for low-flow conditions, as in reversing, and for servo systems in which the time lag associated with the electromagnetic torque motor in the valve is negligible. The second term on the right-hand side of Eq. 4-196 has been included to account for the pressure sensitivity of a hydraulic valve.

The flow from the valve q_v is made up of three components:

1. The flow that produces displacement of the hydraulic motor q_m
2. The leakage flow q_ℓ
3. The compressibility flow q_c.

Accordingly, expressed in mathematical form,

$$q_v = q_m + q_\ell + q_c \tag{4-197}$$

* For more-detailed information on the operation of hydraulic servovalves and other components of hydraulic servos, the reader should consult Section 13-6 of Chapter 13 in Reference 42. The type of servovalve concerned in the present example is a four-way spool valve.

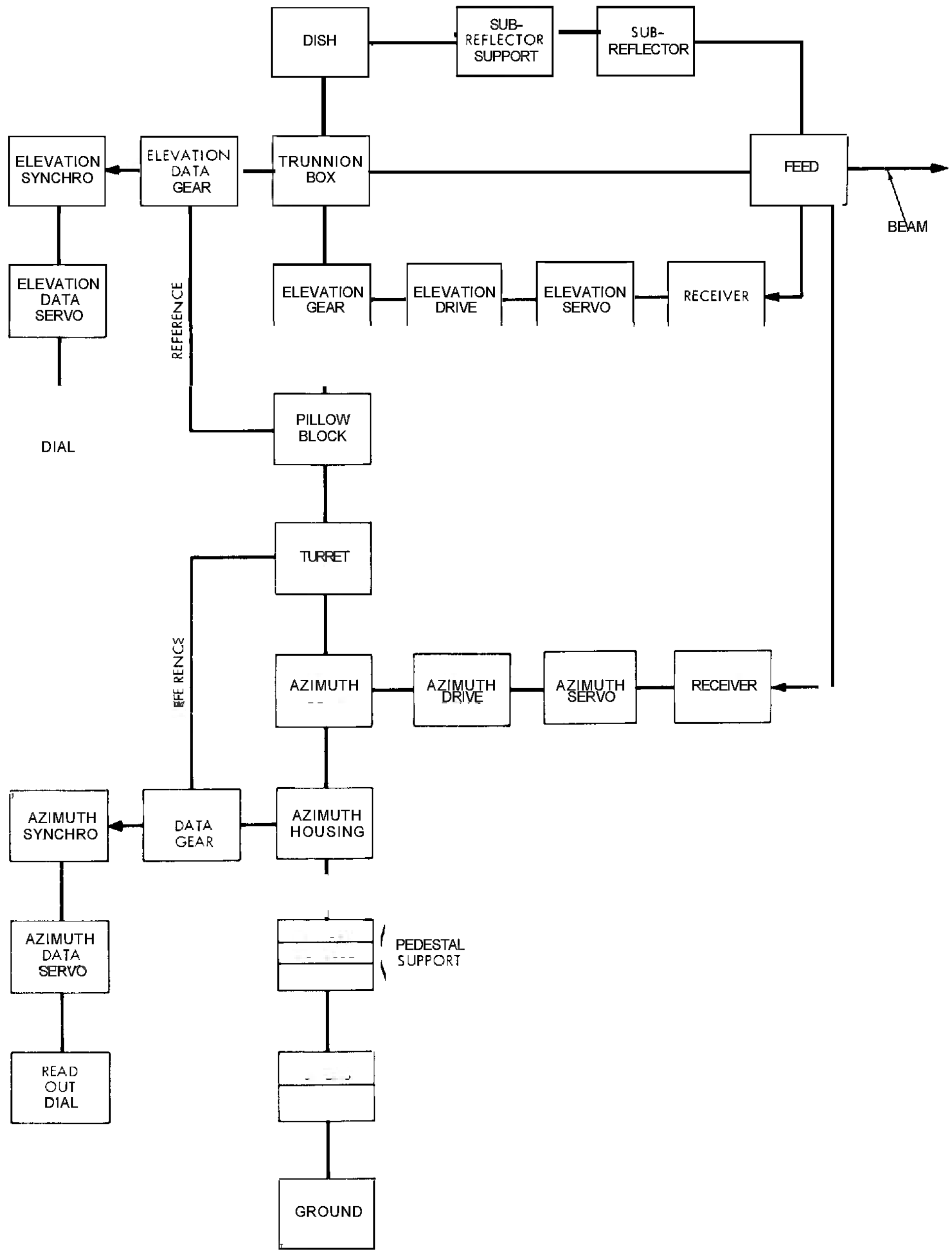
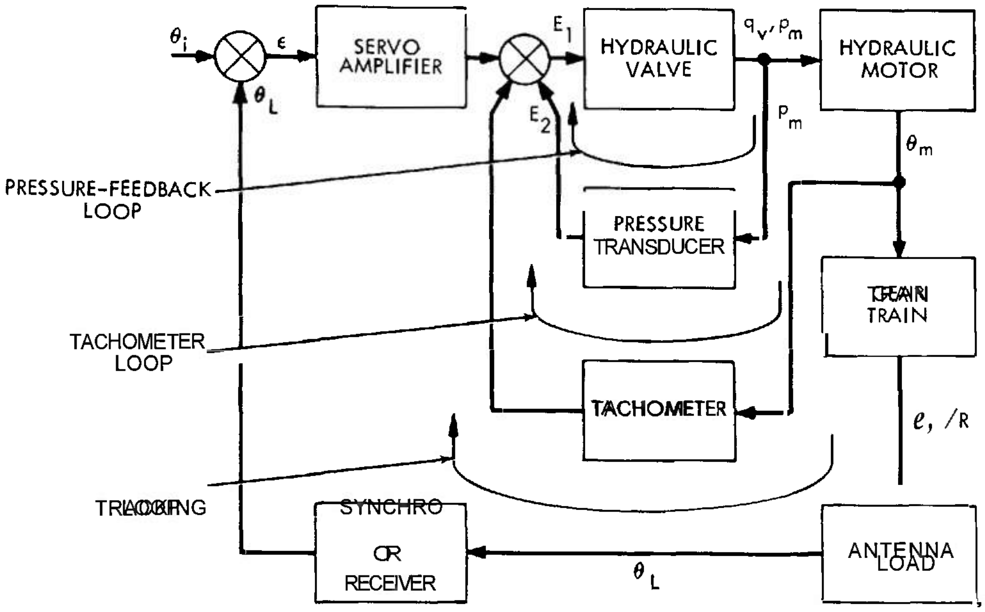


Figure 4-37. Error flow diagram for the automatic-track-with-data-readout mode of operation,

The displacement flow q_m is related — in Laplace-transform notation — to the angular velocity of the motor shaft by the equation

$$q_m = d_m s \theta_m \tag{4-198}$$



DEFINITIONS:

θ_i = INPUT ANGLE OF THE ANTENNA SERVO

θ_L = ANGULAR DISPLACEMENT OF THE ANTENNA

θ_m = ANGULAR DISPLACEMENT OF THE HYDRAULIC MOTOR SHAFT

ϵ = POSITION ERROR OF THE ANTENNA SERVO

E_1 = INPUT VOLTAGE APPLIED TO THE ELECTROMAGNETIC TORQUE MOTOR IN THE HYDRAULIC VALVE

q_v = OUTPUT FLOW FROM THE HYDRAULIC VALVE

p_m = PRESSURE DROP ACROSS THE HYDRAULIC MOTOR, WHICH CONSTITUTES THE LOAD ON THE HYDRAULIC VALVE

E_2 = VOLTAGE OUTPUT OF THE PRESSURE TRANSDUCER

R = GEAR RATIO OF THE GEAR TRAIN

Figure 4-38. Simplified functional diagram of the antenna servo.

where

d_m = motor displacement per radian, in units of volume per radian of shaft rotation

θ_m = angular displacement of the motor shaft, in radians

s = Laplace operator.

The leakage flow q_ℓ is related to the pressure drop across the hydraulic motor p_m by the equation

$$q_\ell = k_\ell p_m \tag{4-199}$$

where

k_ℓ = leakage coefficient of the motor.

The compressibility flow q_c is related to the pressure drop across the hydraulic motor p_m by the equation

$$q_c = k_c s p_m = KV_t s p_m \tag{4-200}$$

where

k_c = compressibility coefficient

$K = \frac{1}{B}$ = compressibility of the hydraulic fluid

B = bulk modulus of the hydraulic fluid

V_t = trapped volume of fluid under pressure.

The torque output of the hydraulic motor T_m is given by the relationship

$$T_m = d_m p_m . \tag{4-201}$$

Also, since the load cn the motor is pure inertia, the torque output is also given by the relationship

$$T_m = J_m s^2 \theta_m \tag{4-202}$$

where

J_m = rotational moment of inertia of the total motor load.

Therefore,

$$p_m = \frac{T_m}{d_m} = \frac{J_m}{d_m} s^2 \theta_m \tag{4-203}$$

Substitution from Eqs. 4-196 and 4-198 through 4-203 into Eq. 4-197 yields, after some rearrangement, the following equation for the operation of the hydraulic system of the antenna servo

$$\frac{\theta_m}{E_1} = \frac{k_1/d_m}{s\left[\frac{KV_t J_m}{d_m^2} s^2 + \frac{(k_2 + k_\ell) J_m}{d_m^2} s + 1\right]} \tag{4-204}$$

for which the standard quadratic form is

$$\frac{\theta_m}{E_1} = \frac{K_i}{s\left(\frac{1}{\omega_m^2} s^2 + \frac{2\zeta_m}{\omega_m} s + 1\right)} \tag{4-205}$$

where

$$K_1 = \frac{k_1}{d_m} \tag{4-206}$$

$$\omega_m = \frac{d_m}{\sqrt{KV_t J_m}} \tag{4-207}$$

= angular natural frequency of the hydraulic system

$$\zeta_m = \frac{k_2 + k_\ell}{2d_m}\sqrt{\frac{J_m}{KV_t}} \tag{4-208}$$

= damping ratio of the hydraulic system.

Since the hydraulic compliance associated with the motor C_m is given by the relationship

$$C_m = \frac{KV_t}{d_m^2} \tag{4-209}$$

Eq. 4-207 can be rewritten in the form

$$\omega_m = \frac{1}{\sqrt{C_m J_m}} \tag{4-210}$$

Thus, the angular natural frequency ω_m of the hydraulic system is determined by the rotational inertia J_m of the motor load and the hydraulic compliance C_m. The damping of the hydraulic system, on the other hand, is a function of the motor leakage coefficient k_ℓ and the pressure-flow constant k_2 of the hydraulic servovalve; this damping is usually quite small.

It is desirable to increase the damping of the hydraulic system by the addition of pressurefeedbackto the servovalve. This feedback can be achieved by use of a pressure transducer, as shown in Fig. 4-38, that measures the pressure drop p_m across the hydraulic-motor load of the valve and generates a voltage output E_2 that is directly proportional to p_m. Thus

$$E_2 = k_t p_m \tag{4-211}$$

where

k_t = voltage-pressure constant of the pressure transducer.

Substitution into this relationship from Eq. 4-203 gives

$$E_2 = \frac{k_t J_m}{d_m} s^2 \theta_m = K_2 s^2 \theta_m \tag{4-212}$$

where

$$K_2 = \frac{k_t J_m}{d_m} . \tag{4-213}$$

Figure 4-39 depicts the pressure feedback loop in terms of the transfer functions that have been derived. The voltage E_o is the input voltage to the pressure feedback loop. From Fig. 4-39, it is evident that

$$E_1 = E_o - E_2. \tag{4-214}$$

Substitution into this relationship from Eqs. 4-205 and 4-212 gives the following equation

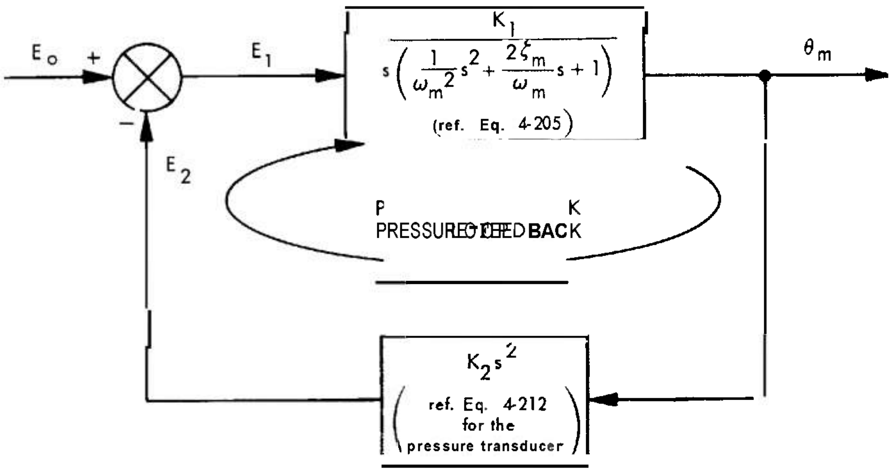


Figure 4-39. Transfer functions associated with the pressure-feedback loop of the antenna servo.

for the operation of the closed pressure-feedback loop:

$$\frac{\theta_m}{E_o} = \frac{K_1}{s\left[\dfrac{1}{\omega_m^2}s^2 + \left(\dfrac{2\zeta_m}{\omega_m} + K_1K_2\right)s + 1\right]} \tag{4-215}$$

which can be written in simplified form as

$$\frac{\theta_m}{E_o} = \frac{K_1}{s\left(\dfrac{1}{\omega_1^2}s^2 + \dfrac{2\zeta_1}{\omega_1}s + 1\right)} \tag{4-216}$$

where

$$\omega_1 = \omega_m \tag{4-217}$$

= angular natural frequency of the closed pressure-feedback loop.

$$\zeta_1 = \zeta_m + \frac{1}{2}K_1K_2\omega_1 \tag{4-218}$$

= damping ratio of the closed pressure-feedback loop.

The addition of the pressure-feedback loop thus provides an adjustable amount of additional

damping for the hydraulic system of the antenna servo.

The load on the antenna servo is a simple inertia-spring-damping system in which (a) the load's moment of rotational inertia J_L is that of the antenna, (b) the spring restraining force of the load is that contributed by the compliance of the antenna structure and associated gearing C_L, and (c) the load damping coefficient B_L is due to the viscous friction of the gears and the bearings. Therefore, the torques acting on the antenna load — in the absence of externally applied moments — can be summed as follows

$$J_L s^2 \theta_L + B_L s \theta_L + \frac{1}{C_L}\left(\theta_L - \frac{\theta_m}{R}\right) = 0 \tag{4-219}$$

where

- θ_L = angular displacement of the antenna
- J_L = rotational moment of inertia of the antenna load
- C_L = compliance associated with the antenna load
- B_L = damping coefficient of the antenna load
- R = gear ratio of the gear train.

Rearrangement of this equation yields

$$\frac{\theta_L}{\theta_m} = \frac{1}{RC_L\left(J_L s^2 + B_L s + \frac{1}{C_L}\right)} \tag{4-220}$$

for which the standard quadratic form is

$$\frac{\theta_L}{\theta_m} = \frac{1}{R\left(\frac{1}{\omega_L^2} s^2 + \frac{2\zeta_L}{\omega_L} s + 1\right)} \tag{4-221}$$

where

$$\omega_L = \frac{1}{\sqrt{J_L C_L}} \tag{4-222}$$

= angular natural frequency of the antenna load

$$\zeta_L = \frac{B_L}{2}\sqrt{\frac{C_L}{J_L}} \tag{4-223}$$

= damping ratio of the antenna load.

The dynamic system comprised of the hydraulic motor and its antenna load, therefore, involves two coupled quadratic lags (see Eqs. 4-216 and 4-221). Comparison of the corresponding parameters in the two quadratics, however, reveals the following information after all quantities concerned are referred to the same point in the gear train:

1. The compliance associated with the antenna load is much larger than the compliance associated with the hydraulic motor.
2. The rotational moments of inertia of the antenna load and of the motor are com-

parable.

3. The damping associated with the antenna load is negligible.

As a result, for the motor-and-load dynamic system under discussion, the system response will be that associated with a single quadratic in which (a) the angular natural frequency ω is slightly less than that of the load and (b) the damping ratio ζ is slightly greater than that of the motor.* Since the motor damping is adjustable (by varying the amount of pressure feedback), it follows that the damping of the motor-and-load dynamic system is similarly adjustable.

A high-pass network was cascaded with the pressure transducer.† This network has a transfer function equal to $T_p s/(T_p s + 1)$, where T_p is the time constant of the network. The network thereby introduces a pole at a complex frequency approximately equal to $-1/T_p$ and a zero at 0. This frequency was selected to be sufficiently low so that the high-pass filter has negligible effect on the dynamics of the pressure-feedback loop. The steady-state effect, however, is to remove the pressure feedback so that the stiffness of the system to disturbing torques is determined solely by the valve and motor leakage and remains at a high value.

The computed value of the natural frequency for the motor-and-load combination was 9.8 cps. A value of 8 cps was employed in the actual design, however, in order to allow a factor of safety. The computed damping ratio without pressure feedback was less than 0.1.

The transfer function of the closed pressure-feedback loop (see Fig. 4-39) can now be obtained for the situation in which the motor and load dynamic systems have been combined. This expression is

$$\frac{\theta_L}{E_o} = \frac{K_1/R}{s\left(\frac{1}{\omega_2^2}s^2 + \frac{2\zeta_2}{\omega_2}s + 1\right)} \tag{4-224}$$

where

$$\omega_2 \approx \omega_L \tag{4-225}$$

= angular natural frequency of the pressure-feedback loop when the presence of the antenna load is taken into account

$$\zeta_2 = \zeta_L + \frac{1}{2} K_1 K_2 \omega_1 \tag{4-226}$$

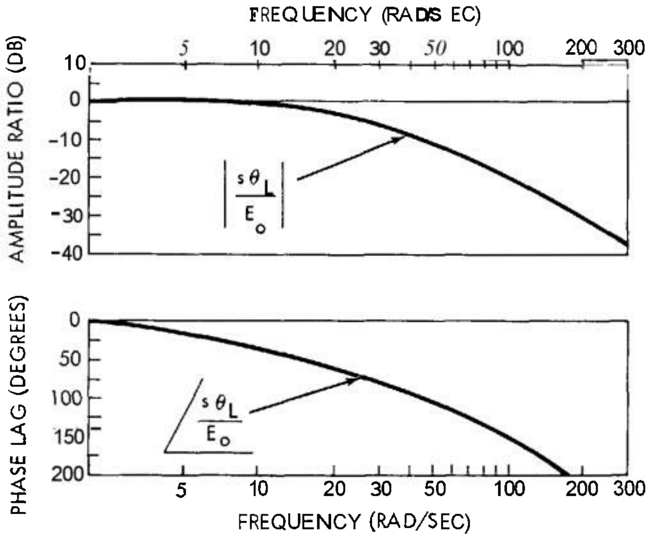
= damping ratio of the pressure-feedback loop when the presence of the antenna load is taken into account

R = gear ratio of the gear train.

The value of ω_2 is 50 rad/sec, corresponding to the value of 8 cps employed for the natural frequency in the actual design. The value of ζ_2 was adjusted to 0.77 by means of the pressure-feedback loop. Figure 4-40 gives a plot of the phase angle and amplitude ratio asso-

* This statement is based on the material presented in Section 6.81 or pages 170 through 174 of Reference 43.

† This network does not appear in Fig 4-38 inasmuch as this diagram represents a considerable simplification of the actual antenna servo concerned.



NOTE:

$\frac{s\theta_L}{E_o}$ IS THE OUTPUT VELOCITY - INPUT VOLTAGE RATIO.

Figure 4-40. Transfer function of the antenna load and hydraulic drive with the pressure-feedback loop closed.

ciated with the transfer function $s\theta_L/E_o$. These quantities were computed by entering the specified parameter values in Eq. 4-224. The transfer function $s\theta_L/E_o$, rather than θ_L/E_o, was plotted since the antenna velocity, rather than the antenna position, is the output quantity of primary interest. The phase-angle plot shows that a phase lag of 45 degrees corresponds to an angular forcing frequency of 15 rad/sec, and a phase lag of 90 degrees corresponds to a forcing frequency of 50 rad/sec.

The amplitude ratios of the open-loop and closed-loop transfer functions of the tachometer loop (see Fig. 4-38) are plotted in Fig. 4-41. The tachometer loop incorporates a high-pass network whose operation is represented by the ratio $Ts/(Ts + 1)$, where T is the time constant of the network. The use of this high-pass network provides a rising gain characteristic between the angular forcing frequencies of $1/\alpha T$ and $1/T$, where α is the gain of the tachometer loop. A loop gain of 18 db was obtained, with a damping ratio of 0.8 and an angular natural frequency of 113 rad/sec.

The transfer function of the closed tachometer loop (see Fig. 4-42) is

$$\frac{\theta_L}{E_3} = \frac{1}{K_3 s}\left(\frac{Ts + 1}{\alpha Ts + 1}\right)\left(\frac{\alpha/(1+\alpha)}{\frac{1}{\omega_t^2}s^2 + \frac{2\zeta_t}{\omega_t}s + 1}\right) \tag{4-227}$$

where

ω_t = angular natural frequency of the closed tachometer loop

ζ_t = damping ratio of the closed tachometer loop

K_3 = tachometer sensitivity

α = gain of the tachometer loop.

For the design value of $\alpha = 8$ (18db), $\alpha/(1 + \alpha) = 0.89$.

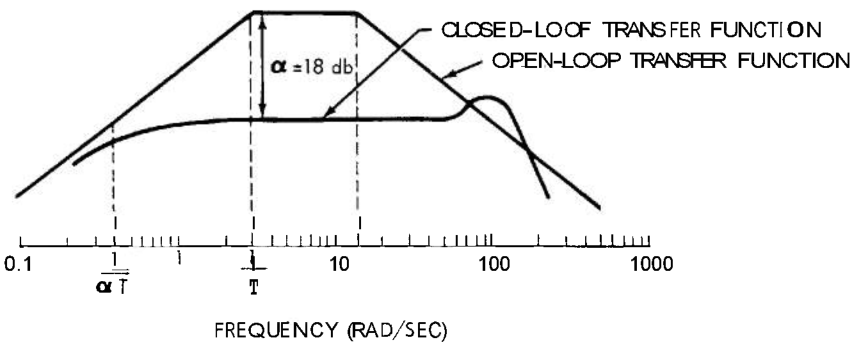


Figure 4-41. Open-loop and closed-loop transfer functions of the tachometer loop.

As shown by the detailed block diagram of Fig. 4-42, the tracking loop is a Type I servo that is formed by cascading the closed tachometer loop with a lag-compensation network. The purpose of this network is to increase the tracking-loop gain to a value of 600 sec^{-1}. (As shown subsequently, a value of 2.5 for the lag-network attenuation α_o was required to achieve this objective.) The overall open-loop transfer function KG of the antenna servo is then given by the relationship

$$\frac{\theta_L}{\epsilon} = KG = \frac{K_o}{K_3 s} \cdot \frac{T_o s + 1}{\alpha T_o s + 1} \cdot \frac{Ts + 1}{\alpha Ts + 1} \cdot \frac{\alpha/(1+\alpha)}{\frac{1}{\omega_t^2} s^2 + \frac{2\zeta_t}{\omega_t} s + 1} \tag{4-228}$$

where

K_o = tracking-loop sensitivity factor
α_o = attenuation of the lag compensation network
T_o = time constant of the lag compensation network
ϵ = $\theta_i - \theta_L$
= position error of the antenna servo
θ_i = input angle of the antenna servo.

Figure 4-43 gives an asymptotic plot of the open-loop transfer function of Eq. 4-228. Figure 4-44 gives the complete set of open-loop and closed-loop amplitude-ratio and phase-angle plots for the antenna servo.

Required for the error analysis are the velocity-error constant K_v and the acceleration-error constant K_a. The acceleration-error constant K_a is found by a Taylor's series expansion of the error/input ratio as follows:

$$\frac{\epsilon}{\theta_i} = \frac{1}{1+KG} = e_o + e_1 s + e_2 s^2 + \cdots = \frac{1}{K_p} + \frac{1}{K_v} s + \frac{1}{K_a} s^2 + \cdots \tag{4-229}$$

where s is the Laplace variable and e_o, e_1, etc. are the coefficients of the Taylor's series and are called error constants. The transfer function, KG, can be expressed in the polynomial form as

$$KG = K_v \frac{n_j s^j + \cdots + n_2 s^2 + n_1 s + 1}{s(d_j s^j + \cdots + d_2 s^2 + d_1 s + 1)} \tag{4-230}$$

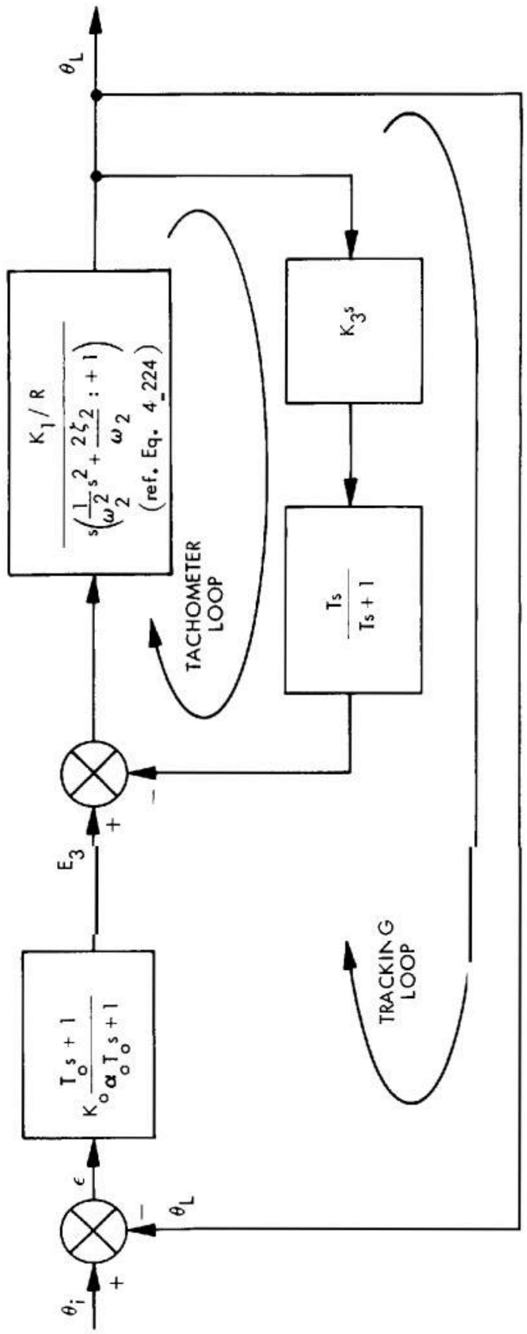


Figure 4-42. Detailed block diagram of the tachometer and tracking loops of the antenna servo.

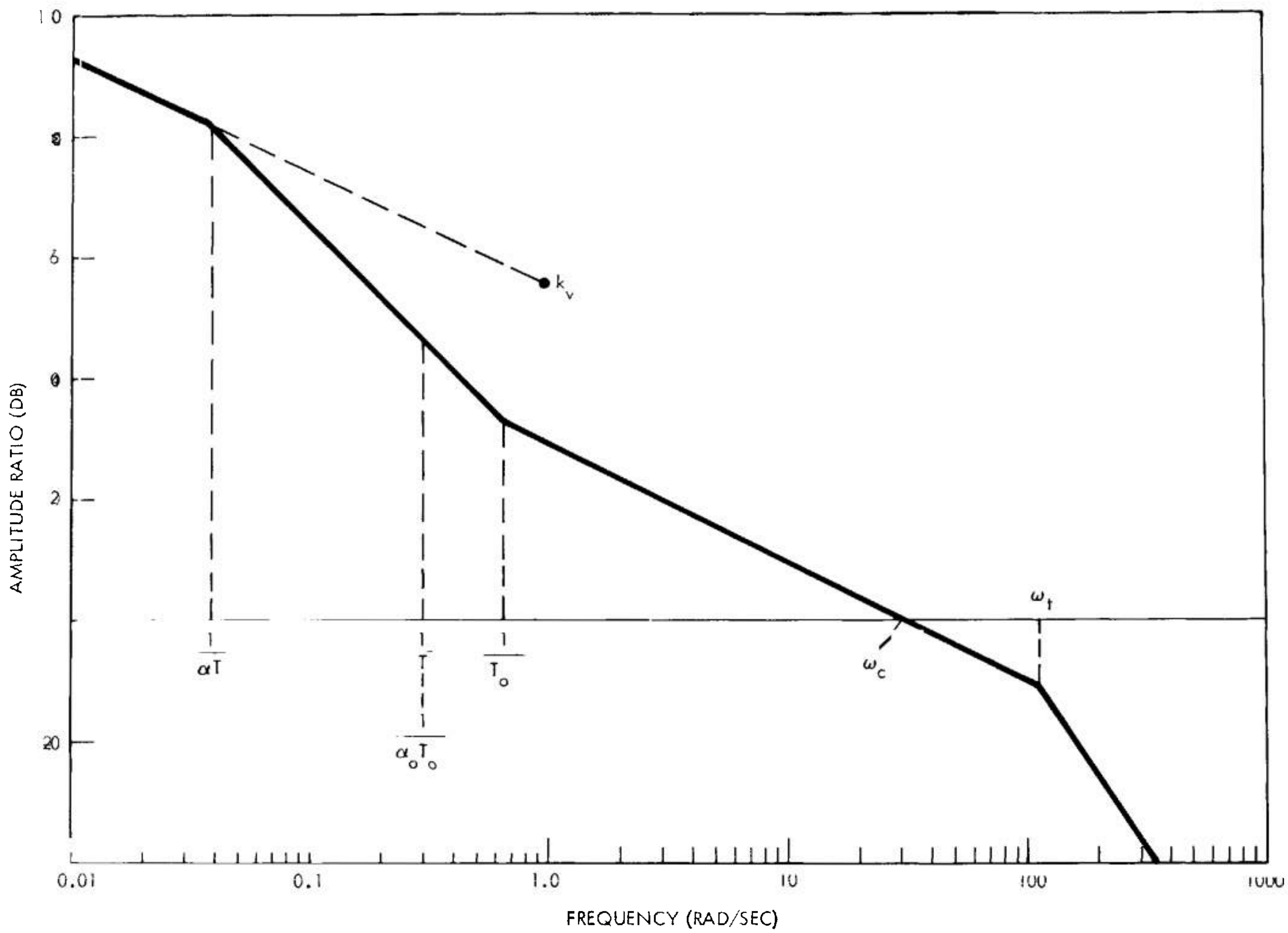


Figure 4-43. Asymptotic open-loop amplitude ratio for the antenna servo.

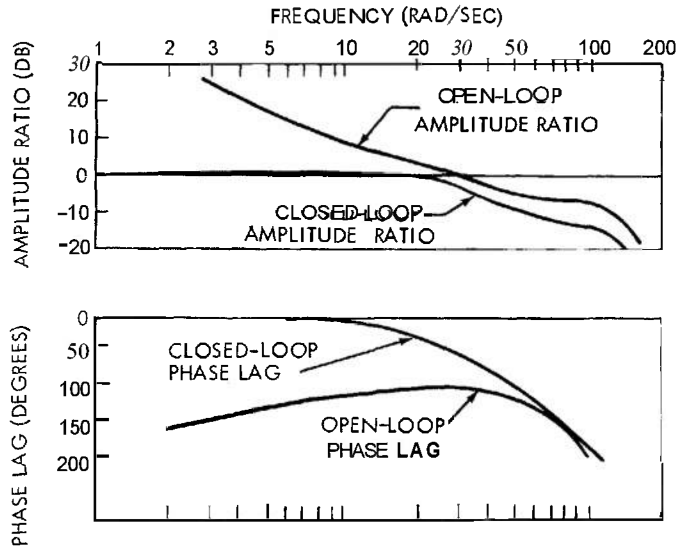


Figure 4-44. Open-loop and closed-loop transfer functions for the complete antenna servo.

Solving Eqs. 4-229 and 4-230 for K_a gives

$$K_a = \frac{K_v^2}{K_v(d_1 - n_1) - 1} \tag{4-231}$$

The expression for KG given by Eq. 4-228 can be expanded in the form

$$KG = K_v \frac{T_o T_s^2 + (T_o + T)\, s + 1}{s\left[1 + \left(\frac{2\zeta_t}{\omega_t} + \alpha T + \alpha_o T_o\right) s + \cdots\right]} \tag{4-232}$$

Therefore

$$K_a = \frac{K_v^2}{\left[\left(\qquad T + \alpha_o T_o\right) - (T_o + T)\right] K_v - 1} \tag{4-233}$$

The velocity-error constant K_v is the servo loop gain and is given approximately by the equation

$$K_v = \alpha_o\, \alpha\, \omega_c \tag{4-234}$$

where

ω_c = servo bandwidth, defined as the angular frequency at which the open-loop gain is unity (gain crossover).

For the 5-cps bandwidth, the various design constants are as follows

$a = 8$	$\omega_t = 113$ rad/sec	$\frac{2\zeta_t}{\omega_t} = 0.016$ sec/rad
$a_o = 2.5$	$\zeta_t = 0.8$	$(a-1)T = 2.19$
$T = \frac{1}{3.2}$ sec	$\omega_c = 30$ rad/sec	$(a_o - 1)T_o = 1$
$T_o = \frac{1}{1.5}$ sec		

Subsititution of these design values into Eqs. 4-234 and 4-233 shows that

$$K_v = a_o\, a\, \omega_c = 2.5 \times 8 \times 30 = 600 \text{ sec}^{-1} \tag{4-235}$$

$$K_o \cong \frac{K_v}{(a-1)T + (a_o - 1)T_o} = \frac{600}{3.19} = 188 \text{ sec}^{-2}. \tag{4-236}$$

The dynamic lag error (DLE) is the servo lagging error that occurs when the antenna is following a moving target. The error at any instant is given by the relationship

$$\text{DLE} \cong \frac{\dot{\theta}_L}{K_v} + \frac{\ddot{\theta}_L}{K_o} \tag{4-237}$$

when higher-order components are neglected.

The maximum angular velocity and angular acceleration of the antenna cannot occur simultaneously about either the azimuth axis or the elevation axis for an actual satellite track. Therefore, for the azimuth axis, it will be assumed that a satellite is passing in a straight line near the zenith, so as to generate a maximum angular velocity of 10 deg/sec. Figure 4-45 shows the azimuth angular velocity and angular acceleration of the antenna for such a course. The equations plotted to obtain the velocity and acceleration curves of Fig. 4-45 are as follows:

$$\text{Velocity} \cong K \cos^2 \phi \tag{4-238}$$

$$\text{Acceleration} \cong 2K^2 \sin \phi \cos^3 \phi \tag{4-239}$$

$$K = \frac{v_s}{h} \tan \theta_o \tag{4-240}$$

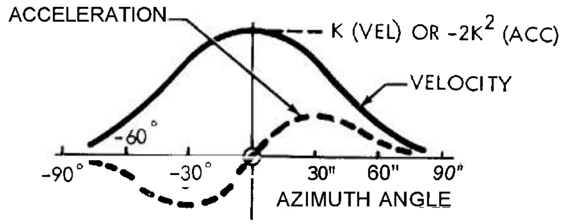


Figure 4-45. Azimuth angular velocity and angular acceleration for a target on a straight-line passing course.

where

- ϕ = azimuth angle
- K = maximum angular velocity of antenna = 10 deg/sec = 0.175 rad/sec
- v_s = satellite orbital velocity
- h = satellite altitude
- θ_o = maximum elevation angle (occurs at ϕ = 0).

The approximations are not valid near the horizon (ϕ = ±90 degrees), but here both theazimuth angular velocity and angular acceleration are small. It should be noted that the acceleration plot in Fig. 4-45 is inverted, so that the total error is indicated by the distance between the two curves.

The dynamic lag errors of the antenna system were computed for a track in which K = 0.175 rad/sec. (For example, θ_o = 80 deg and h = 130 naut. mi. or θ_o = 88 deg and h = 600 naut. mi., or any other combination shown in Fig. 4-46 that corresponds to K = 0.175 rad/sec.) The velocity, acceleration, and total dynamic lag errors were computed for K_v = 600 sec^{-1} and K_a = 188 sec^{-2} (see Eqs. 4-235 and 4-236.) The maximum error occurs at an azimuth angle of 20 degrees.

The worst condition in elevation occurs with a zenith pass. Under these conditions

$$_{max} = \frac{-}{h} \text{ at } \theta = 9 \tag{4-241}$$

and

$$\ddot{\theta}_{max} = 0.65 \left(\frac{v_s}{h} \right)^2 \text{ at } \theta = 60° \tag{4-242}$$

It is impossible to reach 5 deg/sec with any practicable satellite, so a minimum altitude of 130 naut. mi. is assumed to be the worst case. Then

$\theta_{max} \approx 0.032$ rad/sec

and

$\ddot{\theta}_{max} \approx 0.00067$ rad/sec^2

The dynamic lag errors thus found are tabulated in Table 4-4.

The dynamic lag errors discussed in the preceding paragraphs provide an example of errors due to operating elements within a system. An example of a random input error is afforded by the noise in the output of the tracking receiver which feeds its output to the servo. The noise in the receiver output was specified as 10 mv RMS, with a uniform spectrum over a 1000-cps pass band. In accordance with Eq. 4-160, the spectrum of the noise input to the servo is given by the relationship

(4-243)

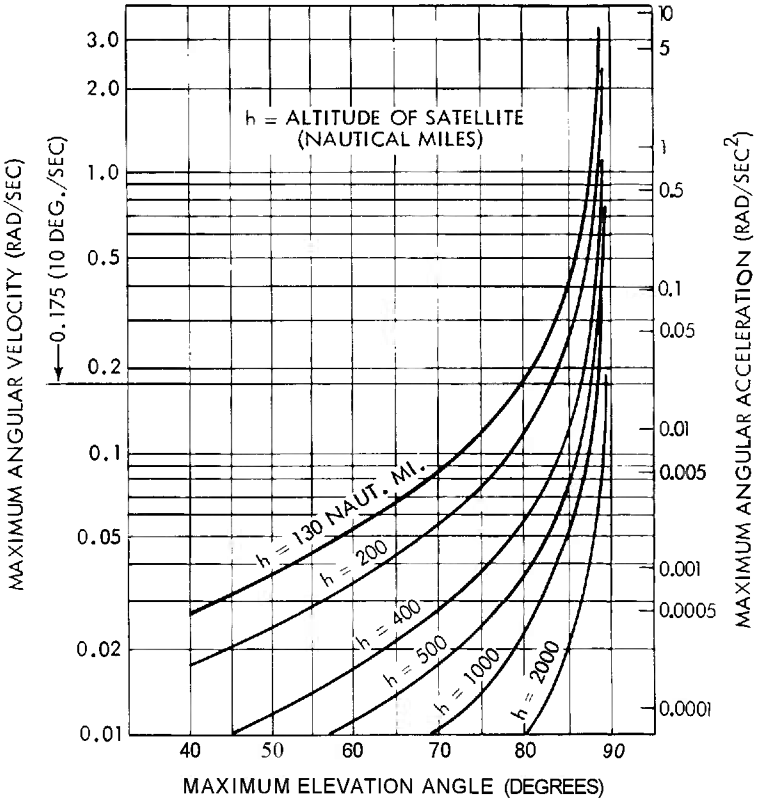


Figure 4-46. Maximum azimuth angular velocity and azimuth angular acceleration as functions of satellite height and maximum elevation angle.

where

$\overline{e_1^2}$ = mean-square noise voltage in the servo input
= $(10 \times 10^{-3})^2$ (volt)^2 = 10^{-4} (volt)^2 by specification

$\Phi_{ii}(j\omega)$ = power spectral density

In the present case, $\Phi_{ii}(j\omega)$ is equal to a constant value, A, between the two cutoff frequencies $-\omega_c$ and ω_c, and is zero elsewhere. Therefore, Eq. 4-243 can be rewritten as

$$\overline{e_i^2} = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} A\,d\omega = \frac{1}{2\pi}[2A\omega_c] = 10^{-4}\,(\text{volt})^2 \qquad (4\text{-}244)$$

where

$\omega_c = 2\pi \times 10^3$ rad/sec = bandwidth.

Solution of Eq. 4-244 for A gives

A = 5×10^{-8} (volt)^2/(rad/sec) = amplitude of the power spectral density between the two cutoff frequencies $-\omega_c$ and ω_c.

Similarly, in accordance with Eqs. 4-186 and 4-191, the mean-square noise voltage of the servo output is given by the relationship

$$\overline{e_o^2} = \sigma_o^2 = \int_{-\infty}^{\infty} |H(j\omega)|^2 \Phi_{ii}(j\omega)\, d\omega \tag{4-245}$$

$$= 2A \int_0^{\omega_c} |H(j\omega)|^2 d\omega \tag{4-246}$$

where

$\overline{e_o^2}$ = mean-square noise voltage of the servo output

σ_o^2 = variance of the servo output voltage

$|H(j\omega)|$ = magnitude of the closed-loop transfer function of the servo.

(Note that $\overline{e_o^2} = \sigma_o^2$ by virtue of Eq. 4-35, inasmuch as the mean value of the noise is zero. The servo output voltage is a fictitious quantity that is related to the actual angular output by a scale factor. This scale factor is the gain of the antenna and microwave receiver. Note also that the integral in Eq. 4-246 is taken for positive values of ω only since $H(j\omega)$ is assumed to be zero for $\omega < 0$).

The amplitude of the closed-loop transfer function was approximated by the asymptotes (see Fig. 4-44) and then squared. The integral of Eq. 4-246 was then evaluated graphically by replotting $|H(j\omega)|^2$ on a linear scale (see Fig. 4-47) and then measuring the area beneath the curve. Equation 4-246 shows that $\overline{e_o^2} = 4.24 \times 10^{-6}\,(\text{volt})^2$. (Alternatively, the integral of Eq. 4-246 might have been evaluated analytically.)

The receiver gain was adjusted to a level of 1.0 volt/degree. Therefore, the RMS error, which is equal to $\sqrt{\overline{e_o^2}}$, is 2.03×10^{-3} degree or 0.036 milliradian.

The receiver measures the target displacement in a plane perpendicular to the antenna boresight axis. Rotation of the antenna, however, is about the azimuth and elevation axes. It can be shown* that, for small displacements, a displacement of the azimuth axis through an angle ϕ induces an output ϕ_x in the azimuth channel of the receiver that is given by the equation

$$\phi_x = \phi \cos\theta \tag{4-247}$$

where

θ = elevation angle.

This relationship may be interpreted as a change in the loop gain of the azimuth servo by the factor $\cos\theta$. This gain variation is compensated by a secant-function potentiometer that is coupled to the elevation axis. The receiver output is connected to the servo input through the secant potentiometer. This potentiometer has the unfortunate effect, however, of increasing the noise amplitude at high elevation angles. At the maximum elevation considered (80 degrees), the noise error in the servo output is $2.03 \times 10^{-3} \times$ secant 80°, which is 11.7×10^{-3} degree or 0.204 milliradian. This value is tabulated in Table 4-4 under the column for azimuth axis, 80° elevation, noise component.

4-4.5 WEAPON-SYSTEM ERRORS THAT ARE BEYOND THE CONTROL OF THE FIRE CONTROL SYSTEM DESIGNER.

4-4.5.1 Introduction

Paragraph 4-4.1.3 summarizes the steps involved in the design of a fire control sys-

* See page 4 of Reference 41.

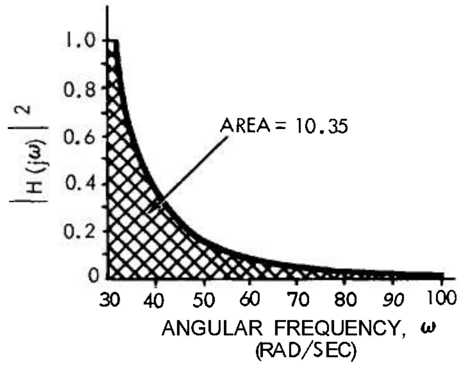


Figure 4-47. $|H(jw)|^2$ plotted to a linear scale.

tem of prescribed accuracy. Thus far, techniques for specifying the single-shot kill probability of a weapon system from the prescribed weapon-system engagement kill probability, and for specifying the allowable overall error of the weapon system from the single-shot kill probability have been described (see par 4-4.2 and par 4-4.3). In addition, the design procedure required for specifying the allowable errors for subsystems and components on the basis of the allowable overall weapon-system error has been detailed in par 4-4.4. As noted, this procedure requires that the designer rearrange the system functional block diagram and/or adjust component specifications as necessary to meet the over-all accuracy specification of the weapon system.

In practice, the errors in certain parts of the weapon system are usually beyond the control of the fire control system designer. Such circumstances might occur for some parts of the weapon system because their performance characteristics were dictated by natural phenomena or because there was a requirement to employ existing components or subsystems in the weapon-system design. Those parts of a weapon system whose errors are not under the control of the fire control system designer are usually associated with the input portion of the weapon system (the fire-control acquisition and tracking subsystem) and the output portion of the weapon system (the weapon-pointing system and the weapon itself). The errors associated with the input portion of the weapon system are considered first (see par 4-4.5.2) and then the errors associated with the output portion of the weapon system (see par 4-4.5.3). The errors associated with the fire control computing system, on the other hand, are considered to be under the control of the fire control system designer; these errors are discussed in par 4-4.6.

4-4.5.2 Errors Associated With the Input Portion of a Weapon System.

The input portion of a weapon system, which comprises the target acquisition and tracking devices, commonly employs either radar, visual-optical, laser-optical, or infrared detection means. In addition, especially if the fire control system is mounted on a moving base (such as would be the case, for example, in a tank fire control system; see Chapter 1), gyroscopes or pendulous elements may also be provided in the input portion of a weapon system.

The target acquisition device detects the presence and approximate positional coordinates of a target in order that the tracking device can initiate tracking of the target when it is so commanded. (The acquisition and tracking devices may be combined or they may be entirely separate subsystems.) The tracking device generates signals that describe the target motion and serve as inputs to the fire control computing system.

For specific types of acquisition devices and tracking devices, tabulations of the errors involved can be made in accordance with various classification schemes. Such a tabulation

for a beacon-tracking antenna is given in Table 4-4 (see par 4-4.4.4). For a skin tracker, i.e., a radar tracking system, additional sources of error are present. Detailed error analyses of typical radar systems are found in References 44 and 45. For gyroscopic elements, Section 11-6 in Chapter 11 of Reference 36 includes a discussion of the sources of error.

Some of the systematic, or bias, errors associated with tracking elements are described in the example of par 4-4.4.4. In general, except for wind loading and other atmospheric effects, these errors are not unlike the bias errors encountered in computer components; accordingly, they can be considered to be under the control of the fire control system designer and can be handled by the techniques already described. Tracking systems, however, are subject to random input errors of much greater amplitudes than are usually encountered in computer components. These random errors are associated with the real or apparent motion of the target about the tracking line and cannot be controlled by the fire control system designer. By analogy with communication systems, these random errors are called "noise". The noise of concern here is of relatively low frequency since high-frequency components are effectively filtered by the fire control system. Such filtering action can be accomplished, for example, by smoothing in the fire control computing system or by filtering directly in the tracking system itself by means of electrical filters or by limiting the pass bands of the tracking servos. The aforenoted noise should be distinguished from the atmospheric noise and resistance noise that determine the maximum range of an acquisition radar, or from the atmospheric effects that in an essentially similar way limit the range of a telescopic acquisition device. Once the target has been acquired and transferred to the tracking device, the signal strength is usually well above the atmospheric noise level. The atmospheric noise level then principally affects the resolution of the tracking device.

The input noise that is of prime importance to the fire control system designer is the real or apparent random motion of the target about the tracking line. This noise may arise from motion of the target, caused either by evasive action or by unintentional motions, or it may be caused by shifts in the apparent center of reflection of the radiation that is providing the tracking intelligence (the so-called glint effect). A further source of noise is the imperfection of the tracking device itself which may be controlled either automatically or by a human operator. Since the glint noise in radar tracking systems is one of the most difficult problems facing the fire control system designer, it will be discussed first and in some detail.

4-4.5.2.1 Radar Glint Noise

The origin of the glint noise in a radar tracking system is extremely complex, and largely defies analytical treatment. To introduce the problem, a simple case, but one that is still of considerable importance, is discussed in the paragraphs which follow.*

Consider a typical lobing radar, tracking two point targets in two dimensions, The radar may be of the conical-scan or lobe-switching variety, with output signals e_A and e_B from its two lobes as shown in Fig. 4-48. For small angles, the curves can be assumed to be linear near the crossover point. Consider first a single target. The output voltages from lobes A and B are then

$$e_A = G\,[1 - p(\theta_T - \theta_o)]\cos\omega t \tag{4-248}$$

and

$$e_B = G\,[1 + p(\theta_T - \theta_o)]\cos\omega t \tag{4-249}$$

*The following discussion is adapted from pages 435 through 444 of Reference 46.

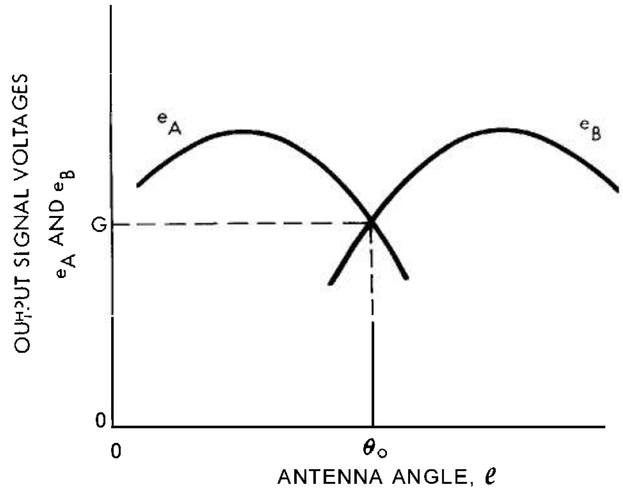


Figure 4-48. Antenna patterns for a lobing radar. (Adapted from GUIDANCE by A. S. Locke et al, D. Van Nostrand Company, Inc., 1955.)

where

e_A and e_B = output voltages from lobes A and B, respectively
G = overall system gain factor, including transmitter power, target size, receiver gain, etc.
p = slope of the gain curves at the crossover point
θ_T = angle of the target
θ_o = boresight angle of the radar antenna
ω = transmitter angular frequency.

Typically, the type of radar system concerned employs a square-law detector, which gives an output error voltage ϵ that is proportional to the difference of the squares of e_B and e_A. A low-pass filter removes the sinusodial components. The error voltage is then given by the relationship.

$$\epsilon = \overline{e_B^2} - \overline{e_A^2} = 2G^2 p(\theta_T - \theta_o) \tag{4-250}$$

which is readily obtained by performance of the indicated operations on Eqs. 4-248 and 4-249.

Now consider two targets that are included in the beam and located respectively at target angles θ_{T1} and θ_{T2}, as in Fig. 4-49. In general, these two targets occur at different ranges, so that the energy received from one differs in phase from the energy received from the other, because of the different path lengths. Also, because of the different ranges and different radar cross sections, the received amplitudes differ. Thus the output voltage from each lobe is made up of the sum of two components from each of the two targets, and may be written as follows:

$$e_A = G[1 - p(\theta_{T1} - \theta_o)]\cos\omega t + aG[1 - p(\theta_{T2} - \theta_o)]\cos(\omega t + \alpha) \tag{4-251}$$

$$e_B = G[1 + p(\theta_{T1} - \theta_o)]\cos\omega t + aG[1 + p(\theta_{T2} - \theta_o)]\cos(\omega t + \alpha) \tag{4-252}$$

where

a = amplitude ratio of the energy received from the two targets
= energy received from target T2/energy received from target T1
α = phase difference of the energy received from the two targets.

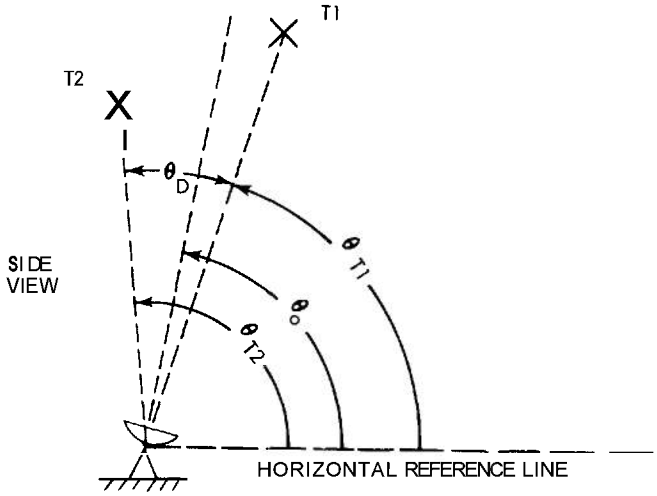


Figure 4-49. The geometry associated with two-target tracking.

In this case, the error signal ϵ is given by the relationship

$$\epsilon = \overline{e_B^2} - \overline{e_A^2} = 2G^2 p\left[(1 + a\cos\alpha)\theta_{T1} + (a^2 + a\cos\alpha)\theta_{T2} - (1 + a^2 + 2a\cos\alpha)\theta_o\right] \tag{4-253}$$

The effects of phase and amplitude differences are made more obvious by assuming an on-target indication (ϵ=o) and making use of the fact that $\theta_D = \theta_{T2} - \theta_{T1}$ (see Fig. 4-49). Equation 4-253 may then be rearranged to show that

$$\theta_o = \theta_{T1} + \frac{a^2 + a\cos\alpha}{1 + a^2 + 2a\cos a}\,\theta_D. \tag{4-254}$$

A number of significant conclusions can be deduced from Eq. 4-254:

1. If the phase difference α is zero and the amplitude ratio a is zero, as would be the case if target T2 had negligible reflecting area, then the antenna boresight coincides with the line of sight to target T1 and there is no noise in the system.

2. If a = 1, as might be the case for two identical aircraft flying in formation, the antenna boresight is displaced from the line of sight to T1 by $\theta_D/2$, i.e., the apparent target center lies halfway between the two real targets. In general, with zero phase difference, the angular displacement is $\left(\frac{a}{a+1}\right)\theta_D$; thus, the apparent target center is shifted toward the stronger target.

3. As the phase angle changes from 0" to 180°, the apparent boresight angle changes by an amount determined by the value of the amplitude ratio a. For a = 0.9, the variation of the boresight angle with phase angle can become extremely large, as shown by Fig. 4-50.

4. The noise error is an angular displacement; thus, if the two targets are a fixed distance apart, the amplitude of the noise error increases with decreasing range.

This simple two-target case indicates the problems that may be encountered in tracking groups of aircraft. Also aircraft flying near ground level reflect an image from the ground plane, which gives similar effects. Even a single aircraft or other vehicle is not a point target, but is made up of multiple reflecting surfaces, each of which may be considered

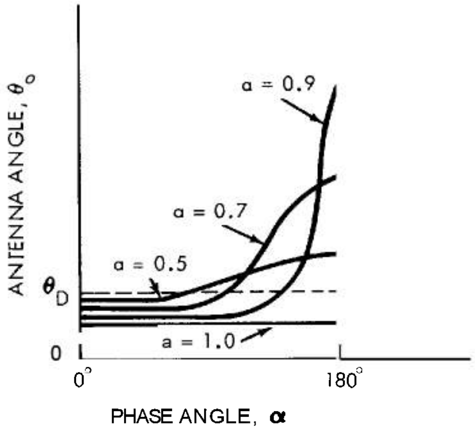


Figure 4-50. Tracking error for two targets. (Adapted from GUIDANCE by A. S. Locke et al, D. Van Nostrand Company, Inc., 1955.)

as one of the targets in an expansion of the two-target case. The multiplicity of reflecting surfaces produces a large number of echoes with randomphase angles and amplitudes, and, as may be seen from the two-element case, the noise amplitude may exceed the dimensions of the target.

4-4.5.2.2 Amplitude Noise

The dependence of angular glint noise on phase angle means that is significant only in systems employing coherent radiation.* such as radar and the new coherent-light systems (lasers), in which the detection is sensitive to the phase angle of the received signal. Systems employing noncoherent radiation, such as infrared and visible light, are affected only by the amplitude portion of the glint noise. For the two-target case when noncoherent radiation is employed, the displacement of the antenna boresight angle is $\left(\frac{a}{a+1}\right)\theta_D$, so that this displacement can never be greater than the target dimensions.

Tracking systems that employ lobing are subject to errors from amplitude modulation that occurs at the lobing frequency. This type of noise is independent of range. The lobing frequency should be selected to minimize the noise; in particular, the fundamental and harmonics of the blade frequencies of propeller-driven aircraft should be avoided.

It must be emphasized that noise amplitudes must be determined experimentally for the particular targets and tracking system under consideration. The brief analysis given here merely shows the significance of the various factors involved.

4-4.5.2.3 Target Motions

Random motions of slowly-moving targets are not of great significance but, in the case of aircraft, both unintentional motion caused by wind gusts and thermal currents† and intentional evasive action may contribute to the noise input. In the case of evasive action, there is little that the fire control system designer can do to counter its effects, other than to keep the computer settling time as short as possible. It might be noted that guided missiles can correct for evasive maneuvers, inasmuch as their trajectories can be continually modified; this fact is one of their major advantages over unguided projectiles. On the other

* Coherent radiation is radiation that is all of the same frequency and is substantially in phase.

† For experimental data on aircraft motion caused by air disturbances, see Reference 47.

hand, if the target aircraft is to take any effective offensive action, it must cease its maneuvers in the vicinity of its objective. Thus, in locations close to enemy objectives, weapons employing unguided projectiles may be advantageous.

4-4.5.2.4 Tracking Noise

Tracking may be accomplished manually, automatically by servos, or by a combined system. Noise is introduced into the tracking system by such mechanical nonidealities as coulomb friction, backlash, gear irregularities, and vibration. If the tracking is accomplished automatically, the design of the servos and the mechanical components should be such as to minimize these nonidealities. Viscous friction and inertia of the tracking head are helpful in attaining smooth tracking but detract from rapid, accurate positioning. If the tracking system must be capable of both smooth tracking and rapid positioning, some compromise in the design must accordingly be made. Since the design of servomechanisms is thoroughly covered elsewhere in the Engineering Design Handbook Series,* no further discussion of the design of automatic tracking systems is presented here.

Most tracking systems employ a human operator at some point in the tracking loop; human operators are also employed in other parts of the fire control system to make decisions, serve as communications links, and to perform computations that do not have to be made rapidly. In contradistinction to the situation in the past, modern fire control systems are designed to make the task of the human operator as simple as possible in order to improve his speed and accuracy, to reduce the effects of fatigue, and to reduce the amount of training required. This portion of the design process – which combines the disciplines of psychology, physiology, and engineering – is known as human engineering. A knowledge of the principles of human engineering will be valuable to the designer of audible and visual data presentations, hand and foot controls, seats and enclosures for human operators. A full treatment of the topic will be found in References 49, 50, and 51; a brief general survey is given in Reference 7. The discussion which follows covers those facets of the subject which contribute to tracking error.

Human operators are employed in tracking systems either (1) because a suitable detector is lacking (as in optical tracking), (2) to distinguish signals from noise (a function that a trained operator can perform better in most respects than a filter circuit), or (3) because an operator is required in any case to make decisions and the designer wishes to economize by using him for the tracking function as well.

The simplest tracking is accomplished by manually pointing a tracking head mounted on pivots; however, a considerable increase in accuracy can be achieved by incorporating a geared drive from a crank or handwheel. Experimental data on handwheel drives shows that a considerable decrease in tracking error occurs with increasing speed of rotation, up to a maximum of 140 to 200 rpm.[50] The diameter of the handwheel may be varied from 2 to 5 inches with little effect. It is desirable to employ as high a ratio as possible between the handwheel rotation and the resulting visual indication since there is an observed tendency to greater speed and reduced error with high ratios. Of course, the physical limit of 200 rpm must not be exceeded.

Coulomb friction causes rapid degradation in performance if it is greater than about two pounds, measured at the crank. Errors caused by coulomb friction are reduced if the inertia or the viscous friction in the system are increased, or if the speed of rotation of the handwheel is increased.

Many measurements on human operators in both positioning and tracking systems indicate that there is an average delay time of 0.5 second, and that this time is substantially independent of the extent of the motion required. Measurements of the sinusoidal response of humans show good response out to about 3 or 4 cps.

* See References 40 and 48.

Efforts to develop a linear mathematical model of the human operator in tracking tasks have been made by Raggazini* who measured the response of an operator who was given position feedback from a hand control to a visible spot on a cathode-ray-tube screen, and by Tustin† who employed a gun carriage with control of angular rate by the operator. The mathematical model in either experiment is the relationship

$$H(s) = \left[\left(as + b + \frac{c}{s}\right)e^{-\tau s}\right] A(s) \tag{4-255}$$

where

$A(s)$ = position of the stimulus (the visible spot)

$H(s)$ = operator's hand position

a, b, c, and τ = constants that were determined for the particular experiment

s = Laplace operator.

Thus, the operator's response is made up of motions proportional to the stimulus, its derivative, and its integral; all delayed by the reaction time τ. The derivative term was found to be quite small and the value of τ was determined to be about 0.3 second. It appears that this mathematical model can be used with considerable confidence.

The tracking of rapidly moving targets can be greatly facilitated by the provision of controls that, by means of servos or gyro precession,** provide a rate of turn of the tracker that is proportional to the motion of the control. Further improvement is secured by aided tracking, in which both a position and rate response are obtained for a given control movement. The ratio between the position and the rate responses is called the aiding ratio; a discussion of optimum values for the aiding ratio can be found in pages 360-368 of Reference 53.

4-4.5.3 Errors Associated With the Output Portion of a Weapon System.

The output portion of a weapon system comprises the weapon itself, i.e., the projectile and its launching system, and the associated weapon-pointing system, To a large degree, the errors associated with this portion of the fire control system have lain outside the control of the fire control system designer. This has been true principally because of the nature of weapon-system development. Traditionally, the fire control system of a weapon system has been designed to match either an existing weapon or one that is already under development. This has meant that the fire control system designer has been forced to adapt his designs to match already existing components, even though it has frequently been apparent that some modification of these components would result in worthwhile improvements in the overall performance of the weapon system.

On the other hand, the modern weapon-system design concept that is now evolving has modified this traditional approach to a considerable degree. The new approach incorporates a system planning stage in which the individual designs of the acquisition and tracking system, the computing system, the weapon-pointing system, and weapon are adjusted so as to obtain an optimum overall weapon-system design within the time and funds allotted. Beyond this planning stage, the development of each major subsystem of the weapon-system is largely independent of the others.

Both systematic and random errors are associated with the output portion of a weapon system. The systematic errors are largely the boresight errors of the weapon. In order to keep these errors to a minimum, provision is made for the accurate initial alignment of the weapon and the tracking system. In addition, since gun tubes may wear unevenly and

* See pages 1329-1330 of Reference 50.

† See pages 190-202 of Reference 52.

** For an example of this application of gyro precession, see the Vigilante Antiaircraft Weapon System example in Par. 4-6.

gun carriages may settle unevenly, provision is usually incorporated somewhere in a fire control system for manual corrections to be made from time to time, based on observations of actual projectile trajectories.

The dispersion errors associated with the output portion of a weapon system arise primarily from such sources as ammunition variations and gun-tube vibration. As pointed out in par 4-4.1.1 and par 4-4.1.2, however, a certain degree of dispersion is desirable in order to increase the engagement hit probability.

It is not desirable, of course, for the probabilistic nature of the dispersion to vary during the course of an engagement – or over longer periods, either. Accordingly, consideration has been given to means for automatically compensating for some of the variations. An example of the application of such automatic compensation is a system employed by a European weapon-system manufacturer, whereby the variations in muzzle velocity are continuously determined and compensated. In order to achieve this compensation, the actual muzzle velocity is measured automatically during the course of firing and this information is fed to a computer that provides corrected aiming data for use by the weapon system. This system provides compensation for the systematic errors in muzzle velocity, and could provide useful data (but, of course, no compensation) for the variance. Initial data that is not yet fully substantiated indicates that a significant increase in hit probability can be achieved by the use of this arrangement.

4-4.6 WEAPON-SYSTEM ERRORS THAT ARE UNDER THE CONTROL OF THE FIRE CONTROL SYSTEM DESIGNER.

4-4.6.1 Introduction

In carrying out the design of a fire control system to meet a prescribed accuracy requirement, the designer has as variables the component accuracy specifications and the arrangement of system components. In order to achieve an optimum design, he must not only be able to determine the propagation of the component errors through the system but must also be aware of the nature of, and limitations imposed by, the errors in the components at his disposal. Means for determining the propagation of errors have been covered in par 4-4.4. It is the purpose of the remaining paragraphs to discuss the errors in the components that are at the disposal of the fire control system designer.

The portion of a weapon system that is under the control of the fire control system designer to a greater degree than any other portion is the fire control computing system. The reason for this follows. At the input end of the weapon system, the design of the acquisition and tracking system is limited to a great degree by the nature of the problem (see par 4-4.5.1 and par. 4-4.5.2). At the output end of the weapon system, on the other hand, the fire control system designer is limited by the fact that he doesn't have any control over the design of the weapon itself and in some instances has no control over the weapon-pointing system. The computing system, by virtue of its central position in the weapon system is, in effect, isolated from these two types of limitations.

A full discussion of the errors in computer components requires a complete understanding of the operation of the components themselves. A detailed treatment of component errors from the standpoint of the component designer must, therefore, be postponed to Section 3 of the Fire Control Series. In the paragraphs below, however, an attempt will be made to outline the types of errors found in computer components and the methods of specifying them that will be of use to the fire control system designer. As an example of such use, consider the following. In preceding parts of Chapter 4 (see par 4-4.3 and par 4-4.4), means have been described for determining, given a hit probability requirement, the permissible errors in the components of the system. This process is reversible; thus, from an error in a single component the resulting error in the system output can be computed and its effect on the hit probability can then be determined.

The errors of computer components that are of concern can be divided into the classi-

fications of (1) bias errors and (2) random errors. A third group of errors are those that can be removed at calibration and are, therefore, not generally of concern. An example of such a calibration error is the mechanical offset at the end of a potentiometer winding from the corresponding zero mark of its dial.

Both bias and random errors may be either dependent on, or independent of, the input quantities and time. Dependent errors are commonly found in computer circuits; e.g., a potentiometer used as a multiplier in an analog computer has errors (such as those due to nonlinearities in the winding) that are functions of the shaft angle. Other sources of input-dependent errors include eccentricity of shafts carrying gearing or cams, saturation in computing amplifiers, and backlash in gearing. In each case, the error produced by the particular nonlinearity concerned can be expressed as a function of the input, the function being determined either experimentally or by analysis. The expression for the error as a function of input can then be introduced into the set of Equations 4-116. While this process is very neat theoretically, the complications introduced into the equations are such that this method is employed only when the error in question is large. Usually, it is sufficient to assume some probability that the error will have a specific value. The choice of this probability will require a good deal of judgment on the part of the designer. A conservative approach would be to determine the peak (3σ) error under all possible conditions. The most rigorous approach would be to determine the probability of each value of the error; the error could then be characterized by the moments. A practical approach that is commonly employed (see the example of par 4-4.4.4) is to assume a normal distribution of the values of the error. Since the mean is zero if calibration errors have been removed, the error is characterized by the standard deviation σ.

Certain computer errors are functions of time. Two types of time-variable errors must be distinguished. The first type is characterized by time variations that are slow in comparison with the solution time of the computer; drift in operational amplifiers (see par 4-4.6.2.6) is an example of this type of error. These drift-type errors are handled exactly like the independent, or assumed independent, bias errors noted in the preceding paragraph, i.e., a normal distribution is assumed for all values of error within the range of the drift error and the standard deviation of this distribution is determined. The second type of time-variable error has variations that are fast in comparison with the solution time of the computer. This type can be assumed to be a random error. The gear noise mentioned subsequently in par 4-4.6.2.1 is an example of such a random time-variable error.

The errors introduced by effects such as backlash, dead zone, and magnetic hysteresis (such as found in electric motors) or mechanical hysteresis (such as found in springs) are particularly interesting, as well as important. A typical hysteresis curve is shown in Fig. 4-51(A) where the arrows indicate the direction of change of the input signal. It is evident that the output signal is a function of the direction in which the input is changing, as well as its magnitude. In computer components, the opening of the hysteresis loop is small compared with the range of the input and output variables. Under this condition, the output may be represented by a function that is proportional to the input but has a small. offset whose direction is determined by the sign of the input (see Fig. 4-51 (B)). Such a representation is also a good mathematical model of the mechanical backlash encountered in computer components.

For most purposes, hysteresis-type nonlinear errors may be lumped with other signal-variant and signal-invariant errors to give a total invariant uncertainty for the component. However., in a closed-loop portion of the computer, these errors can contribute to instability of the overall system, thereby generating errors that are much greater than the magnitude of the hysteresis. Methods are available for analyzing the dynamic effects of this and other types of nonlinearity in closed-loop systems. Either the describing-function technique, which is a sinusoidal method of analysis in which the harmonics generated by the nonlinear element are neglected, or the graphical phase-plane method can be employed. Both methods are treated in Reference 25 (see pages 566-663) and Reference 40 (see Chapter 10).

The dynamic properties of errors must be considered when the components that follow

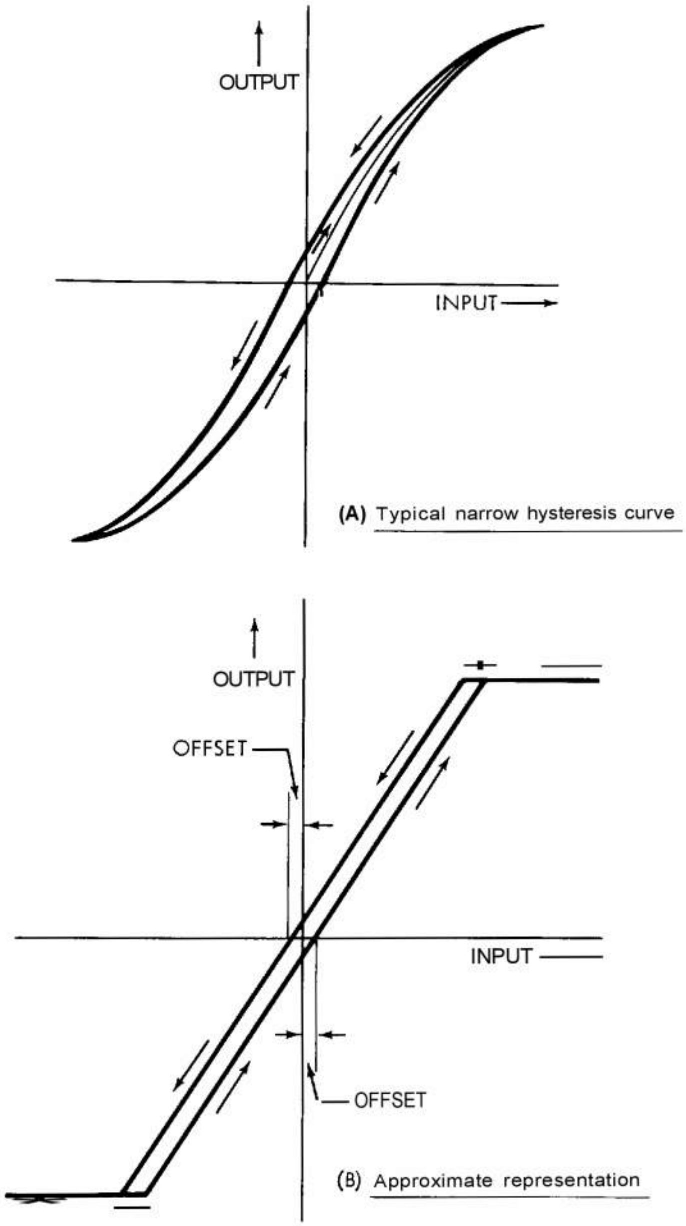


Figure 4-51. A typical narrow hysteresis curve and its approximate representation.

in a chain have a significant dynamic response or when the solution of a differential equation is being performed by the computer. As an example of error propagation in a system having dynamic response, consider a uniform random (white) noise error that is applied to an amplifier having a band-pass characteristic. The components that follow the amplifier will then receive an error signal that has a uniform distribution over the pass band and is zero for all frequencies outside the pass band. Such a limitation of noise error by the band-pass characteristic of a tracking receiver and servo system is shown by the tracking-system example of par 4-4.4.4.

Another example of the propagation of dynamic errors is afforded by a computing servo that has an underdamped quadratic response, producing a peak in the frequency response at

the damped natural frequency. If the input error has components in the vicinity of the peak frequency, these components will be amplified in passing through the servo.

In some cases, the dynamic response of a computer element may be such as to amplify even a static error at its input. For example, rate-servo or feedback-amplifier integrators are commonly employed in differential analyzers. A fixed error applied to the input of such an integrator will induce a constant rate of change in the output; if the error persists, a very large output error is generated. In general, the output error of an integrator is proportional to the area under the curve of the input error signal and is also inversely proportional to the frequency.

The general method for determining the propagation of errors in such. dynamic systems is developed in par 4-4.4.3. In cases such as those just given as examples, it would be necessary to employ this method.

The paragraphs which follow give suggestions as to the specification of errors in some of the more significant computer components. No attempt has been made to be exhaustive but the ideas are generally applicable to other components. Analog and digital components are discussed separately. The analog-component discussion emphasizes the unique problems of a-c analog computers which are particularly applicable to fire control problems.

4-4.6.2 Errors in Analog Components

The reduction of uncertainties and nonlinearities in analog devices can be accomplished by either the refinement of design and reduction of manufacturing tolerances or the ingenious application of new methods. The attainment of better levels of accuracy is, therefore, a slow and painful process. At the present state of development, under laboratory conditions maximum errors can be held to the order of magnitude of ±0.01% with some of the less complex components. Under field conditions, errors having the order of magnitude of ± 0.05% are more realistic.

A major problem in most analog devices is deterioration during use; for example, the wear in bearings and rubbing contacts, and the change of characteristics with age that is experienced with vacuum tubes, transistors, and, to a lesser extent, resistors and other components. Not only the initial error but also the error at the end of the useful life of the component must be considered.

4-4.6.2.1 Mechanical Elements

The significance of backlash, which may be caused by loose bearings or inaccuracy in gear cutting, has been mentioned previously. Differential gears, because of the many meshes and bearings, are particularly subject to backlash. The effect may be mitigated by operating the differential at a high gear ratio. Spring-loaded antibacklash gearing is sometimes employed in the more-sensitive locations.

Springs employed for summing or other computing functions are subject to hysteresis, particularly if improperly mounted. However, the uncertainty in a well-designed spring system can be lower than that in the most carefully built gearing.

Eccentricities in shafts, bearings, cams, gearing, etc. introduce nonlinearity errors, and may also contribute to backlash in gearing. Inaccurate gear cutting also produces both backlash and nonlinearity errors. Inaccuracies in high-speed gears may be treated as noise since the variations are generated at high frequencies.

Distortion of mechanical members under load may contribute to errors in those parts of the system, such as the weapon-pointing servos, that undergo heavy loading. This is seldom a problem in the computing elements, however, which tend to be lightly loaded. The weapon-pointing system may also have dynamic errors, caused by vibrations induced by the rapid motion of heavy masses. These problems will be discussed in Section 4. The only dynamic errors likely to be troublesome in mechanical computer elements are the effects of accelerations on linkages and cams in parts of the system that must follow rapidly chang-

ing signals. Design techniques for avoiding these acceleration errors have been worked out in considerable detail.[54]

4-4.6.2.2 Servos

Instrument servos are widely used to convert signals from electrical to mechanical form, and for the generation of such functions as multiplication and integration. The servo performance may be specified by the permissible static error and the required bandwidth, or, alternatively, by the position, velocity, and acceleration error constants, which may be thought of as determining the error components generated by the derivatives of the input; thus

$$\epsilon(t) = \frac{1}{1 + K_p} x(t) + \frac{1}{K_v} \dot{x}(t) + \frac{1}{K_a} \ddot{x}(t) + \ldots \qquad (4\text{-}256)$$

where

$\epsilon(t)$ = servo error
$x(t)$ = input signal
K_p = position error constant
K_v = velocity error constant
K_a = acceleration error constant.

Since the servos are lightly loaded, it is usually not difficult to achieve the desired performance, unless extremely rapid response is required. Information on servo design techniques will be found in other parts of the Engineering Design Handbook Series.[40,48]

It is usually desirable to specify separately the errors of potentiometers, tachometers, resolvers, and similar elements used for feedback or function generation in connection with servos. The sections which follow present a brief survey of significant sources of error in some of the more important analog-computer components.

4-4.6.2.3 Potentiometers*

Wirewound potentiometers are limited in their accuracy by the number of turns in the winding, which determines the resolution or uncertainty level, The development of helical winding techniques has made it possible to compress a very long winding into a small volume, making practicable potentiometers of high resolution and excellent linearity. Improved housings have reduced the eccentricity errors and improved contact materials have increasedthe life to several million revolutions. Great care must be given to the computer design so that the resistive load on the potentiometer that stems from the circuit into which it feeds does not introduce nonlinearity errors; circuits designed to reduce the loading effect will be described in Section 3.

Nonlinear functions can be generated by (1) potentiometers with shaped windings, (2) tapped linear potentiometers with resistive loads connected to the taps, or (3) tapped linear potentiometers with voltage sources connected to the taps. In addition to the errors inherent in any potentiometer, nonlinear potentiometers have errors introduced by inaccuracies in the shaping of the winding. In the case of tapped potentiometers, errors are introduced by inaccuracies in the location of the taps and by the approximation of the function by straight-line or parabolic segments.

4-4.6.2.4 Resolvers and Synchros†

A resolver is a device that generates voltages proportional to the sine and cosine of its

* See Section 11-2 in Chapter 11 of Reference *36*.

† See Section 11-3 in Chapter 11 of Reference **36**.

shaft angle. When shaped potentiometers are employed as resolvers, the considerations given in the preceding paragraph apply. Electromagnetic resolvers and synchros are quite similar to one another, differing only in the arrangment of the windings. They have errors that are invariant with shaft angle, caused by transformer coupling; such errors may be compensated by summing the error with a voltage having the same magnitude but opposite phase angle. Errors that cannot be so compensated are those, caused by magnetic anomalies and winding inaccuracies, that vary with shaft angle. In addition, the magnetic circuit induces odd harmonic voltages and an error voltage proportional to shaft speed is generated.

4-4.6.2.5 Tachometers*:

Tachometers employed in analog computers are either permanent-magnet-field d-c generators, or drag-cup-type a-c generators. The d-c type suffers from voltage fluctuations (caused by commutation) at low speeds. The a-c types also have low-speed fluctuations but at lower levels. Fixed errors are produced by transformer coupling, and small voltages proportional to shaft acceleration are generated.

4-4.6.2.6 Operational Amplifiers

High-gain d-c feedback amplifiers are employed for summation, integration, isolation, and for other functions in analog computers. The major source of error in such amplifiers is drift of the output when the input voltage is zero. Drift is minimized by the use of chopper stabilization, by good regulation of supply voltages, and by temperature control of critical components.

4-4.6.2.7 Voltage Supplies for Analog Components

If the reference voltage — i.e., the source of the analog signal voltages in an electrical analog computer — varies, errors may be introduced simultaneously in many points of the computation. In addition, power supplies that provide electrode voltages for electronic computer components must be extremely well regulated in order to avoid the generation of drift errors.

4-4.6.2.8 A-C Computers

Analog computers employing a-c signals are subject to errors caused by phase shift in the signal voltages. If two nearly-equal signals having a small phase difference are subtracted, the error in the in-phase component of the output will be very small but there will be a residual quadrature component. This component can be removed from the computer output by means of a quadrature-elimination circuit but care must be taken so that the dynamic range of amplifiers in high-gain parts of the computer is not severely reduced by the presence of large amounts of quadrature signal. For this reason, a-c computer circuits are customarily provided with phase-compensating adjustments, and quadrature-elimination circuits may be introduced at points of high gain.

4-4.6.2.9 Gyroscopes

Gyroscopes (usually referred to simply as gyros) are employed in fire control systems to measure the angular rate of a tracking device, or to provide a stable vertical reference when a mobile base is present. Gyros must be classed as analog devices, although a digital encoder may be employed in order to provide a digital output signal.

* See Section 11-5 in Chapter 11 of Reference 36.

Two types of gyros have been employed in fire control systems. The first is the two-degree-of-freedom type, which serves as a vertical reference or as a directional reference, depending on the orientation of its axes. The second type is the single-degree-of-freedom rate-integrating type of gyro. This type can be employed to measure the angular rate about a tracking axis, or three units can be arranged on a platform with accompanying platform servos to form a stable reference. Gyros with an attached pendulous mass have been employed as accelerometers in guided-missile applications. Future uses for this type of device will possibly be found for fire control systems.

The principal sources of error in gyros are mass unbalance, friction in pivots, vibration, and nonidealities in the electrical pickoffs. In addition, all types of gyros sense the component of the earth's angular velocity that is directed along an input axis; usually, however, the resulting error is insignificant in fire control systems. The other sources of error noted are discussed in the paragraphs which follow.

Mass unbalance introduces disturbing torques whenever the unbalanced mass is accelerated, either by motion of the gyro base or by gravity, At constant temperature, the unbalance can be adjusted to a minimum determined by the precision of measurement that is available. (The difficulty of balancing two-degree-of-freedom gyros is much greater than for single-degree-of-freedom gyros.) A major source of mass unbalance is the shifting of the rotor position that arises chiefly from end-play in the rotor bearings. Even with preloaded bearings, the rotor position may shift somewhat under acceleration loading against the elasticity of the bearings. Gyro balance will also shift with temperature because of the unequal temperature coefficients of expansion of the various gyro parts. Many precision gyros are accordingly temperature-compensated.

The other major source of disturbing torques is the frictional or elastic coupling arising from the gimbal pivots and the electrical connections to the gimbals. Usually of the greatest magnitude is the friction torque of the gimbal pivots. Conventional ball bearings are inexpensive but have poor frictional characteristics for this application. Pivot bearings are better but are easily damaged. Flotation of the gimbal has been employed with pivot bearings. This achieves both protection of the bearing and a reduction of the load on the bearing, thus further reducing the friction level. A large reduction in the error due to pivot friction can be secured if fluid bearings are introduced. Either air or hydraulic fluid is employed. Recent gyro developments have seen the introduction of magnetic supports, in which an electromagnetic field is arranged so as to provide a uniform radial force on the gimbal, directed toward the support axis. Electrostatic fields are also employed. The only error-producing torques then remaining are the residual tangential magnetic fields of the support coils and of any magnetic pickoffs, friction torques from slip-rings or potentiometer pickoffs, and elastic torques that might arise from pigtail leads and from the non-Newtonian behavior of damping and flotation fluids.

Vibration, due principally to unbalance and bearing defects in the rotor, introduces noise errors. Roughness of potentiometer windings and magnetic anomalies in magnetic pickoffs are also sources of noise error. A major effect of all these noise errors is to mask the small gyro output signal near a null. This null error is the limit of resolution for the gyro. Pickoff nonlinearities can produce large errors; however, many applications employ the gyro in a null-seeking system — e.g., the stable-platform systems. In such systems, nonlinearity of the pickoff is of minor significance.

4-4.6.3 Errors in Digital Components

Digital computing elements are not subject to error in the same sense that analog computing elements are. The fundamental digital computing elements are either storage elements or logical elements, and all such elements have the property that they can assume only two states — 0 or 1. Consequently, the only error that can occur in such an element is the loss of a bit. Depending on the significance of the digit in which it occurred, such an error might be either negligible or catastrophic. Therefore, much effort has gone into the

development of error-detection and error-correcting codes and circuits, and of redundant circuitry, all of which are more fully described in Chapter 5 of Section 3 of the Fire Control Series,

The use of error-checking means is *so* essential and so highly developed that the errors present in the fundamental digital computing elements do not merit further discussion here. The error sources that will be discussed in connection with digital computing elements involve (1) problems of providing inputs to, and obtaining outputs from, an element — giving rise to sampling errors in time-varying data; (2) problems arising from the approximation of continuous functions by discrete elements, thereby introducing truncation errors; and (3) similar problems. These error sources are found in both simple and complex digital computing elements. They are discussed in the paragraphs which follow under the headings Dynamic Errors and Static Errors.

4-4.6.3.1 Dynamic Errors

The use of digital computing elements in fire control systems requires that the input signals, initially analog in nature, be converted to digital form. This conversion is performed by sampling the electrical or mechanical signal at regular intervals of time and then quantizing the samples to yield a digital representation. For example, a mechanical shaft angle can be digitized by coupling to it a brush that rides over a fixed commutator. At regular intervals, a voltage pulse is applied to the brush, and the pulse then appears at a particular commutator bar which is determined by the shaft angle at that instant. Since each commutator bar is assigned a numerical value, the device described produces a digital representation of shaft angle.

That the sampling process introduces dynamic errors is evident since, for a fixed sampling rate, the amount of information lost between sampling intervals is a function of the frequency of the input signal, If the sampling frequency is less than twice the maximum frequency of importance in the system, serious distortion will result. On the other hand, the digital computer must complete a set of calculations on one sample before the end of the sampling interval in order that the registers be cleared to accept the next sample. A higher sampling rate thus requires that the designer employ either higher-speed computer components, or more duplication of components in order to reduce the amount of time-sharing; in either case, the computer becomes more expensive. Therefore, a compromise must be made between the conflicting desires to minimize the size and complexity of the computer, and to minimize the dynamic error introduced by the sampling interval. In the analytical work associated with this design problem, the digital computer can be represented by a simple system composed of an impulse modulator (representing the sampling process) and a pure time delay.*

4-4.6.3.2 Static Errors

In contrast to the analog computer elements, digital elements can be designed to have any accuracy desired, subject only to limitations of computation speed and circuit complexity. The task of the system designer is, therefore, to achieve a compromise between accuracy, speed, and complexity.

The digital computation is also subject to round-off errors which occur at every stage of the computation because of the limited capacity of the registers.† Depending on the equipment design, individual round-off errors will be at most the value of the least significant digit but successive round-offs may build up the error. If the difference of two large

* See pages 861-869 of Reference 1.

† As is discussed fully in Section 3, computation in a digital computer is carried out in an arithmetic unit that performs the basic operations of addition, subtraction, multiplication, and division. The results of each individual computation are stored in registers for later use, while the arithmetic unit is employed in other computational steps.

rounded-off numbers is taken, the error in the result may be excessive. Many of these difficulties can be corrected by improvements in the computer program.

A continuous function can be represented in a digital computer by the storage of a series of stepwise values or, more efficiently, by the computation in a subroutine of a series approximation to the function. Differential equations are solved in a digital computer by programming one of several methods of numerical integration.* These methods involve the computation of small increments in the function, employing series expansions, and, in some cases, a knowledge of the preceding increments. Naturally, as few terms as possible are retained in the series; therefore, the errors caused by truncation of the series must be given serious consideration. Truncation errors are minimized by reducing the interval in the input variable or by increasing the number of terms in the series. Care must be exercised, however, that the greater complexity of the program does not increase the round-off errors.

The following simple example will serve to illustrate the variation in the truncation error achieved by adjusting the interval in the input variable. Figure 4-52 illustrates a continuous function f(x) plotted as a function of the variable x. In the simplest digital computing scheme, this continuous function is represented in a stepwise manner, with the value of f(x) occurring at the beginning of any interval, Ax, held constant throughout the interval. The difference between the continuous curve and the approximation (this difference is shown shaded in Fig. 4-52) is the truncation error. It is evident from the figure that a reduction in **Ax** will decrease the truncation error, while an increase in **Ax** will increase this error.

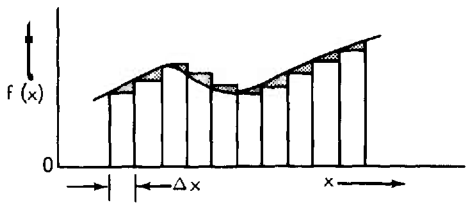


Figure 4-52. A simple example of truncation error.

4-5 MECHANIZATION OF THE MATHEMATICAL MODEL†

4-5.1 NATURE OF THE MECHANIZATION PROBLEM

4-5.1.1 General Considerations

The mechanization of the mathematical model of a fire control system involves, in the broad sense, the embodiment in physical hardware of the equations that describe the system. Presumably, previous study of this mathematical model (see par 4-3) has satisfied the system designer that the proposed system is capable of meeting the specified requirements. The actual process of mechanization includes the following steps:

1. Selecting suitable standard components
2. Preparing detailed drawings and parts list
3. Fabricating the necessary nonstandard components
4. Assembling the complete system
5. Testing the complete system.

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* See pages 232–246 of Reference 1.

† By W. W. Seifert.

If the fire control system concerned involves any significant research or development effort, it is seldom possible to mechanize it into its final form in one step. Instead, those concerned with the physical design – i.e., the equipment designers – should work closely with the system designers – particularly the mathematical analysts – to examine those portions of the system that may present difficulties, or at least need to be verified because they incorporate new techniques or are pushing the state of the art in some respect.

4-5.1.2 Departure from Nominal Procedure

In many systems, major elements in the mathematical model are dictated by the selection of major physical components. This situation results when the use of particular components is spelled out in the specifications or the number of existing alternatives is so small as to narrow the selection to a few possibilities and to permit essentially no control of the parameters of these units. Typical examples are the requirement that a fire control system be built around a certain gun with a particular power drive or the requirement that a fire control system derive tracking data from a particular radar. If one or more major physical components are specified and their performance characteristics are well understood, the mathematical analyst may be able to formulate a suitable ideal mathematical model based upon existing data (see par 4-3) and then proceed in the development of the mathematical model to optimize the overall system while leaving unchanged the specified elements of the fire control system. Frequently, additional data must be obtained before an adequate mathematical model can be completely established; particularly, if the performance of the overall fire control system is dependent in a major way upon the characteristics of the particular element involved. Thus, the testing of the components to be included in a system frequently becomes a necessary phase of the mechanization of a mathematical model because components may be used in an unconventional manner or may be *so* new that their characteristics are not well documented. The problems of formulating and mechanizing the mathematical model become completely intertwined in such cases.

4-5.1.3 Synthesis Problems

The overall task of mechanizing a fire control system may be subdivided into the mechanization of its three subsystems, i.e.: (1) the acquisition and tracking system, (2) the computing system, and (3) the weapon-pointing system. However, since the overall mechanization is a systems problem, the interaction between these three subsystems must be considered. Accordingly, system modifications that might be effected by combining into a single unit functions of components that might normally be thought of as falling into different subsystems should be examined. This is a synthesis problem and, like many synthesis problems, no unique relationship exists that leads from the mathematical description of a system to the physical embodiment of that system. It is exactly this problem of nonuniqueness that makes this portion of the system-design problem difficult and that places such a premium on engineering know-how. The inverse problem of developing a mathematical description for an existing physical system, on the other hand, is purely an analysis problem and is completely deterministic, i.e., there is only one solution to the problem.

A typical synthesis problem that might arise in connection with the realization of a fire control system is the determination of the most appropriate means of obtaining tracking-rate data. Such a problem would involve judicious consideration of the following questions:

1. Should tachometers be installed on each axis of the tracking system to derive relative-rate information?
2. Should rate gyros be installed to measure rates with respect to an inertial frame of reference?
3. Should only angular-position transducers be provided and tracking-rate data be generated by suitable arrangements within the computer?

Each of these alternatives utilizes different instrumentation and each imposes different

requirements on the computing system. Only be carefully appraising the accuracies of the available instruments, the over-all demands upon the computing system, and the economics involved can the analysts and those responsible for the mechanization of the system, working closely together, arrive at a truly optimum system.

In many fire control systems relatively little interaction occurs between the tracking, computing, and weapon-pointing systems. Therefore, except for the immediate interface problems associated with seeing that the signals supplied by the tracking system are acceptable for use by the computing system and that the computer output is satisfactory for activating the weapon-pointing equipment, the design of each of these major subsystems can be separated to a large degree. Division of responsibility along these major-system subdivision lines expedites execution of the overall job but imposes a responsibility on the manager of the overall project to see that changes introduced by one team do not impose different requirements on the work of one of the other teams.

4-5.2 DESIGN CONSIDERATIONS ASSOCIATED WITH ACQUISITION AND TRACKING SYSTEMS AND WITH WEAPON-POINTING SYSTEMS

In general, it can be stated that both acquisition and tracking systems, and weapon-pointing systems are essentially following devices. Accordingly, servomechanism principles[55] are primarily involved in achieving designs for these two fire-control subsystems that will meet the specified speed and accuracy requirements. The particular design problems that relate to the mechanization of acquisition and tracking systems are covered in detail in Section 2 of the Fire Control Series.'!' Those relating to the mechanization of weapon-pointing systems, on the other hand, are covered in Section 4 of the Fire Control Series.† Inasmuch as servomechanism principles as they apply to ordnance design have been so adequately documented in the Servomechanism Series[55] and because no major fire-control design problems are expected to occur in this area, further discussion of these concepts here is not deemed necessary.

4-5.3 DESIGN CONSIDERATIONS ASSOCIATED WITH COMPUTING SYSTEMS

4-5.3.1 General Considerations

While the design principles of two of the subsystems of a fire control system are reasonably straightforward and well documented, the same cannot be said of computing systems. It is in this area of technology that most of the mechanization problems of a fire control system arise. Therefore, although detailed consideration of the circuitry and operation of computers is covered in Section 3 of the Fire Control Series, it is nevertheless appropriate to point out here some of the major considerations that influence the mechanization of the computing system for any particular fire control system.

It is taken for granted that whatever other characteristics the computing system has, it must be capable – at least to a good approximation – of performing the mathematical operations described by the mathematical model. Although the detailed design considerations involved if the computer employed is of the analog type are quite different from those arising if a digital computer is to be used, three major considerations arise in either case.

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* With certain exceptions; e. g. , the detailed design of radar tracking equipment is not covered in Section 2, inasmuch as this type of equipment comes under the cognizance of the Electronics Command.

† The following decisions have been made in regard to the "gray areas" between the subject of gun carriages and mounts and the subject of fire control equipment:

(a) The subject of power controls used for controlling the elevating and traversing mechanisms of various weapons will be covered in Section 4 (Weapon-Pointing Systems) of the Fire Control Series.

(b) The subject of the cumulative errors due to misalignments, backlash, etc., in carriages and mounts that can cause trouble for the fire-control designer are covered in Ref. 68.

These can be classified broadly as follows:

1. Accuracy considerations
2. Speed considerations
3. Logical-arrangement considerations.

4-5.3.2 Accuracy Considerations

It is rather obvious that unless the computing system is capable of providing commands to the weapon-pointing system with sufficient accuracy to permit hits, the overall fire control system will not be satisfactory. On the other hand, with the present state of the art, precise estimation of the manner in which errors propagate through either an analog computer or a digital computer is very difficult to determine. With presently available components, relatively little difficulty is experienced in limiting the errors in any single operation on an analog computer to the order of 0.1 to 0.25 percent; in spite of great effort, however, reduction of individual errors below 0.005 percent is extremely difficult. The precise manner in which individual errors affect an overall solution depends implicitly on the computer configuration and discussion of this problem is postponed until Section 3.

In the case of a digital computer, errors are introduced as a result of round-off and truncation. The term "round-off" refers to the fact that the number of bits or digits that can be carried in any digital computer is limited, and hence errors are introduced whenever it is necessary to round a number off to the machine word length. When only a few operations are required, the problem of round-off is usually not serious. When many calculations are involved, however, round-off errors can accumulate to a serious degree — even in machines having quite long word lengths (32 to 36 binary bits). Truncation errors result from the fact that digital computations are carried on in a step-by-step manner. As described in par 4-4.6.3.2, a stepwise approximation is, in mathematical terms, an approximation by a truncated series. The truncation errors are affected both by the number of terms retained in the series and by the length of the interval. Although the errors introduced in a computer solution by round-off and truncation are quite difficult to compute, some attempt to appraise their effects should be made before any design is accepted because these effects can become substantial.

4-5.3.3 Speed Considerations

Speed and accuracy requirements are intimately interrelated, particularly in a digital computer. Nonetheless, in many applications, one or the other of these problems takes precedence, As an example of this, consider the fact that the computer in a fire control system must operate in real time, even if this results in some reduction in accuracy. This means that the computing interval and the integration or extrapolation rules must be selected in such a manner as to permit real-time operation with the computing circuitry that has been selected. In the analog case, the slower elements — such as servos, function generators, and multipliers — must not introduce dynamic-response delays that will render the computation ineffective. If it is suspected that the computing delays may have an important effect on system accuracy, a proper account of them should be included in the mathematical model for the system and their effect evaluated.

4-5.3.4 Logical-Arrangement Considerations

The problem of the logical arrangement of a computer is tied up with each of the two aforenoted factors: accuracy and speed. Nonetheless, several alternative means may exist for mechanizing specified mathematical functions. The logical arrangement used to carry out coordinate transformations is a particularly good example of this type of problem as it occurs in the design of a fire control system. Transformations may be instrurnented by any one of the following three means:

1. By direct mechanical analogy
2. By computations employing Euler angles
3. By computations involving direction cosines.

The first means would employ a suitably gimbaled and instrumented stable platform, Basically, this would be an analog device, even though digital pick-offs might be employed. The second means — computations based upon Euler angles — lends itself well to mechanization by means of instrument servos and resolvers. For the third means — computations employing direction cosines — the use of multipliers, which may be either analog or digital devices, would be required. The method most suitable for a particular application can be determined only in the light of the specific type of equipment and requirements associated with that application.

The introduction of ballistic-correction data illustrates another situation where several alternatives may exist. Here, since nonlinear functions are involved, some method of approximation may be introduced to permit direct analytical approximation of the functions in some simple form — such as a power series or a piecewise linear approximation based upon storage of information concerning the function and possibly its derivative at particular points. Selection of the more advantageous of these two methods in an actual computing system would depend on the characteristics of the particular nonlinear function to be represented and on the characteristics of the computer itself,

4-5.4 OPERATIONAL CONSIDERATIONS

In the preceding discussion, the attempt has been made to develop the idea that several alternative means may exist for mechanizing many of the operations called for in the mathematical model of a system, and that choice of the best possible alternative depends upon the characteristics of the specific system being developed and on the characteristics of the specific physical components that might be used in the mechanization of the system. Thus far, attention has been directed primarily at those aspects of the mechanization that influence the mathematical performance of the system. However, the ultimate system must meet the requirements imposed by operational use in the field, Frequently, it is advantageous to relax the operational requirements on the preliminary model for a fire control system and to concentrate on demonstrating the basic feasibility of the system. Thus, it is very likely that the first complete system will bear little resemblance to a system that would be acceptable for actual operational use in the field. This preliminary model, or prototype, might employ some components that would not meet all the environmental requirements that would be imposed on the final system, or it might lack some of the test or checkout features that would be essential to its satisfactory performance in combat use, However, such a prototype could be evaluated and any necessary design changes worked out before incurring the expense concomitant with meeting these broader requirements. The next step may involve production of a limited number of complete units for actual field evaluation by service personnel. Only after these units have been pronounced satisfactory or suitable design changes have been incorporated would production of the final operational systems be begun and mechanization of the mathematical model be brought to its ultimate conclusion.

4-6 ERROR ANALYSIS OF THE VIGILANTE ANTIAIRCRAFT WEAPON SYSTEM *

4-6.1 INTRODUCTION

Preceding paragraphs of Chapter 4 have indicated the usefulness of mathematical models and have described in considerable detail analytical methods for determining the hit probability, or accuracy, predicted by the use of such models of fire control systems. In order to relate this material more closely to the problems faced by the fire control system designer,†

*By E. St. George, Jr. This material is based primarily on References 58 through 67.

†Or team of system designers; *see* par 4-1.

an example of the application of these techniques to the design of an actual fire control system is presented in the paragraphs which follow.

As a first step, it is desirable to review the stages through which a fire-control-system design normally passes. The design of a fire control system follows a regular progression from initial military requirement to delivery of production units. It is important that the fire-control-system designer understand these steps and also the relationship of his part of the weapon-system design to the whole weapon-system design effort.

The development of a weapon system starts with the issuance of a Military Requirement by one of the Using Arms. This is a largely nontechnical description of a military need, and does not specify equipment. Sometimes an engagement kill probability will be specified, which can then be converted to an engagement hit probability by the procedures described in par 4-4.13. In general, however, no model* will have been worked out at this time.

The engineering phases of the development start with the preparation of an Engineering Specification by one of the AMC agencies. The Engineering Specification converts the Military Requirement into engineering terms, but still without specifying actual equipment. Simplified models, particularly of the geometry of the military problem, are employed at this stage.

With the basic specifications determined, the project comes to the stage of a Feasibility Study. This study is either performed at an AMC agency, or contracted to industry under AMC supervision. It is common to institute parallel, competitive programs at this stage.

The Feasibility Study is an engineering design study in the course of which (1) the design problem is analyzed, (2) systems are proposed and their predicted performances compared, and (3) the system finally chosen is analyzed in detail. Mathematical models are employed extensively, and become more detailed as the design progresses. The final result of the Feasibility Study is a proposal (or set of proposals) to develop the hardware of a specific system.

Accuracy studies† are made in the course of the Feasibility Study, and at the close of this phase a thoroughgoing accuracy evaluation is made as part of the overall evaluation of the proposals. As a result of this evaluation, one system is chosen for further development. At this point, bids for the development of the chosen system may be solicited. Frequently, however, considerations such as the reduction of development time or cost to the Government, or the inclusion of proprietary items in the winning proposal, may dictate the negotiation of a contract with the winner of the feasibility competition.

The next two development stages, Engineering Design and Component Development, are to a great extent carried out simultaneously. As test results on the components become available, they are used in designing improvements to the original system. In addition, these test results permit the formulation of more-realistic mathematical models and better estimates of the hit probability that can be achieved.

If these estimates of hit probability indicate a good likelihood that a successful design has been achieved, a test prototype is assembled and delivered by the contractor for the phases of System Demonstration and Engineering Tests. The engineering tests are performed by one of the Army proving grounds. Although the system designer is less actively concerned with this phase, his advice is of great importance in formulating the tests so that (1) any potential deficiencies may be exposed, and (2) tests can be designed to provide data to support other programs, e.g., reliability. If deficiencies are uncovered, further analysis and testing may be required. Finally, a comparison of the test results with the system analysis should provide the designer with ideas for improved analysis in future projects.

The final phases of the system development will be only briefly noted since they rarely involve the system designer. These phases are as follows:

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*See Par 4-2 for a discussion of the various types of models employed during the course of developing any complex system.

†It should be noted that the term "accuracy study" is equivalent to the term "error analysis" and these terms can therefore be used interchangeably.

1. User Test, i.e., testing by or for the using arm of the service
2. Release for Industry
3. Industrial Engineering
4. Production

The fire control portion of the Vigilante Antiaircraft Weapon System provides an excellent example of the application to a relatively complex fire control system of the various error-analysis techniques discussed earlier in this chapter (see par 4-4.4). The Vigilante Antiaircraft Weapon System is a highly mobile, self-contained system designed primarily to operate in defense against low-flying,* high-speed aircraft in forward areas. A secondary mission is employment against ground targets. The system had its inception in a 37 mm weapon system. Design studies for this system were prepared by selected contractors; following which, one of these contractors was chosen to develop the system.

A thorough preliminary system analysis — including the formulation of mathematical models and the performance of accuracy studies — was, of course, carried out as a major part of the Feasibility Study phase. As the development proceeded, numerous analyses were made on special aspects of the system. For example, a simplified dynamic analysis of the tracking system and the computer was performed,† with the tracking operator simulated by a simple integration. As a result of this study, which was performed on an analog computer, the computer gain factors were optimized to provide minimum settling time with good damping and low sensitivity to noise.

As the development of a prototype system proceeded, some concern was felt for the accuracy of the system. Accordingly, Frankford Arsenal, as the responsible Government agency, undertook an accuracy study — using experimental data on the system components — in order to predict the overall engagement hit probability. This study was well documented[59] and is discussed in detail in the paragraphs which follow. With the incorporation of improvements suggested by the accuracy study, the prototype of the Vigilante Antiaircraft Weapon System was successfully completed.

The paragraphs below describe the general characteristics of the Vigilante Antiaircraft Weapon System and of the fire control system employed therein. They also describe the general methods of the Frankford Arsenal error analysis. For the details of the analysis and the computations involved, reference should be made to the original report (see Reference 59).

4-6.2 A BRIEF DESCRIPTION OF THE VIGILANTE ANTIAIRCRAFT WEAPON SYSTEM**

The Vigilante Antiaircraft Weapon System basically comprises a mobile 37 mm antiaircraft gun, a radar acquisition system, an optical tracking system, a computer, and a power supply. The gun is of the Gatling type, with six barrels and a magazine feed, making possible the firing of three one-second bursts of 48 rounds each before reloading. Either contact-fuzed ammunition or proximity-fuzed ammunition can be employed.

The entire Vigilante weapon system is mounted on either a tracked vehicle or a trailer. The trailer-mounted version is illustrated in Fig. 4-53.

4-6.3 THE FIRE CONTROL SYSTEM USED IN THE VIGILANTE ANTIAIRCRAFT WEAPON SYSTEM

During the search mode, the fire-control-system operator looks for possible targets on the plan-position-indicator (PPI) presentation of the radar which is of the pulsed-doppler

*The maximum effective range of the gun is 4500 yards.

† See Section VII of Reference 58.

**For more-detailed descriptive information relating to the Vigilante Antiaircraft Weapon System, see References 58, 60 and 61. For general background information, see par 1-3.3.7 of Chapter 1.

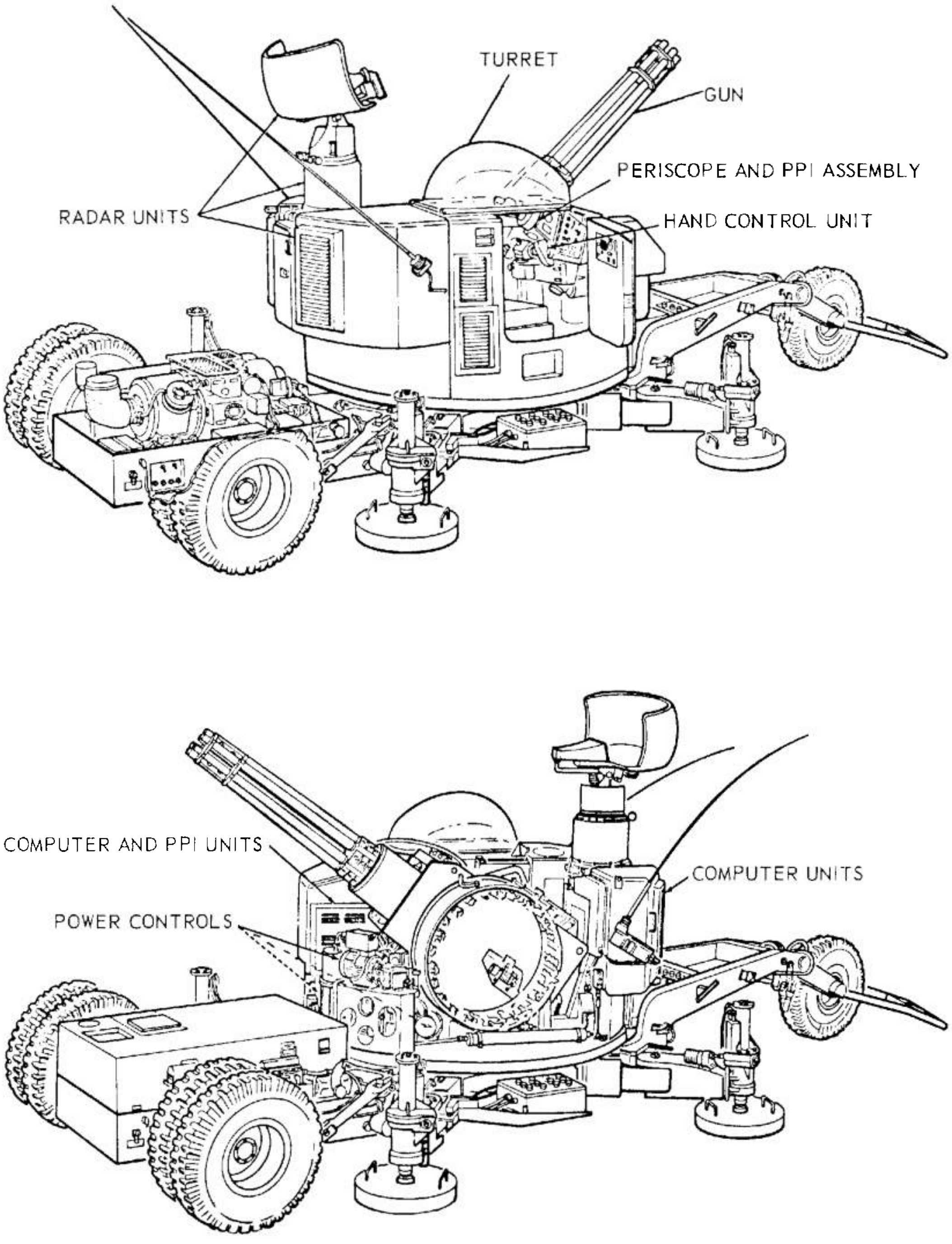


Figure 4-53. The trailer-mounted version of the Vigilante Antiaircraft Weapon System

type. When the radar detects a target, an earphone alarm alerts the operator who then acquires the target in azimuth by slewing the gun mount with his hand controls until the PPI cursor bisects the target. The PPI presentation of the radar and the periscope field of view of the optical tracking system are optically superposed. This arrangement enables the operator to search optically in elevation for the target as the track-while-scan circuitry of the radar provides automatic azimuth tracking information to the gun mount.

When the operator has acquired the target optically, he switches the weapon system to the "track" mode. The PPI is then blanked, leaving only the periscope view, but the radar does supply range-only information to the computer. The computer now begins computing the appropriate prediction-angle components based on the range data from the radar, tracking data from the optical tracking system, and ballistic data set in by hand by the operator. After the operator has tracked the target for a time long enough for the computer to settle (about 3 seconds), he may fire one 1-second burst of 48 rounds. During firing, the sight is shuttered to protect the operator from the flash of the gun, and the computer is switched to a regenerative tracking mode that continues the tracking rates last sensed.

The fire control system is shown diagrammatically in Fig. 4-2* as part of the overall weapon system. Not shown are such possible modifications considered for the system as the addition of infrared tracking and operation without radar range information.

The computer inputs are azimuth and elevation tracking data from the optical tracking system, range from the radar, and preset ballistic corrections. When switched to the regenerative mode, the computer operates without inputs for the 1-second firing period.

The heart of the computer is a two-degree-of-freedom gyroscope which is provided with torquers on both axes. During tracking, the gyro spin axis is constrained to follow the line of sight. The torquer signals then represent components of the velocity of the line of sight. Two auxiliary gyros are also provided: the first, a single-degree-of-freedom gyro, is used to determine the angular velocity about the spin axis of the primary gyro. The second auxiliary gyro provides a vertical reference for ballistic corrections; its use avoids the necessity for leveling the mount.

The operator views the target through a periscopic sight and, by means of hand controls, superposes the periscope reticle on the target. The periscope is driven by signals obtained from the combination of the hand-control signals and the position of the gyro spin axis; thus, when the operator is on target, the hand-control signals provide the angular tracking error signals to the computer. The operator is an inherent part of the tracking loop. For smoothing, the tracking error is integrated twice: first by rate servos, and secondly by the gyro precession induced by the torquers.

Once the line-of-sight tracking is established, the computer determines a vector that represents the future target position at the time of impact of the projectile. Components of this vector — combined with ballistic corrections — are used to position the gun in azimuth and elevation.

When firing is initiated by the operator, the hand controls are first disabled, and the computer operates in the regenerative mode, in which it reproduces the last tracking rates received. After 1/3 sec, firing commences, and continues for 1 sec, after which the normal tracking mode may be reestablished.

4-6.4 THEORY OF OPERATION OF THE COMPUTER

The computer, which is of the electromechanical analog type (see Chapter 6 of Section 3), may be considered to be made up of a tracking section and a prediction section. The operation of each section is described in the paragraphs which follow.

The mechanical components of the fire control system require the resolution of vectors between a number of coordinate systems. These resolutions will be described as they appear.

*Figure 4-2 appears in par 4-2 as an example of the category of models that is known as pictorial representations.

4-6.4.1 Tracking Section

The function of the tracking section is to generate a tracking line (coincident with the tracking vector or smooth range vector to the target $\vec{D}_s$ in Fig. 4-54) that will closely approximate the line of sight to the target (coincident with the instantaneous range vector to the target $\vec{D}_o$). The tracking line in the Vigilante Antiaircraft Weaponsystem might be represented by numerous mechanizations, but is in fact represented by the spin axis of a two-degree-of-freedom gyroscope. This gyroscope* is mounted on a gimballed platform that is servo-driven to follow the gyro spin axis. Componentsof the tracking vector $\vec{D}_s$ can thus be obtained from measurements of the platform orientation. The operator's handgrips control sight displacement, with only a coordinate transformation interposed, and the handgrip displacements are used as a measure of the instantaneous tracking error vector— $\vec{E}$ in Fig. 4-54.

The convention used in establishing the vector relationship between E, $\vec{D}_o$, and $\vec{D}_s$ that is shown in Fig. 4-54 is explained in Derivation 4-8 in the Appendix to Chapter 4. An alternative convention is also discussed therein. As emphasized, whichever convention is initially employed in an error analysis, the remainder of the analysis must rigidly adhere to this choice. Particular attention must be paid to this point when the system being analyzed is of such complexity that more than just a few people — of possibly differing backgrounds and preferences — are working on the analysis,

The simplest tracking system would be one in which the rate of change of the tracking line is proportional to the error. However, this system is characterized by a steady-state error that is proportional to the tracking rate. This error can be removed by the addition of an error-integrating term. Therefore, a tracking system in which tracking velocity is proportional to the tracking error plus the integral of the tracking error was employed. Such tracking systems are often described as second-order tracking systems, aided tracking systems, or aided laying systems.

As noted earlier (see par 4-6.3), the operator's vision is obscured during firing to protect him from gun flash, and a regenerative mode is provided in the tracking section in order to continue the tracking rate last sensed. The basis of this operation is as follows: (1) as the error is nulled in normal tracking, the operator returns his handgrips to zero, and (2) when the handgrips are at zero and a switch-over to regenerative tracking is made, the system will continue to track the target as long as the line of sight to the target continues to have an approximately constant angular velocity.

The paragraphs below derive the expressions for the second-order regenerative tracking system, starting with the vector equation of normal second-order (error plus integral of error) tracking. This relationship is

$$\frac{d}{dt}\vec{D}_s = S_1\vec{E} + S_2\int\vec{E}\,dt \tag{4-257}$$

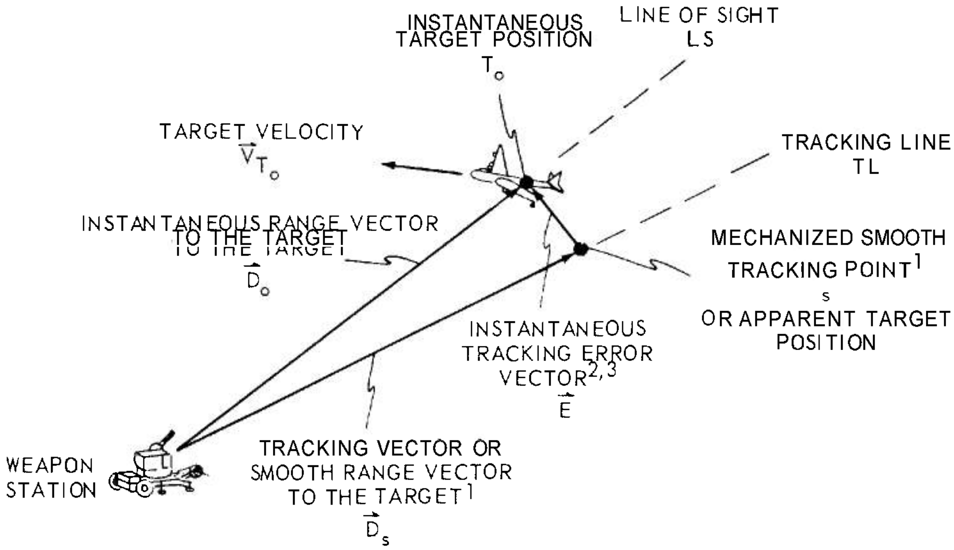
where, see Fig. 4-54, $\vec{D}_s$ = smooth range vector to the target

$$\vec{E} = \vec{D}_o - \vec{D}_s \tag{4-258}$$

= instantaneous tracking error vector

and S_1 and S_2 are constants whose values are so chosen that the tracking system will have a response time compatible with the requirements of a human operator and an optimum compromise between fast settling and noise attenuation.

*The theory of operation of the two-degree-of-freedom gyroscope (as well as that of the single-degree-of-freedom rate gyroscope also employed in the Vigilante Antiaircraft Weapon System) is discussed in par 11-6 of Reference 62. As indicated therein, a gyroscope is also commonly referred to as simply a gyro. This nomenclature will now be employed here for convenience.



THE FOLLOWING REMARKS KEY IN WITH THE SUPERSCRIPT NUMBERS ON THE NOMENCLATURE ABOVE

1. BECAUSE OF RANDOM INPUT ERRORS (NOISE) ASSOCIATED WITH TRACKING SYSTEMS (SEE PAR. 4-4.5.2), SMOOTHING IS GENERALLY REQUIRED IN THE TRACKING LOOP BY MEANS OF WHICH THE INSTANTANEOUS TARGET POSITION T_o IS TRACKED BY A MECHANIZED SMOOTH TRACKING POINT s. SEE AMCP 706-328 (SECTION 2 OF THE FIRE CONTROL SERIES) FOR A DISCUSSION OF SMOOTHING.
2. THE LENGTH OF THE INSTANTANEOUS TRACKING ERROR VECTOR $\vec{E}$ IS EXAGGERATED HERE FOR PURPOSES OF ILLUSTRATION. BY THE NATURE OF THE OPERATION OF A TRACKING SYSTEM $\vec{E}$ IS, OF COURSE, ACTUALLY A NULLED QUANTITY AND CAN THEREFORE BE EXPECTED TO BE QUITE SMALL – ESPECIALLY IN COMPARISON WITH THE RANGE VECTORS $\vec{D}_o$ AND $\vec{D}_s$.
3. AS SHOWN BY THE DIAGRAM, THE VECTOR RELATIONSHIP BETWEEN $\vec{E}$, $\vec{D}_o$ AND $\vec{D}_s$ IS

$$\vec{E} = \vec{D}_o - \vec{D}_s$$

THE CONVENTION UPON WHICH THIS TRACKING ERROR EQUATION IS BASED IS DISCUSSED IN DERIVATION 4-8 IN THE APPENDIX TO CHAPTER 4. THAT DERIVATION ALSO DISCUSSES AN ALTERNATIVE CONVENTION FOR ESTABLISHING THE DIRECTION OF THE TRACKING ERROR VECTOR $\vec{E}$. SEE AMCP 706-328 (SECTION 2 OF THE FIRE CONTROL SERIES) FOR A MORE DETAILED DISCUSSION OF TRACKING AND THE NATURE OF THE TRACKING ERROR VECTOR $\vec{E}$ IN PARTICULAR.

Figure 4-54. Vector relationships associated with tracking.

The integral term in Eq. 4-257 defines the smooth target velocity $\vec{V}_S$. Thus, for small values of $\vec{E}$,

$$\frac{d}{dt}\vec{D}_s \cong S_2 \int \vec{E}\, dt \stackrel{\Delta}{=} \bar{V}_s \qquad (4-259)$$

Also,

$$\vec{V}_s = S_2 \vec{E} \qquad (4-260)$$

The coordinate system employed for the vector tracking equation is shown in Fig. 4-55. As noted, the unit vector $\vec{2a}$ is chosen to be coincident with the tracking vector $\vec{D}_S$. The components of $\vec{E}$ in this coordinate system are defined by the relationship

$$\vec{E} = E_{1a}\,\vec{1a} + E_{2a}\,\vec{2a} + E_{3a}\,\vec{3a} \tag{4-261}$$

As shown by Fig. 4-55, E_{2a} is the range component of the tracking error vector $\vec{E}$. It represents the error in the range-only information supplied to the computer by the radar (see Fig. 4-2). The error components E_{1a} and E_{3a}, on the other hand, stem respectively from the components Δ_{1a} and Δ_{3a} of the instantaneous tracking error angle A, which is the misalignment between the tracking vector $\vec{D}_S$ and the instantaneous range vector $\vec{D}_O$ to the target. This misalignment results from the normal operation of the optical tracking system in conjunction with its human operator. See AMCP 706-328 (Section 2 of the Fire Control Series) for a more detailed discussion of the tracking process.

The gyro coordinate system shown in Fig. 4-55 rotates with respect to an inertial coordinate system with vector angular velocity $\vec{\omega}_a$ whose coordinates in the gyro coordinate frame are given by the relationship

$$\vec{\omega}_a = \omega_{1a}\,\vec{1a} + \omega_{2a}\,\vec{2a} + \omega_{3a}\,\vec{3a} \tag{4-262}$$

As already noted,

$$\vec{D}_s = D_s\,\vec{2a} \tag{4-263}$$

Similarly, the components of the velocity vector $\vec{V}_S$ in the gyro coordinate frame can be specified by the relationship

$$\vec{V}_s = V_{1a}\,\vec{1a} + V_{2a}\,\vec{2a} + V_{3a}\,\vec{3a} \tag{4-264}$$

For convenience, the components of $\vec{V}_S$ will be designated $D_S\,v_{1a}$, $D_S\,v_{2a}$, $D_S\,v_{3a}$, respectively; thus,

$$\vec{V}_s = D_s\,v_{1a}\,\vec{1a} + D_s\,v_{2a}\,\vec{2a} + D_s\,v_{3a}\,\vec{3a} \tag{4-264A}$$

where

$$v_{1a} = V_{1a}/D_s \tag{4-265}$$

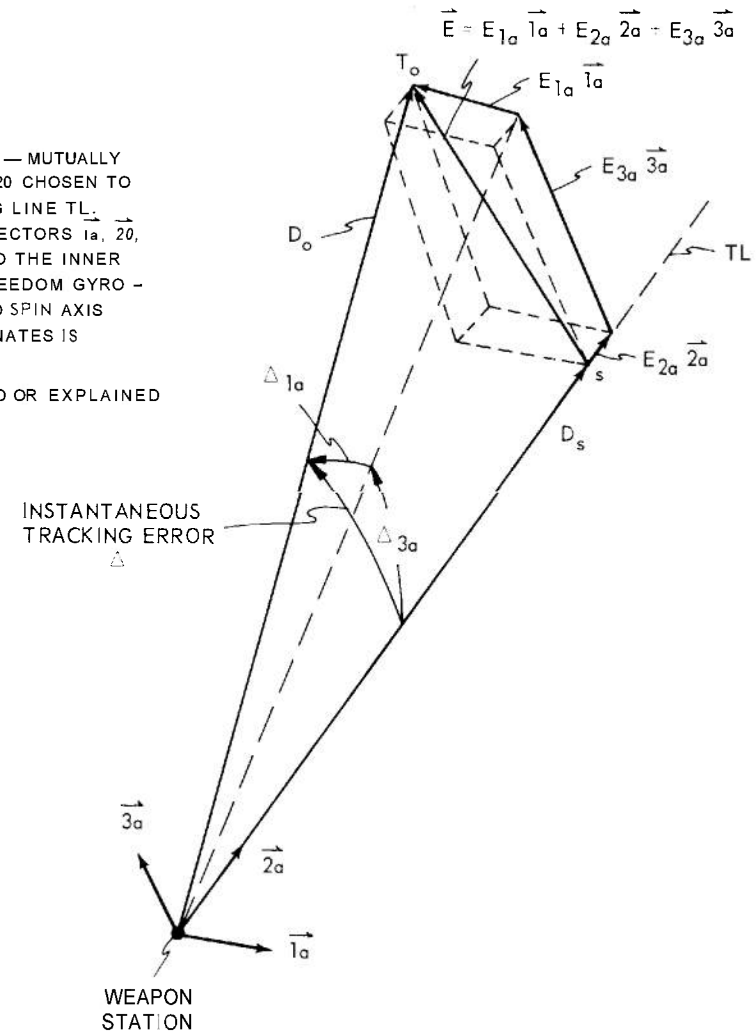
$$v_{2a} = V_{2a}/D_s \tag{4-266}$$

$$v_{3a} = V_{3a}/D_s \tag{4-267}$$

The quantities v_{1a}, v_{2a}, and v_{3a}, which have the dimensions of reciprocal time, are employed since they serve in effect to reduce the range of the velocity variables and thereby simplify the mechanization of the computer.

NOTES:

1. $\vec{1a}$, $\vec{2a}$, AND $\vec{3a}$ CONSTITUTE A SET OF — MUTUALLY ORTHOGONAL UNIT VECTORS, WITH 20 CHOSEN TO BE COINCIDENT WITH THE TRACKING LINE TL. SINCE (AS SHOWN IN FIG. 4-57) THE VECTORS $\vec{1a}$, $\vec{2a}$, AND $\vec{3a}$ ARE FIXED WITH RESPECT TO THE INNER GIMBAL OF THE TWO-DEGREE-OF-FREEDOM GYRO – WITH THE TRACKING LINE AND GYRO SPIN AXIS COINCIDENT – THIS SET OF COORDINATES IS CALLED THE GYRO COORDINATES.
2. QUANTITIES THAT ARE NOT DEFINED OR EXPLAINED HERE ARE DEFINED IN FIG. 4-54.



EXPLANATION:

E_{1a}, E_{2a}, AND E_{3a} ARE THE COMPONENTS OF THE INSTANTANEOUS TRACKING ERROR VECTOR $\vec{E}$ THAT ARE PARALLEL, RESPECTIVELY, TO THE UNIT VECTORS $\vec{1a}$, $\vec{2a}$, AND $\vec{3a}$. (FOR THE PARTICULAR TRACKING ERROR VECTOR SHOWN ABOVE, E_{1a} IS A NEGATIVE QUANTITY, WHILE E_{2a} AND E_{3a} ARE BOTH POSITIVE.) SINCE $\vec{2a}$ LIES ALONG THE TRACKING LINE, E_{2a} IS THE RANGE COMPONENT OF $\vec{E}$ AND IS EQUAL TO $D_o \cos \Delta - D_s$. THE TRACKING ERROR COMPONENTS E_{1a} AND E_{3a} STEM, RESPECTIVELY, FROM THE COMPONENTS Δ_{1a} AND Δ_{3a} OF THE INSTANTANEOUS TRACKING ERROR ANGLE Δ, WHICH IS THE MISALIGNMENT BETWEEN THE TRACKING VECTOR $\vec{D}_s$ AND THE INSTANTANEOUS RANGE VECTOR TO THE TARGET $\vec{D}_o$. INASMUCH AS THE MAGNITUDES OF $\vec{E}$ AND ITS COMPONENTS ARE VERY SMALL COMPARED WITH D_s THE COMPONENT Δ_{1a} IS APPROXIMATELY EQUAL TO E_{1a}/D_s AND Δ_{3a} IS APPROXIMATELY EQUAL TO E_{3a}/D_s.

Figure 4-55. Geometrical relationships associated with tracking error vector, the tracking error angle, their respective components, and the coordinate system used for the vector tracking equation.

Equation 4-257 can now be separated into components along $\overrightarrow{1a}$, $\overrightarrow{2a}$, and $\overrightarrow{3a}$. Substituting Eqs. 4-259, 4-261, and 4-264 into Eq. 4-257 shows that

$$\frac{d}{dt}\vec{D}_s = S_1\vec{E} + \vec{V}_s = (S_1E_{1a} + D_s v_{1a})\,\overrightarrow{1a} + (S_1E_{2a} + D_s v_{2a})\,\overrightarrow{2a} + (S_1E_{3a} + D_s v_{3a})\,\overrightarrow{3a} \tag{4-268}$$

Also, from Eq. 4-262,

$$\frac{d}{dt}\vec{D}_s = \frac{d}{dt}(D_s\,\overrightarrow{2a}) = \dot{D}_s\,\overrightarrow{2a} + D_s\,\dot{\overrightarrow{2a}} \tag{4-269}$$

where $\dot{\overrightarrow{2a}}$ is the time rate of change of the unit vector $\overrightarrow{2a}$.* Setting the scalars of $\overrightarrow{2a}$ equal in Eqs. 4-268 and 4-269 yields

$$\dot{D}_s = S_1E_{2a} + D_s v_{2a} \tag{4-270}$$

See par 4-6.5.2 for the manner in which this equation is mechanized to generate D_S.

The remaining term of Eq. 4-269 must equal the remaining terms of Eq. 4-268; therefore,

$$D_s\,\dot{\overrightarrow{2a}} = (S_1E_{1a} + D_s v_{1a})\,\overrightarrow{1a} + (S_1E_{3a} + D_s v_{3a})\,\overrightarrow{3a} \tag{4-271}$$

Note that $D_S\dot{\overrightarrow{2a}}$ has components along only the $\overrightarrow{1a}$ and $\overrightarrow{3a}$ unit vectors. It is, therefore, the component of the velocity of point s in Fig. 4-54 that is perpendicular to the tracking line TL which lies along the unit vector $\overrightarrow{2a}$ (see Fig. 4-55).

Next, Eq. 4-264 will be differentiated, yielding

$$\dot{\vec{V}}_s = V_{1a}\,\dot{\overline{1a}} + \dot{V}_{1a}\,\overline{1a} + V_{2a}\,\dot{\overrightarrow{2a}} + \dot{V}_{2a}\,\overrightarrow{2a} + V_{3a}\,\dot{\overrightarrow{3a}} + \dot{V}_{3a}\,\overrightarrow{3a} \tag{4-272}$$

where, as shown by Eqs. 4-265 through 4-267, $V_{1a} = D_S v_{1a}$, $V_{2a} = D_S v_{2a}$, and $V_{3a} = D_S v_{3a}$. The components of $\dot{\vec{V}}_S$ are defined by the relationship

$$\dot{\vec{V}}_s = (\dot{V}_s)_{1a}\,\overrightarrow{1a} + (\dot{V}_s)_{2a}\,\overrightarrow{2a} + (\dot{V}_s)_{3a}\,\overrightarrow{3a} \tag{4-273}$$

In order to separate Eq. 4-272 into components, specific expressions for the time derivatives of the three unit vectors are required. Since the unit-vector coordinate set rotates at

*In general, a vector can change with time in both magnitude and direction; i.e., the changes can be both along the vector and perpendicular to it. The length of a unit vector is fixed, however; hence, as shown shortly in specific mathematical relationships, $\dot{\overrightarrow{2a}}$ can have components along $\overrightarrow{1a}$ and $\overrightarrow{3a}$ but cannot have a component along $\overrightarrow{2a}$. (For a discussion of the derivative of a vector of constant length and the mathematics applicable thereto, see Section k in Definition Summary 1-1 of Chapter 1 of Reference **4.**)

the angular velocity $\vec{\omega}_a$ with respect to an inertial coordinate system, and since the unit vectors are invariant in magnitude, the unit vector time derivaties are of the form $\dot{\overrightarrow{1a}} = \vec{\omega}_a \times \overrightarrow{1a}$.* Thus,

$$\dot{\overrightarrow{1a}} = \vec{\omega}_a \times \overrightarrow{1a} = \begin{vmatrix} \overrightarrow{1a} & \overrightarrow{2a} & \overrightarrow{3a} \\ \omega_{1a} & \omega_{2a} & \omega_{3a} \\ 1 & 0 & 0 \end{vmatrix} = \begin{vmatrix} \overrightarrow{2a} & \overrightarrow{3a} \\ \omega_{2a} & \omega_{3a} \end{vmatrix} = \omega_{3a}\overrightarrow{2a} - \omega_{2a}\overrightarrow{3a} \tag{4-274}$$

where ω_{1a}, ω_{2a}, and ω_{3a} are, respectively, the components of the angular velocity $\vec{\omega}_a$ about $\overrightarrow{1a}$, $\overrightarrow{2a}$, and $\overrightarrow{3a}$. Similarly,

$$\dot{\overrightarrow{2a}} = \omega_{3a}\overrightarrow{1a} - \omega_{1a}\overrightarrow{3a} \tag{4-275}$$

and

$$\dot{\overrightarrow{3a}} = \omega_{2a}\overrightarrow{1a} - \omega_{1a}\overrightarrow{2a} \tag{4-276}$$

However, since $D_S\dot{\overrightarrow{2a}}$ is given in terms of the error components by Eq. 4-271, Eq. 4-272 can be evaluated by substituting from Eqs. 4-271, 4-274, and 4-276. This procedure yields

$$\begin{aligned}
\dot{\vec{V}}_s &= V_{1a}(\omega_{3a}\overrightarrow{2a} - \omega_{2a}\overrightarrow{3a}) + \dot{V}_{1a}\overrightarrow{1a} + v_{2a}(S_1 E_{1a} + D_s v_{1a})\overrightarrow{1a} \\
&\quad + v_{2a}(S_1 E_{3a} + D_s v_{3a})\overrightarrow{3a} + \dot{V}_{2a}\overrightarrow{2a} + V_{3a}(\omega_{2a}\overrightarrow{1a} - \omega_{1a}\overrightarrow{2a}) \\
&\quad + \dot{V}_{3a}\overrightarrow{3a} \\
&= (\dot{V}_{1a} + S_1 E_{1a} v_{2a} + D_s v_{1a} v_{2a} + V_{3a}\omega_{2a})\overrightarrow{1a} \\
&\quad + (V_{1a}\omega_{3a} + \dot{V}_{2a} - V_{3a}\omega_{1a})\overrightarrow{2a} \\
&\quad + (-V_{1a}\omega_{2a} + S_1 E_{3a} v_{2a} + D_s v_{2a} v_{3a} + \dot{V}_{3a})\overrightarrow{3a}.
\end{aligned} \tag{4-277}$$

The components of $\dot{\vec{V}}_S$ are then

$$\left.\begin{aligned}
(\dot{\vec{V}}_s)_{1a} &= \dot{V}_{1a} + S_1 E_{1a} v_{2a} + D_s v_{1a} v_{2a} + V_{3a}\omega_{2a} \\
(\dot{\vec{V}}_s)_{2a} &= \dot{V}_{2a} + V_{1a}\omega_{3a} - V_{3a}\omega_{1a} \\
(\dot{\vec{V}}_s)_{3a} &= \dot{V}_{3a} + S_1 E_{3a} v_{2a} + D_s v_{2a} v_{3a} - V_{1a}\omega_{2a}
\end{aligned}\right\} \tag{4-278}$$

*See, for example, Section k of Definition Summary 1-1 of Chapter 1 of Reference 4.

From the definition of V_{1a} (see Eq. 4-265) and substitution from Eq. 4-270,

$$\dot{V}_{1a} = \frac{d}{dt}(D_s v_{1a}) = \dot{D}_s v_{1a} + D_s \dot{v}_{1a}$$

$$= S_1 E_{2a} v_{1a} + D_s v_{1a} v_{2a} + D_s \dot{v}_{1a} \tag{4-279}$$

Similarly,

$$\dot{V}_{2a} = S_1 E_{2a} v_{2a} + D_s v_{2a}^2 + D_s \dot{v}_{2a} \tag{4-280}$$

and

$$\dot{V}_{3a} = S_1 E_{2a} v_{3a} + D_s v_{2a} v_{3a} + D_s \dot{v}_{3a} \tag{4-281}$$

Substitution from Eqs. 4-279 through 4-281 into Eqs. 4-278 yields

$$\left.\begin{aligned} (\dot{\vec{V}}_s)_{1a} &= 2D_s v_{1a} v_{2a} + D_s(\dot{v}_{1a} + v_{3a}\omega_{2a}) + S_1(E_{1a} v_{2a} + E_{2a} v_{1a}) \\ (\dot{\vec{V}}_s)_{2a} &= D_s v_{2a}^2 + D_s(\dot{v}_{2a} + v_{1a}\omega_{3a} - v_{3a}\omega_{1a}) + S_1 E_{2a} v_{2a} \\ (\dot{\vec{V}}_s)_{3a} &= 2D_s v_{2a} v_{3a} + D_s(\dot{v}_{3a} - v_{1a}\omega_{2a}) + S_1(E_{2a} v_{3a} + E_{3a} v_{2a}) \end{aligned}\right\} \tag{4-282}$$

Utilizing Eq. 4-261 in connection with Eq. 4-260 gives

$$\vec{V}_s = S_2 E_{1a} \overrightarrow{1a} + S_2 E_{2a} \overrightarrow{2a} + S_2 E_{3a} \overrightarrow{3a} \tag{4-283}$$

as a second expression for the components of $\vec{V}_S$. Equating the corresponding components of Eqs. 4-282 and 4-283 yields the relationships

$$\left.\begin{aligned} S_2 E_{1a} &= 2D_s v_{1a} v_{2a} + D_s(\dot{v}_{1a} + v_{3a}\omega_{2a}) + S_1(E_{1a} v_{2a} + E_{2a} v_{1a}) \\ S_2 E_{2a} &= D_s v_{2a}^2 + D_s(\dot{v}_{2a} + v_{1a}\omega_{3a} - v_{3a}\omega_{1a}) + S_1 E_{2a} v_{2a} \\ S_2 E_{3a} &= 2D_s v_{2a} v_{3a} + D_s(\dot{v}_{3a} - v_{1a}\omega_{2a}) + S_1(E_{2a} v_{3a} + E_{3a} v_{2a}) \end{aligned}\right\} \tag{4-284}$$

or, solving for $\dot{v}_{1a}$, $\dot{v}_{2a}$, and v_{3a}:

$$\left.\begin{aligned}
\dot{v}_{1a} &= \frac{S_2 E_{1a}}{D_s} - 2v_{1a}v_{2a} - v_{3a}\omega_{2a} - \frac{S_1}{D_s}(E_{1a}v_{2a} + E_{2a}v_{1a}) \\
\dot{v}_{2a} &= \frac{S_2 E_{2a}}{D_s} - v_{2a}^2 - v_{1a}\omega_{3a} + v_{3a}\omega_{1a} - \frac{S_1}{D_s}E_{2a}v_{2a} \\
\dot{v}_{3a} &= \frac{S_2 E_{3a}}{D_-} - 2v_{2a}v_{3a} + v_{1a}\omega_{2a} - \frac{S_1}{D}(E_{2a}v_{3a} + E_{3a}v
\end{aligned}\right\} \tag{4-285}$$

Equations 4-285 provide the basis for achieving the desired second-order regenerative tracking. (See par 4–6.5.3 for the overall mechanization means employed.) The components of the error vector E can be determined from the tracking error angle components Δ_{1a} and Δ_{3a} and the magnitudes of $\vec{D}_o$ and $\vec{D}_s$ (see par 4-6.5.2 and par 4-6.5.4). Also, a rate gyro is provided in order to determine the tracking-line angular-velocity component ω_{2a} (seepar 4-6.5.1). Given these inputs, plus the knowledge that S_1 and S_2 are constants, and assuming for the moment that D_s and v_{2a} can be obtained, then the first and third equations of Equation Set 4-285 can be integrated to determine v_{1a} and v_{3a}.

The components v_{1a} and v_{3a} can then be entered in Eq. 4-271 in order to generate the angular velocity components ω_{1a} and ω_{3a}. The basis for this procedure is made more obvious by using Eq. 4-275 to rewrite Eq. 4-271 in terms of its components in the rotating axis system as follows:

$$D_s \overrightarrow{2a} = D_s(\omega_{3a}\overrightarrow{1a} - \omega_{1a}\overrightarrow{3a}) \tag{4-286}$$

A comparison of Eqs. 4-271 and 4-286 then shows that the angular velocity components ω_{1a} and ω_{3a} are given by the relationships

$$\left.\begin{aligned}
\omega_{1a} &= -\frac{S_1 E_{3a}}{D_s} - v_{3a} \\
\omega_{3a} &= \frac{S_1 E_{1a}}{D_s} + v_{1a}
\end{aligned}\right\} \tag{4-287}$$

The angular velocity components ω_{1a} and ω_{3a} can now be entered in the second equation of Equation Set 4-285, which can then be integrated to obtain v_{2a}. The parameter v_{2a} is then fedbackintothe first and third equations of Equation Set 4-285 and also is used in Eq. 4-270 to compute D_s, which can then be fed back into Equation Set 4-285.

With the proper gain settings, S_1 and S_2, the computer will settle out at zero error for a nonaccelerating target and will then track this target regeneratively, i.e., without any further tracking correction being required by the operator.

4-6.4.2 Gyro-to-Platform Coordinate Transformation

As previously described (par 4-6.4.1), the gimballed platform on which the two-degree-of-freedom gyro is mounted is constrained by its servos to follow the gyro spin axis. How-

ever, servo errors do exist, and a transformation from gyro coordinates (identified by the subscript a) to platform coordinates (identified by the subscript p) must therefore be made.

The relationships between the gyro and platform coordinate systems are shown in Fig. 4-56. In conformance with the notation used in the development of the tracking equations, the gyro coordinate set is defined by the unit vectors $\vec{1a}$, $\vec{2a}$, $\vec{3a}$, with $\vec{2a}$ directed along the spin axis of the gyro (ref. Fig. 4-55). The platform coordinate set is defined by the unit vectors $\vec{1p}$, $\vec{2p}$, $\vec{3p}$, which are displaced with respect to the corresponding gyro axes by the error angles ψ and ϕ. With the two coordinate sets initially coincident, the angle ψ is produced by rotation of the gyro coordinates with respect to the platform coordinates about the $\vec{3p}$ axis. As a result, the unit vector $\vec{2a}$ arrives at the intermediate position $\vec{2i}$. The angle ϕ is then produced by an additional rotation of the gyro coordinates about the $\vec{1a}$ axis, with the result that the two coordinate sets reach the relative orientation with respect to one another that is shown in Fig. 4-56.

In terms of the physical elements of the fire control system, the angle ψ represents the angle between the outer gyro gimbal and the platform (see Fig. 4-57, which is a pictorial diagram of an illustrative two-degree-of-freedom gyro). It thereby defines the unit vector $\vec{2i}$ which represents the plane of the outer gimbal. Angle ϕ is the angle between the gyro spin axis and the gimbal vector $\vec{2i}$.

Angles ψ and ϕ are measured by the gyro pickoffs and control the platforrn servos. They define an error vector $\vec{\delta}$ which is the difference between $\vec{2a}$ and $\vec{2p}$ and consists of the components $\vec{\delta}_t$ and $\vec{\delta}_e$ which lie in the planes of and $, respectively. In vector notation

$$\vec{\delta} = \vec{\delta}_t + \vec{\delta}_e = \vec{2a} - \vec{2p} \tag{4-288}$$

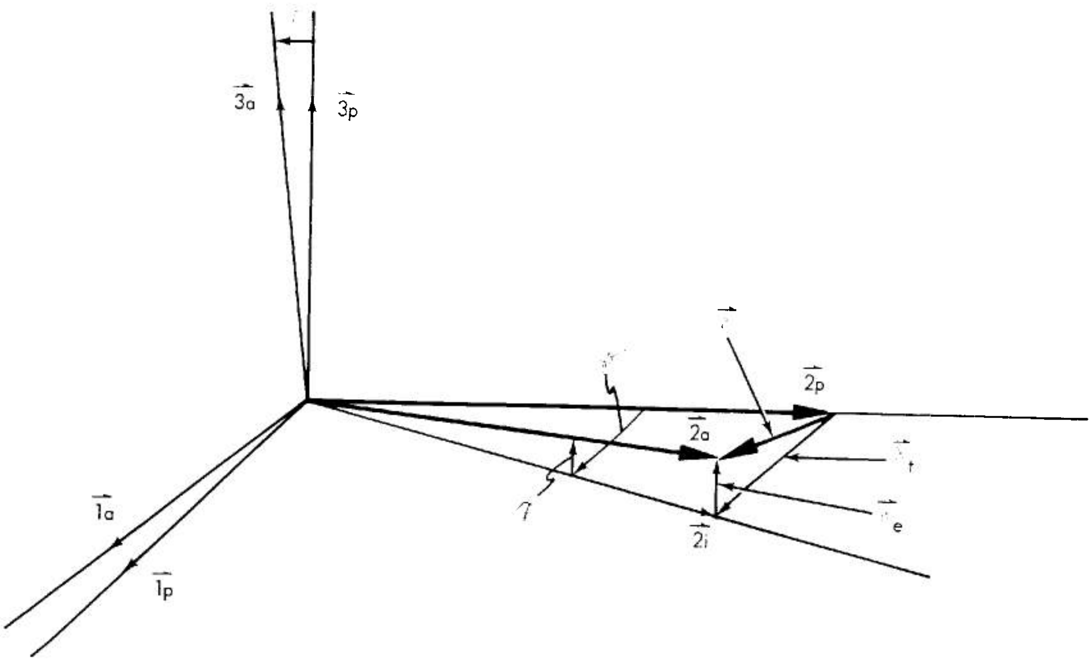
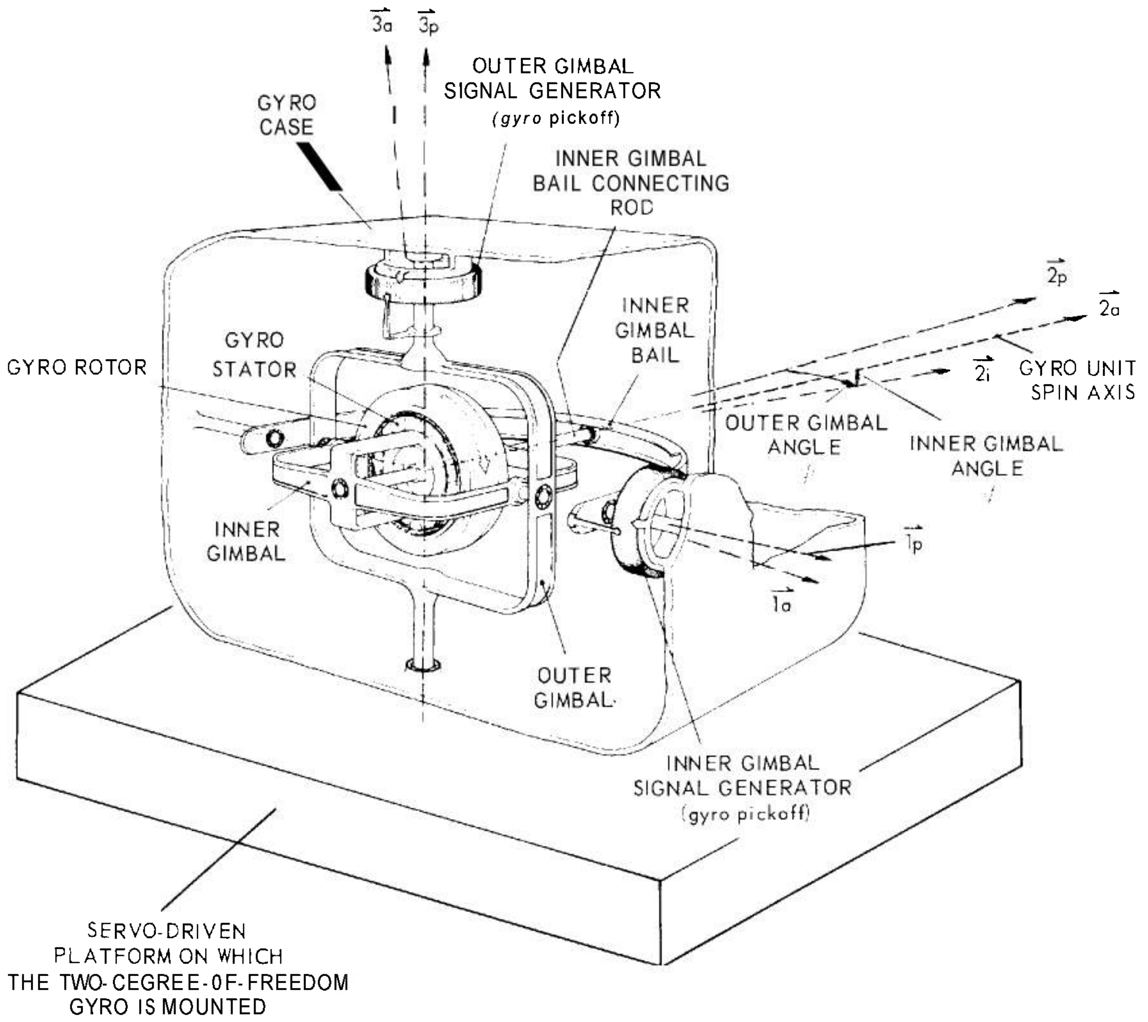


Figure 4-56. The gyro and platform coordinate systems.



NOTES:

1. THE PLATFORM COORDINATES ($\vec{1}p$, $\vec{2}p$, $\vec{3}p$) ARE FIXED WITH RESPECT TO THE PLATFORM AND HENCE WITH RESPECT TO THE GYRO CASE.
2. THE GYRO COORDINATES ($\vec{1}a$, $\vec{2}a$, $\vec{3}a$) ARE FIXED WITH RESPECT TO THE INNER GIMBAL.
3. BOTH COORDINATE SETS ARE CENTERED AT THE CENTER OF THE GYRO ROTOR.
4. THE TWO-DEGREE-OF-FREEDOM GYRO SHOWN HERE IS FOR ILLUSTRATIVE PURPOSES ONLY AND IS NOT INTENDED TO REPRESENT ACTUAL CONSTRUCTION.

Figure 4-57. Pictorial diagram of an illustrative two-degree-of-freedom gyro. (Adapted from THE FLOATING INTEGRATING GYRO AND ITS APPLICATION TO GEOMETRICAL STABILIZATION PROBLEMS ON MOVING BASES by C. S. Draper, W. Wrigley, and L. R. Grohe; published as S. M. F. Fund Paper No. FF-13 by the Institute of the Aeronautical Sciences, New York, N. Y,, 1955.)

The platform is driven by its servos such that

$$\overrightarrow{2p} = k\,\vec{\delta} \tag{4-289}$$

where k is the servo gain.

The rotation that occurs around the gyro spin axis is not measurable, by virtue of the operation of the gyro. Since ω_{2a} — the angular velocity about this spin axis — is required (see Eqs. 4-285), the platform angular velocity component ω_{p2p} is measured by a rate gyro and ω_{2a} is then computed from ω_{p2p} by an appropriate coordinate transformation, as described subsequently in par 4-6.5.1.

The transformation equations may be written by inspection in matrix form — using the relationships developed in Chapter 7 of Section 3, but setting $\theta = 0$ and reversing the sign of ψ to conform to the convention of Fig. 4-57. The first transformation, from the set $\overrightarrow{1p}$, $\overrightarrow{2p}$, $\overrightarrow{3p}$ to the set $\overrightarrow{1a}$, $\overrightarrow{2i}$, 3p, is

$$\begin{bmatrix} u_{1a} \\ u_{2i} \\ u_{3p} \end{bmatrix} = \begin{vmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{bmatrix} u_{1p} \\ u_{2p} \\ u_{3p} \end{bmatrix} = \mathfrak{A} \begin{vmatrix} u_{1p} \\ u_{2p} \\ u_{3p} \end{vmatrix} \tag{4-290}$$

where u_{1a}, etc., are components of any vector $\vec{u}$, along the axis indicated by the subscript. The transformation from the set $\overrightarrow{1a}$, $\overrightarrow{2i}$, $\overrightarrow{3p}$ to the set $\overrightarrow{1a}$, $\overrightarrow{2a}$, $\overrightarrow{3a}$ is given by

$$\begin{bmatrix} u_{1a} \\ u_{2a} \\ u_{3a} \end{bmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{vmatrix} \begin{vmatrix} u_{1a} \\ u_{2i} \\ u_{3p} \end{vmatrix} = \mathfrak{B} \begin{bmatrix} u_{1a} \\ u_{2i} \\ u_{3p} \end{bmatrix} \tag{4-291}$$

The complete transformation is obtained by matrix multiplication, yielding the following set of equations — expressed in both the matrix form and the conventional form:

$$\begin{bmatrix} u_{1a} \\ u_{2a} \\ u_{3a} \end{bmatrix} = \mathfrak{B}\mathfrak{A} \begin{bmatrix} u_{1p} \\ u_{2p} \\ u_{3p} \end{bmatrix} \tag{4-292}$$

$$\left.\begin{aligned} u_{1a} &= u_{1p}\cos\psi - u_{2p}\sin\psi \\ u_{2a} &= u_{1p}\sin\psi\cos\phi + u_{2p}\cos\psi\cos\phi + u_{3p}\sin\phi \\ u_{3a} &= -u_{1p}\sin\psi\sin\phi - u_{2p}\cos\psi\sin\phi + u_{3p}\cos\phi \end{aligned}\right\} \qquad (4\text{-}293)$$

The transformation from gyro to platform coordinates is obtained by inverting the matrices, yielding

$$\begin{bmatrix} u_{1p} \\ u_{2p} \\ u_{3p} \end{bmatrix} = \mathfrak{A}^{-1}\mathfrak{B}^{-1} \begin{vmatrix} u_{1a} \\ u_{2a} \\ u_{3a} \end{vmatrix} \qquad (4\text{-}294)$$

$$\left.\begin{aligned} u_{1p} &= u_{1a}\cos\psi + u_{2a}\sin\psi\cos\phi - u_{3a}\sin\psi\sin\phi \\ u_{2p} &= -u_{1a}\sin\psi + u_{2a}\cos\psi\cos\phi - u_{3a}\cos\psi\sin\phi \\ u_{3p} &= u_{2a}\sin\phi + u_{3a}\cos\phi \end{aligned}\right\} \qquad (4\text{-}295)$$

As shown in Fig. 4-56, $\vec{\delta}_t$ and $\vec{\delta}_e$ are the components of the platform/gyro error vector $\vec{\delta}$ in the planes of ψ and ϕ, respectively, where ψ and ϕ are the angular components of the total angular error that corresponds to the platform/gyro error vector δ. Thus, ψ and ϕ are respectively equal to the platform lateral servo error, designated δ_t, and the platform elevation servo error, designated δ_e. For small servo errors: $\sin\psi \cong \psi$, $\sin\phi \cong \phi$, $\cos\psi \cong 1$ and $\cos\phi \cong 1$. With these relationships applied to Eqs. 4-293 and 4-295, and with the second-order error terms dropped, the resulting transformations in terms of the platform servo errors are

$$\left.\begin{aligned} u_{1a} &= u_{1p} - u_{2p}\delta_t \\ u_{2a} &= u_{1p}\delta_t + u_{2p} + u_{3p}\delta_e \\ u_{3a} &= -u_{2p}\delta_e + u_{3p} \end{aligned}\right\} \qquad (4\text{-}296)$$

$$\left.\begin{aligned} u_{1p} &= u_{1a} + u_{2a}\delta_t \\ u_{2p} &= -u_{1a}\delta_t + u_{2a} - u_{3a}\delta_e \\ u_{3p} &= u_{2a}\delta_e + u_{3a} \end{aligned}\right\} \qquad (4\text{-}297)$$

4-6.4.3 Prediction Section

Reference to Eqs. 4-257 and 4-259 shows that when the tracking error $\vec{E}$ has been reduced to zero, Eq. 4-257 becomes

$$\frac{d}{dt}\vec{D}_s = S_2 \int \vec{E}dt = \vec{V}_s \tag{4-298}$$

so that $\vec{V}_s$ is equal to the velocity of the smooth tracking point s (see Fig. 4-54). Reduction of the tracking error to zero makes the smooth tracking point s coincide with the present target position T_o (see Fig. 4-58). Thus, $\vec{V}_s$ is then identical with $\vec{V}_{T^o}$ and the predicted impact point is given by

$$\vec{D}_p = \vec{D}_s + (TF)\vec{V}_s \tag{4-299}$$

where

$\vec{D}_p$ = vector from the weapon station to the predicted point p in Fig. 4-58

TF = time of flight of the projectile.

The time of flight is given for any $\vec{D}_p$ by ballistic tables, taking into account atmospheric conditions, estimated muzzle velocity, etc. Conversion of $\vec{D}_p$ to platform coordinates permits its computation. This step is accomplished by the conversion of $\vec{D}_s$ and $\vec{V}_s$ to platform coordinates-. Equation 4-262 gives $\vec{D}_s$ in component form. Since there is a component only along the 2a axis, substitution of Eq. 4-262 into the axis transformation set given by Eqs. 4-297 — by replacing the general vector $\vec{u}$ by the specific vector $\vec{D}_s$ — yields

$$\left.\begin{aligned} D_{s1p} &= D_s\,\delta_t \\ D_{s2p} &= D_s \\ D_{s3p} &= D_s\,\delta_e \end{aligned}\right\} \tag{4-300}$$

Similarly, the combination of Eqs. 4-264 and 4-297 yields

$$\left.\begin{aligned} V_{s1p} &= D_s v_{1a} + D_s v_{2a}\,\delta_t \\ V_{s2p} &= -D_s v_{1a}\,\delta_t + D_s v_{2a} - D_s v_{3a}\,\delta_e \\ V_{s3p} &= D_s v_{2a}\,\delta_e + D_s v_{3a} \end{aligned}\right\} \tag{4-301}$$

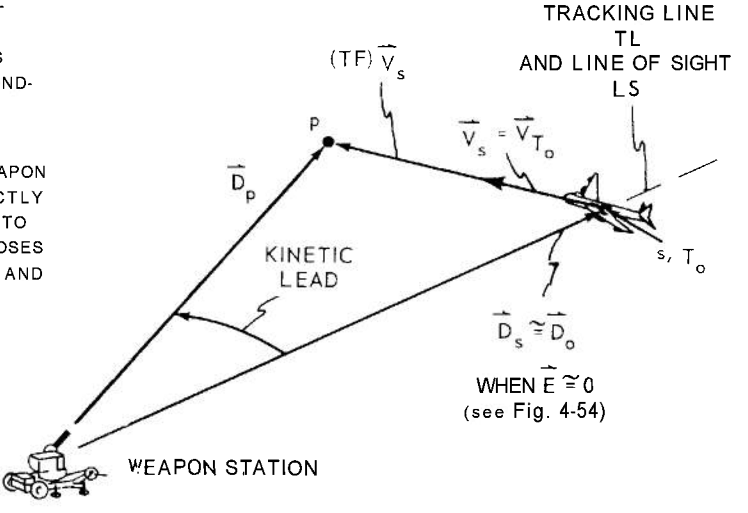
The platform components of $\vec{D}_p$ which are designated D_{p1p}, D_{p2p}, and D_{p3p} are obtained by substitution of Eqs. 4-300 and 4-301 into Eq. 4-299:

$$\left.\begin{aligned} D_{p1p} &= D_s\,\delta_t + (TF)(D_s v_{1a} + D_s v_{2a}\,\delta_t) \\ D_{p2p} &= D_s + (TF)(-D_s v_{1a}\,\delta_t + D_s v_{2a} - D_s v_{3a}\,\delta_e) \\ D_{p3p} &= D_s\,\delta_e + (TF)(D_s v_{2a}\,\delta_e + D_s v_{3a}) \end{aligned}\right\} \tag{4-302}$$

NOTES:

1. FOR ANTIAIRCRAFT FIRING, THE CROSS-OVER POINT (THE POINT ON A TARGET'S COURSE AT WHICH THE TARGET IS CLOSEST TO THE WEAPON STATION AND THEREFORE CHANGING FROM AN INCOMING TARGET TO AN OUTGOING TARGET) IS THE LEAST DESIRABLE FOR FIRING BECAUSE THE ANGULAR RATE OF THE TRACKING LINE THEN HAS ITS HIGHEST VALUE AND THERE IS A CORRESPONDING INCREASE IN THE ERRORS OF THE FIRE CONTROL SOLUTION.

2. THE RADAR, THE SIGHT, AND THE WEAPON CANNOT BE PHYSICALLY LOCATED AT EXACTLY THE SAME POINT, BUT CAN BE CONSIDERED TO BE SO LOCATED FOR ALL PRACTICAL PURPOSES AS A RESULT OF BORESIGHTING THE RADAR AND THE SIGHT TO THE WEAPON.



SYMBOL DEFINITIONS

T_o = PRESENT POSITION OF THE TARGET; I.E., THE TARGET POSITION AT THE INSTANT OF FIRING

s = TRACKING POINT, THE APPARENT PRESENT POSITION OF THE TARGET

p = PREDICTED IMPACT POINT OF THE PROJECTILE ON THE TARGET

$\vec{D}_o$ = ACTUAL RANGE VECTOR TO THE TARGET

$\vec{D}_s$ = TRACKING VECTOR

$\vec{D}_p$ = VECTOR FROM THE WEAPON STATION TO THE PREDICTED IMPACT POINT p

TF = TIME OF FLIGHT OF THE PROJECTILE CORRESPONDING TO THE FUTURE RANGE VECTOR $\vec{D}_p$

$\vec{V}_s \triangleq S_2 \int \vec{E}\, dt$ (Ref. Eq. 4-259)

= SMOOTH TARGET VELOCITY

$\vec{E}$ = TRACKING ERROR

$\vec{V}_{T_o}$ = ACTUAL TARGET VELOCITY

Figure 4-58. Vector relationships associated with prediction.

See par 4-6.6 for the manner in which these equations are mechanized to yield the required gun angles.

4-6.5 TRACKING-SECTION MECHANIZATION

The tracking section comprises four main subsections, as follows:

1. The roll-correction subsection.
2. The range subsection.
3. The integrating-servo subsection.
4. The sight subsection.

The paragraphs which follow briefly describe each of these subsections in turn.

4-6.5.1 Roll-Correction Subsection

As indicated in par 4-6.4.2, the angular velocity of the gyro coordinate system with respect to an inertial reference frame about the tracking—line ω_{2a} is obtained b,y the following means: the angular velocity of the platform about the 2p axis ω_{p2p} is measured by means of a rate gyro, and ω_{2a} is then determined by applying correcting terms to ω_{p2p} to account for the difference between $\vec{2p}$ and $\vec{2a}$. As the first step in explaining this mechanization, the angular velocity ω_a of the gyro coordinate system with respect to an inertial coordinate system will be defined as

$$\vec{\omega}_a = \vec{\omega}_p + \vec{\omega}_d \qquad (4\text{-}303)$$

where

$\vec{w}_p$ = angular velocity of the platform coordinate system with respect to an inertial coordinate system

$\vec{\omega}_d$ = angular velocity of the gyro coordinate system with respect to the platform coordinate system.

As shown by Fig. 2-57, the vector $\vec{2p}$ is rotated about $\vec{3p}$ through the angle δ_t to form the gyro gimbal vector 2i. In accordance with the right-hand rule for the relationship between a rotational quantity and the vector representing that quantity,;: the vector representing the angle δ_t is parallel to $\vec{3p}$ but opposite in direction. Therefore, it is evident that

$$\omega_{d3p} = -\dot{\delta}_t \qquad (4\text{-}304)$$

where ω_{d3p} is the component of $\vec{\omega}_d$ about $\vec{3p}$. Note that, because of the gimballing arrangement,

$$\omega_{d2i} = 0. \qquad (4\text{-}305)$$

The final rotation is about $\vec{1a}$, through angle δ_e, and yields

$$\omega_{d1a} = \dot{\delta}_e \qquad (4\text{-}306)$$

In gyro coordinates, the components of $\vec{\omega}_p$ are derived from transformation Eqs. 4-296:

*See Section a of Definition Summary 1-2 of Chapter 1 of Reference 4.

$$\left.\begin{aligned}\omega_{p1a} &= \omega_{p1p} - \omega_{p2p}\,\delta_t\\ \omega_{p2a} &= \omega_{p1p}\,\delta_t + \omega_{p2p} + \omega_{p3p}\,\delta_e\\ \omega_{p3a} &= -\omega_{p2p}\,\delta_e + \omega_{p3p}\end{aligned}\right\} \tag{4-307}$$

Similarly, the components of $\vec{\omega}_d$ are obtained from Eqs. 4-304 through 4-306 by means of transformation Eqs. 4-291:

$$\left.\begin{aligned}\omega_{d1a} &= \dot{\delta}_e\\ \omega_{d2a} &= \omega_{d2i} + \omega_{d3p}\,\delta_e = 0 - \dot{\delta}_t\,\delta_e\\ \omega_{d3a} &= -\omega_{d2i}\,\delta_e + \omega_{d3p} = -\dot{\delta}_t\end{aligned}\right\} \tag{4-308}$$

Now, the components of $\vec{\omega}_a$ can be obtained from Eqs. 4-303, 4-307, and 4-308 as follows:

$$\left.\begin{aligned}\omega_{1a} &= \omega_{p1a} + \omega_{d1a} = \omega_{p1p} - \omega_{p2p}\,\delta_t + \dot{\delta}_e\\ \omega_{2a} &= \omega_{p2a} + \omega_{d2a} = \omega_{p1p}\,\delta_t + \omega_{p2p} + \omega_{p3p}\,\delta_e - \dot{\delta}_t\,\delta_e\\ \omega_{3a} &= \omega_{p3a} + \omega_{d3a} = -\omega_{p2p}\,\delta_e + \omega_{p3p} - \dot{\delta}_t\end{aligned}\right\} \tag{4-309}$$

Use of these three relationships shows that

$$\omega_{2a} = \omega_{p2p} + \delta_t(\omega_{1a} + \omega_{p2p}\,\delta_t - \dot{\delta}_e) + \delta_e(\omega_{3a} + \omega_{p2p}\,\delta_e + \dot{\delta}_t) - \dot{\delta}_t\,\delta_e \tag{4-310}$$

Since δ_t, δ_e, $\dot{\delta}_t$, and $\dot{\delta}_e$ are small quantities, the second-order terms can be dropped, giving

$$\omega_{2a} \cong \omega_{p2p} + \omega_{1a}\,\delta_t + \omega_{3a}\,\delta_e \tag{4-311}$$

The quantities on the right-hand side of Eq. 4-311 are all measurable or calculable: ω_{p2p} by the rate gyro, ω_{1a} and ω_{3a} from Eqs. 4-287, and δ_t and δ_e by the error signals of the platform servos.

The rate gyro is mounted on the platform with its input axis along $\vec{2p}$, its output axis along $\vec{3p}$, and its spin axis along $\vec{1p}$. An input rate ω_{p2p} causes the gyro to precess about $\vec{3p}$ through an angle δ_r that is proportional to ω_{p2p}. A signal proportional to δ_r and its integral is fed back to the gyro torquer. Since the input axis is displaced by the angle δ_r, a component of ω_{p1p} is measured, as well as ω_{p2p}. The torque equilibrium equation for the gyro is then

$$\omega_{p2p} - \omega_{p1p}\,\delta_r = k_1\,\delta_r + k_2 \int \delta_r\,dt \tag{4-312}$$

Ideally, a correction term $(-\omega_{p1p}\,\delta_r)$ should be introduced by the gyro torquer, thereby modifying Eq. 4-312 to the expression

$$\omega_{p2p} = k_1\,\delta_r + \omega' \tag{4-313}$$

where

$$\omega' = k_2 \int \delta_r\,dt. \tag{4-314}$$

The correction term actually employedin the computer is an approximation to $-\omega_{p1p}\,\delta_r$ and is obtained as follows:

(1) The first of Eqs. 4-287 is set equal to the first of Eqs. 4-309 and the terms rearranged, thereby yielding

$$\omega_{p1p} = \omega_{p2p}\,\delta_t - \dot{\delta}_e - \frac{S_1 E_{3a}}{D_s} \tag{4-315}$$

(2) Multiplying both sides of Eq. 4-315 by δ_r and dropping products of errors then gives

$$\omega_{p1p}\,\delta_r \cong \omega_{p2p}\,\delta_t\,\delta_r - \dot{\delta}_e\,\delta_r - \frac{S_1 E_{3a}}{D_s}\,\delta_r - v_{3a}\,\delta_r \cong -v_{3a}\,\delta_r \tag{4-316}$$

The mechanization of this equation is shown in the upper right-hand corner of Fig. 4-59.

A similar approximation of Eqs. 4-287 permits the mechanization of Eq. 4-311 in the form

$$\omega'' = k_1\,\delta_r + \omega' - v_{3a}\,\delta_t + v_{1a}\,\delta_e \tag{4-317}$$

where ω'' is approximately equal to ω_{2a}. Figure 4-59 in its entirety represents the mechanization of this equation.

4-6.5.2 Range Subsection

The mechanization of Eq. 4-270 is accomplished by the introduction of target range D_o from the tracking radar. As shown in Fig. 4-60, an integrating servo generates D_s which is compared with D_o in a summing network. From Fig. 4-55, it is evident that

$$E_{2a} = D_a \cos\Delta - D_s \tag{4-318}$$

where Δ is the tracking error angle between D_s and D_o. For small values of Δ, $E_{2a} \cong D_o - D_s$. Then, as shown by the block diagram of Fig. 4-60, the servo solves the equation

$$\dot{D}_s = D_s\,v_{2a} + S_1(D_a - D_s) \tag{4-319}$$

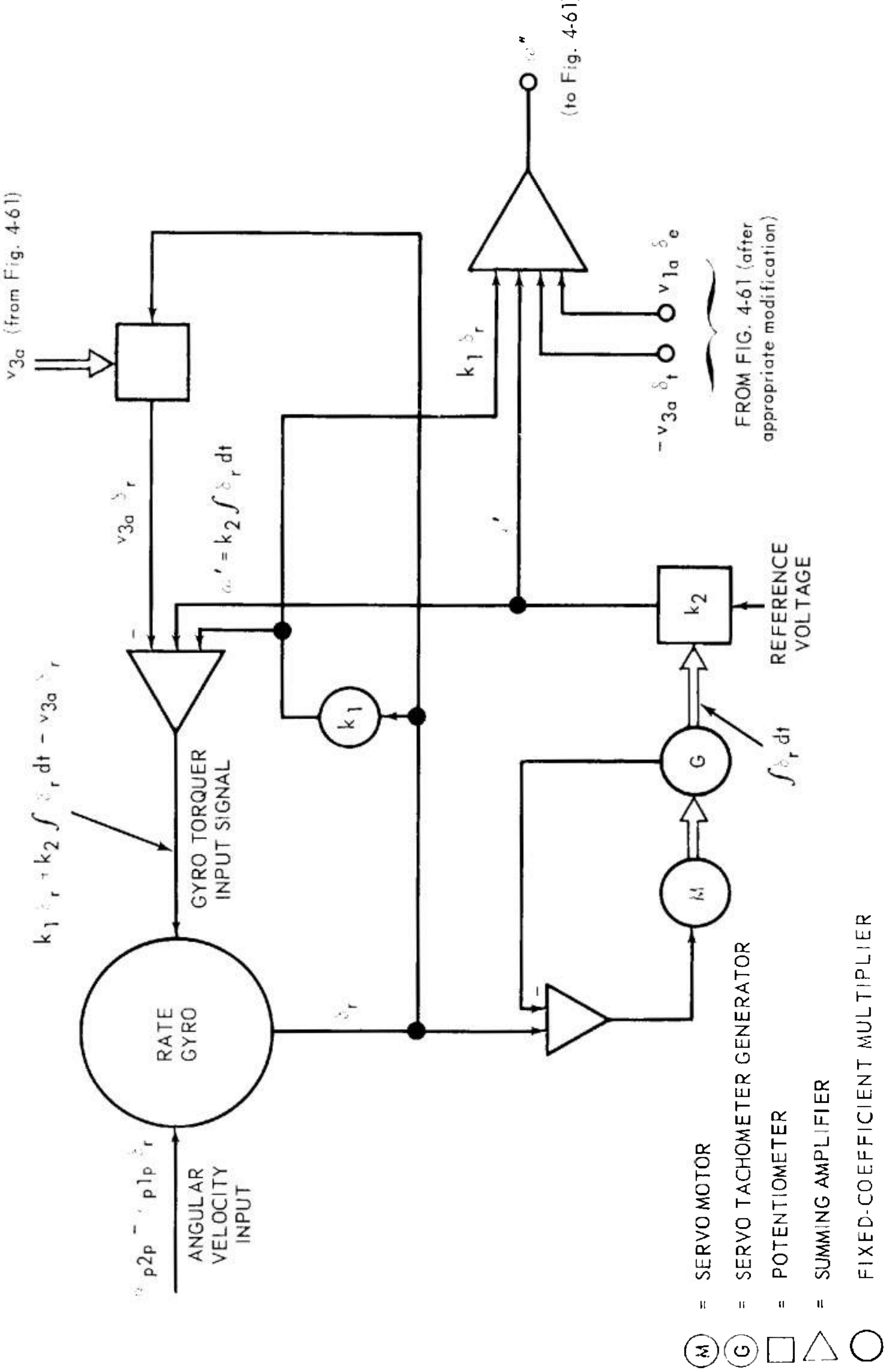
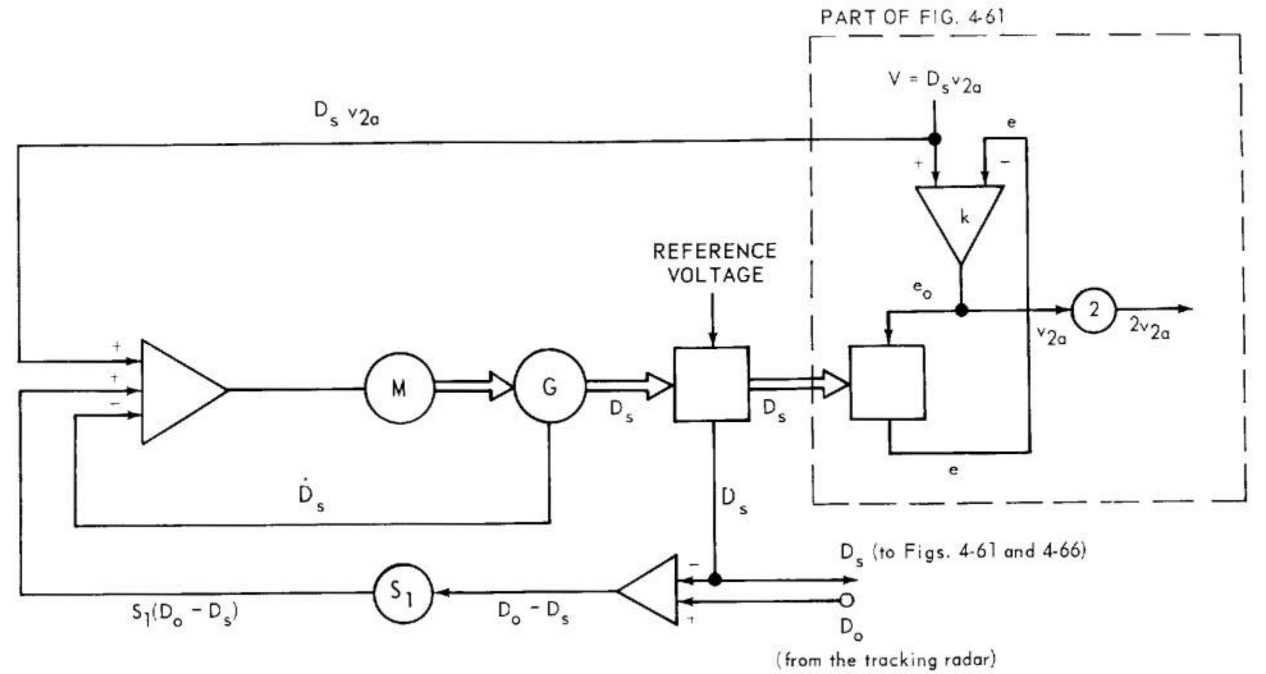


Figure 4-59. Mechanization of the roll-correction equations.



NOTE:

THE COMPONENTS ENCLOSED IN THE DASHED-LINE BOX ARE EMPLOYED IN THE MECHANIZATION OF THE REGENERATIVE TRACKING EQUATIONS (SEE FIG. 4-61) AND ARE NOT DIRECTLY A PART OF THE RANGE-EQUATION MECHANIZATION. THEY ARE INCLUDED HERE IN ORDER TO CLARIFY THE INTERRELATIONSHIPS BETWEEN THE TWO MECHANIZATIONS. THE OPERATION OF THE BOXED SECTION CAN BE EXPLAINED ON THE BASIS OF THE TWO FOLLOWING RELATIONSHIPS WHICH ARE EVIDENT FROM THE DIAGRAM:

$$k(V - e) = e_o$$

$$e_o D_s = e$$

WHERE k IS THE GAIN OF THE AMPLIFIER, AND e AND e_o ARE INPUT AND OUTPUT SIGNALS OF THE AMPLIFIER AND POTENTIOMETER. THEREFORE,

$$kV - k e_o D_s = e_o$$

AND

$$e_o = \frac{kV}{1 + k D_s}$$

FOR A LARGE AMPLIFIER GAIN,

$$e_o \cong \frac{V}{D_s} = v_{2a}$$

Figure 4-60. Mechanization of the range equation.

Substitution of E_{2a} for $D_o - D_s$ yields

$$\dot{D}_s \cong D_s v_{2a} + S_1 E_{2a} \tag{4-320}$$

which shows that Eq. 4-270 has been mechanized.

4-6.5.3 Integrating-Servo Subsection

The tracking computer, in addition to the elements described in the preceding paragraphs, includes three integrating servos that are employed to solve Eqs. 4-285. As noted in par 4-6.4.1, these three equations provide the basis for achieving the desired second-order regenerative tracking. A worthwhile simplifying approximation is to neglect the last term in each of these equations. These are error terms, and tend toward zero as the tracking error decreases. With these changes, Eqs. 4-285 become

$$\left.\begin{aligned}
\dot{v}_{1a} &= \frac{S_2 E_{1a}}{D_s} - 2v_{1a} v_{2a} - v_{3a}\,\omega_{2a} \\
\dot{v}_{2a} &= \frac{S_2 E_{2a}}{D_s} - v_{2a}^2 - v_{1a}\,\omega_{3a} + v_{3a}\,\omega_{1a} \\
\dot{v}_{3a} &= \frac{S_2 E_{3a}}{D_-} - 2v_{2a} v_{3a} + v_{1a}\,\omega_{2a}
\end{aligned}\right\} \tag{4-321}$$

The mechanization of these equations that is employed in order to obtain the quantities v_{1a}, v_{2a}, and v_{3a} is shown in Fig. 4-61. In the mechanization used to obtain v_{1a}, the terms on the right-hand side of the first equation are summed to obtain $\dot{v}_{1a}$; the result is then integrated by the servo depicted at the top of Fig. 4-61. The mechanization used to obtain v_{3a} is similar, but that for v_{2a} requires further manipulation of the equation being mechanized. This involves the introduction of the range rate V, where $V = V_{2a} = D_s v_{2a}$ (ref. Eq. 4-266). If the error terms in Eqs. 4-287 are neglected, the expression for $\dot{v}_{2a}$ given by Eqs. 4-321 may be rewritten in the form

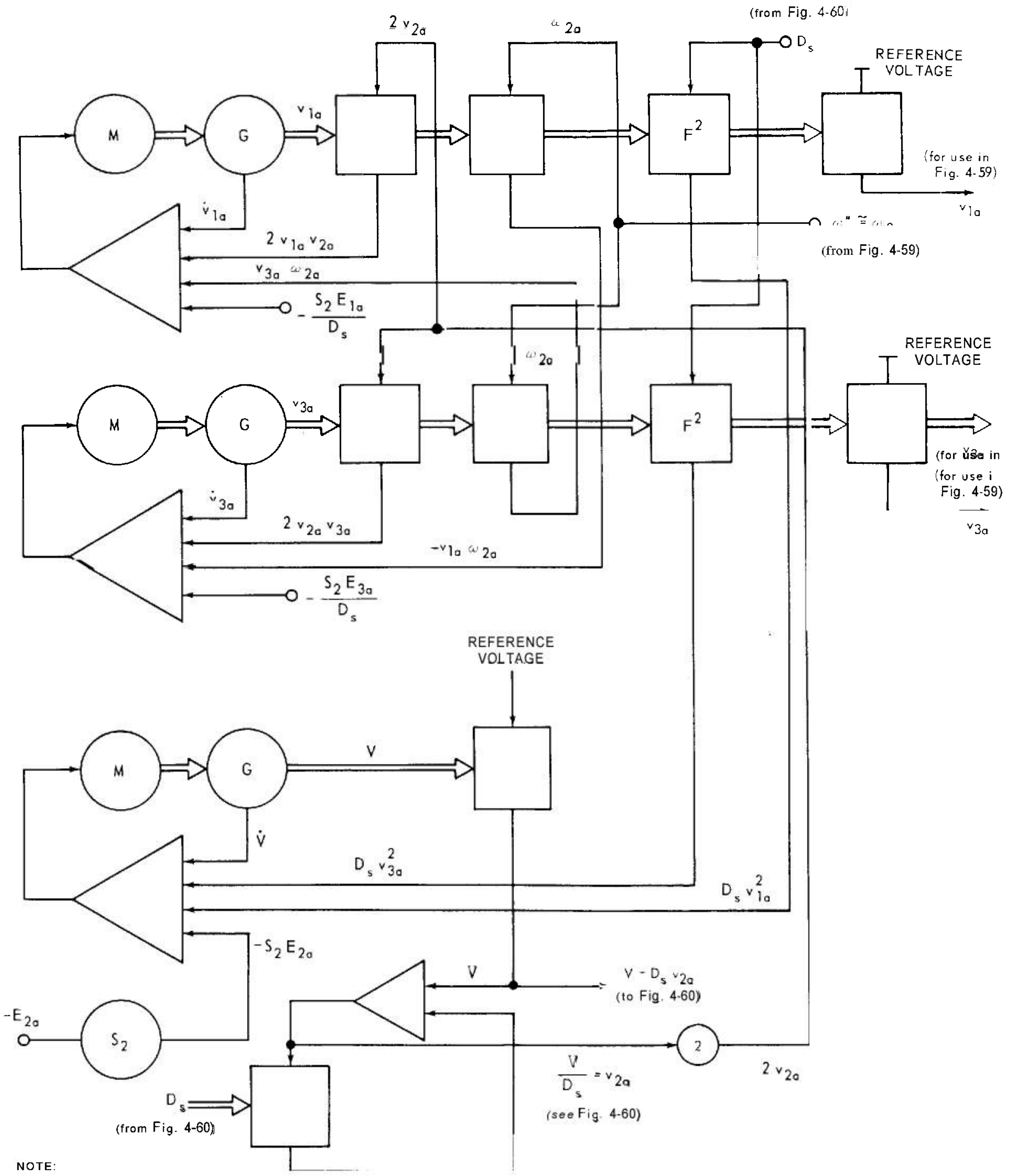
$$\dot{v}_{2a} = \frac{S_2 E_{2a}}{D_s} - v_{2a}^2 - v_{1a}^2 - v_{3a}^2 \tag{4-322}$$

Also,

$$\dot{V} = \dot{V}_{2a} = \frac{d}{dt}(D_s v_{2a}) = D_s \dot{v}_{2a} + v_{2a}\dot{D}_s \tag{4-323}$$

In addition, neglecting the error term in Eq. 4-270 shows that

$$D, \cong D, v_{2a} \tag{4-324}$$



NOTE:

$\boxed{F^2}$ REPRESENTS A FUNCTION-SQUARED POTENTIOMETER, THAT is, A POTENTIOMETER WHOSE OUTPUT IS DIRECTLY PROPORTIONAL TO THE FUNCTION REPRESENTED BY THE INPUT SHAFT ROTATION.

Figure 4-61. Mechanization of the regenerative tracking equations.

The combination of Eqs. 4-322 through 4-324 provides the final equation as mechanized:

$$V = S_2 E_{2a} - D_s (v_{1a}^2 + v_{2a}^2 + v_{3a}^2) + D_s v_{2a}^2$$

$$= S_2 E_{2a} - D_s (v_{1a}^2 + v_{3a}^2) \qquad (4\text{-}325)$$

In the servo shown at the bottom of Fig. 4-61, the position of the servo shaft represents the quantity V.

4-6.5.4 Sight Subsection

The purpose of the sight servo drives is to make the tracking line coincident with the line of sight to the target; i.e., to drive Δ_{1a} and Δ_{3a} to zero. As shown by Fig. 4-55, these two quantities are respectively equal to E_{1a}/D_s and E_{3a}/D_s.

The input signals to the servos that drive the reticle in the optical sight stem from the hand controlunit (see Fig. 4-53) which produces output signals proportional to the operator's handgripdisplacements. The signals from the hand control unit — identified by the symbols Δ_{1p} and Δ_{3p} — differ from the theoretical quantities Δ_{1a} and Δ_{3a}, respectively, only by the limitations of a human operator to make measurements and by the servo errors of the platform drives. This difference is always small and approaches zero upon computer settling.

Although range itself is not of concern in the sight mechanization, the accuracy of the vector resolutions that are required to control the sight is maximized if the range error E_{2a} is given its maximum value, i.e., $E_{2a} = D_s$. This, in effect, postulates a sight error vector $\vec{\Delta}$ having components Δ_{1a} and Δ_{3a} (as derived from the hand control unit) and $\Delta_{2a} \cong E_{2a}/D_s = 1$. The first step of the computation is to convert this sight error vector from gyro coordinates to platform coordinates by introducing the platform errors δ_e and δ_t in accordance with Eqs. 4-297. The results are

$$\left.\begin{aligned} M_{1p} &= \Delta_{1a} + \delta_t \\ M_{2p} &= 1 - \Delta_{1a}\delta_t - \Delta_{3a}\delta_e \cong 1 \\ M_{3p} &= \delta_e + \Delta_{3a} \end{aligned}\right\} \qquad (4\text{-}326)$$

where M_{1p}, M_{2p}, M_{3p} are the components in platform coordinates of the sight error vector just defined.

The sight servos are mounted in the turret. By means of a gimbal system, they rotate the reticle through the sight lateral (LL) and sight elevation (ES) angles, with the ES gimbal being the outer one. The actuating signals must therefore first be resolved from platform coordinates to turret coordinates, and then to sight coordinates. The turret carries the gun as well as the sight; however, the gun has its own elevation drive for firing elevation (FE). The turret azimuth, or firing azimuth, FA, differs from the platform azimuth* AP by the azimuth lead angle AL. Thus, conversion from platform to turret coordinates requires resolution through the azimuth lead angle **AL** and the platform elevation angle EP. The equations for transforming a vector from gyro to platform coordinates are given in general form in Eqs. 4-295. For the present transformation, the equations have the same form and

*That is, the azimuth of the gimballed platform on which the two-degree-of-freedom tracking gyro is mounted (ref. par 4-6 4. 1).

may be obtained by substituting AL for ψ, EP for ϕ, subscript p for subscript a, and subscript t for subscript p. The resulting set of resolution equations, mechanized by a pair of resolvers, is

$$\left.\begin{aligned} M_{1t} &= M_{1p}\cos AL + M_{2p}\sin AL\cos EP - M_{3p}\sin AL\sin EP \\ M_{2t} &= -M_{1p}\sin AL + M_{2p}\cos AL\cos EP - M_{3p}\cos AL\sin EP \\ M_{3t} &= M_{2p}\sin EP + M_{3p}\cos EP \end{aligned}\right\} \tag{4-327}$$

The next step is to resolve M_{1t}, M_{2t}, and M_{3t} into sight coordinates. Recall that the M's are the components of the sight error vector $\vec{A}$; then the components of $\vec{A}$ in sight coordinates can be defined as ϵ_{LL} and ϵ_{ES} with the third component — which lies along the tracking line — being unnecessary. The angles LL and ES are obtained by the sight lateral and sight elevation servos which employ resolver feedback to solve the transformation equations.

$$\left.\begin{aligned} \epsilon_{LL} &= M_{1t}\cos LL - M_{2t}\sin LL\cos ES - M_{3t}\sin LL\sin ES \\ \epsilon_{ES} &= -M_{2t}\sin ES + M_{3t}\cos ES \end{aligned}\right\} \tag{4-328}$$

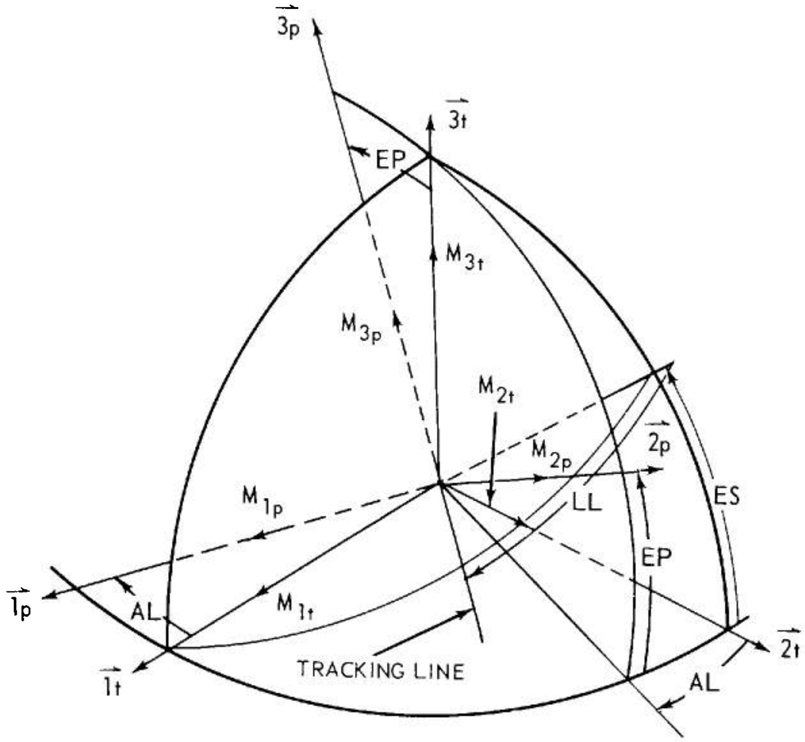
These equations are mechanized by employing ϵ_{LL} as the error of the sight lateral servo, and ϵ_{ES} as the error of the sight elevation servo. Equations 4-328 are based on the first and third of Eqs. 4-295 (the basic transformation equations), with the following required substitutions:

(a) (-LL) for ψ
(b) (-ES) for ϕ
(c) subscript t for subscript a
(d) ϵ_{LL} for u_{1p}
(e) ϵ_{ES} for u_{3p}

The definitions for the positive directions of LL and ES are illustrated by Fig. 4-62 which also shows the complete system of vectors associated with the sight mechanization. The block diagram of the sight mechanization is shown in Fig. 4-63.

4-6.6 PREDICTION-SECTION MECHANIZATION

The paragraphs immediately following describe the mechanization of Eqs. 4-302 which specify the vector D_p — the vector from the weapon station to the predicted impact point p — in platform coordinates. First, it will be assumed that the time of flight TF is known, and Eqs. 4-302 will be normalized with respect to it. The terms $-D_s v_{1a}\delta_t$ and $-D_s v_{3a}\delta_e$ in the second equation will be neglected since they are small compared with the first term. The first term of the first equation becomes $\frac{D_s}{TF}\delta_t$. For δ_t small, $\frac{D_s}{TF}$ may be replaced by a constant V_o which is approximately equal to the muzzle velocity of the gun. This approximation is also made in the third equation, but not in the second, since here the $\frac{D_s}{TF}$ term is not small. With the normalization and approximation carried out, Eqs. 4-302 become



DEFINITIONS

M_{1p}, M_{2p}, M_{3p} = COMPONENTS OF THE SIGHT ERROR VECTOR $\vec{A}$ IN PLATFORM COORDINATES

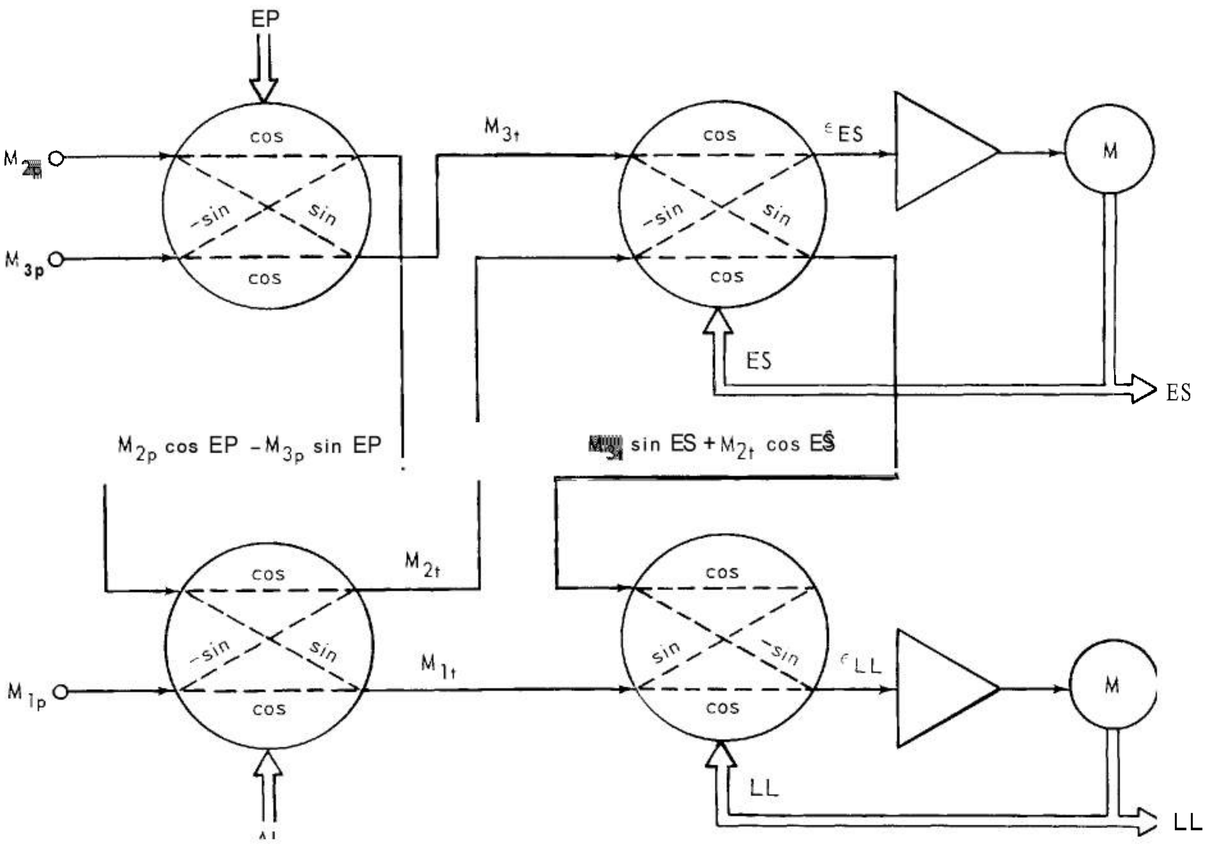
M_{1t}, M_{2t}, M_{3t} = COMPONENTS OF THE SIGHT ERROR VECTOR $\vec{A}$ IN TURRET COORDINATES

$\vec{1p}$, $\vec{2p}$, $\vec{3p}$ = UNIT VECTORS DEFINING THE PLATFORM COORDINATE SYSTEM

$\vec{1t}$, $\vec{2t}$, $\vec{3t}$ = UNIT VECTORS DEFINING THE TURRET COORDINATE SYSTEM

NOTE. THE UNIT VECTORS ARE SHOWN AT THE PERIPHERY OF THE DIAGRAM FOR THE PURPOSE OF CLARITY.

Figure 4-62. The complete system of vectors associated with sight mechanization.



EQUATIONS SOLVED:

$$
\left.
\begin{aligned}
M_{1t} &= M_{1p} \cos AL + {}_{2p} \sin AL \cos EP - M_{3p} \sin AL \sin EP \\
M_{2t} &= -M_{1p} \sin AL + M_{2p} \cos AL \cos EP - M_{3p} \cos AL \sin EP \\
M_{3t} &= M_{2p} \sin EP + M_{3p} \cos EP
\end{aligned}
\right\} \quad (4\text{-}327)
$$

$$
\left.
\begin{aligned}
\epsilon_{LL} &= M_{1t} \cos LL - M_{2t} \sin LL \cos ES - M_{3t} \sin LL \sin ES \\
\epsilon_{ES} &= -M_{2t} \sin ES + M_{3t} \cos ES
\end{aligned}
\right\} \quad (4\text{-}328)
$$

Figure 4-63. Block diagram of the sight mechanization.

$$\left.\begin{aligned} U_{p1p} &= V_o\,\delta_t + D_s\,v_{1o} + D_s\,v_{2a}\,\delta_t \\ U_{p2p} &= \frac{D_s}{TF} + D_s\,v_{2a} \\ U_{p3p} &= V_o\,\delta_e + D_s\,v_{2a}\,\delta_e + D_s\,v_{3o} \end{aligned}\right\} \qquad (4\text{-}329)$$

where

$$U_{p1p} = \frac{D_{p1p}}{TF}, \qquad U_{p2p} = \frac{D_{p2p}}{TF}, \qquad U_{p3p} = \frac{D_{p3p}}{TF} \qquad (4\text{-}330)$$

and

$$U_{p1p}\,\vec{1p} + U_{p2p}\,\vec{2p} + U_{p3p}\,\vec{3p} = \vec{U}_p = \frac{\vec{}}{TF} \qquad (4\text{-}331)$$

Equations 4-329 must now be resolved into turret coordinates in order to generate the required gun angles: the firing azimuth angle FA, and the firing elevation angle FE. The coordinate systems involved are shown in Fig. 4-64. The transformed U_p components, based on the application of Eqs. 4-295, are

$$\left.\begin{aligned} U_{p1t} &= U_{p1p}\cos AL + U_{p2p}\sin AL\cos EP - U_{p3p}\sin AL\sin EP \\ U_{p2t} &= -U_{p1p}\sin AL + U_{p2p}\cos AL\cos EP - U_{p3p}\cos AL\sin EP \\ U_{p3t} &= U_{p2p}\sin EP + U_{p3p}\cos EP \end{aligned}\right\} \qquad (4\text{-}332)$$

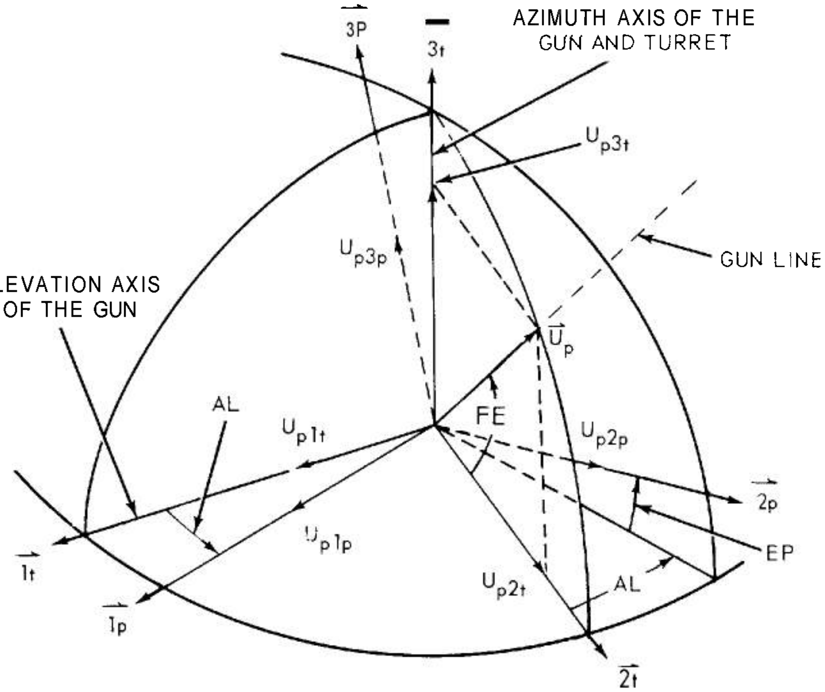
where U_{p1t}, U_{p2t}, U_{p3t} are the components of $\vec{U}_p$ in turret coordinates. This transformation is shown in block-diagram form in Fig. 4-65.

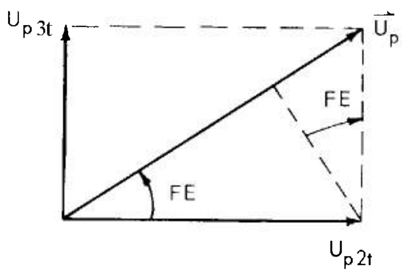
Ballistic corrections are then applied to the components U_{p1t}, U_{p2t}, and U_{p3t} in order to correct them for mount tilt, gravity drop, wind, and projectile drift. For the sake of clarity, the application of these corrections is not depicted in Fig. 4-65.

The gun and the turret are driven by hydraulic servos. The turret azimuth servo rotates both turret and gun through the firing azimuth angle FA since the turret carries the gun. The gun has its own elevation servo that rotates the gun through the firing elevation angle FE.

In the absence of an azimuth lead angle, the firing azimuth angle FA and the platform azimuth angle **AP** are the same because the turret serves as a base for the platform. The platform-gyro error δ_t is therefore fed to the turret azimuth servo error point so that gun, turret, and platform follow the tracking gyro in azimuth. As a lead angle is introduced by the **AL** servo, the platform is offset from the turret by the angle AL; the turret then shifts by an equal and opposite angle to maintain $\delta_t = 0$.

The gun elevation servo carries a resolver. Figure 4-64 shows that the gun line rotates about the gun elevation axis $\vec{1t}$ through the firing elevation angle FE. The normalized predicted range U_p must lie along the gun line. This is achieved by making U_{p1t} the error signal





$$\tan FE = \frac{\sin FE}{\cos FE} = \frac{U_{p3t}}{U_{p2t}}$$

THEREFORE,

$$U_{p3t} \cos FE - U_{p2t} \sin FE = 0 \qquad (4\text{-}333)$$

ALSO,

$$U_p = U_{p2t} \cos FE + U_{p3t} \sin FE$$

NOTE AS DISCUSSED IN THE TEXT, U_{p1t} IS DRIVEN TO ZERO BY THE AZIMUTH SERVO.

Figure 4-64. Geometrical relationships between the platform and turret coordinate systems.

for the AL servo, which then drives U_{p1t} to zero by computing the azimuth lead angle AL required to accomplish this. Since the AL servo maintains U_{p1t} at zero, as shown in Fig. 4-65, the components of U_p in turret coordinates (see Fig. 4-64) are $U_{p2t} = U_p \cos FE$ and $U_{p3t} = U_p \sin FE$. Thus, FE is computed by the resolver servo shown at the bottom of Fig. 4-65 on the basis of the following equations, which are evident from Fig. 4-64:

$$\left.\begin{aligned} \epsilon_{FE} = 0 &= U_{p3t} \cos FE - U_{p2t} \sin FE \\ U_p &= U_{p2t} \cos FE + U_{p3t} \sin FE \end{aligned}\right\} \tag{4-333}$$

where ϵ_{FE} is the error signal for the FE servo and is driven to zero. As ϵ_{FE} approaches zero in the first of Eqs. 4-333, the value of FE thus determined yields the magnitude of U_p from the second equation.

The time of flight TF of the projectile is the predicted range D_p divided by the average velocity of the projectile Vp. Thus, from the definition of U_p given in Eq. 4-331,

$$TF = \frac{D_p}{V_P} = \frac{(TF)\, U_p}{V_P} \tag{4-334}$$

In order to simplify the time-of-flight computation, Vp is approximated as the sum of a constant term V_{oP} and correction functions of air temperature T, round characteristics RD, muzzle velocity MV, and gun elevation angle FE, where the constant term is much larger than the correction functions. The approximation can be expressed in the form

$$V_P = V_{oP} + f_1(T) + f_2(RD) + f_3(MV) + f_4(FE) \tag{4-335}$$

Equation 4-334 can therefore be rewritten as

$$(TF)\left[V_{oP} + f_1(T) + f_2(RD) + f_3(MV) + f_4(FE)\right] = (TF)\, U_p = D_p \tag{4-336}$$

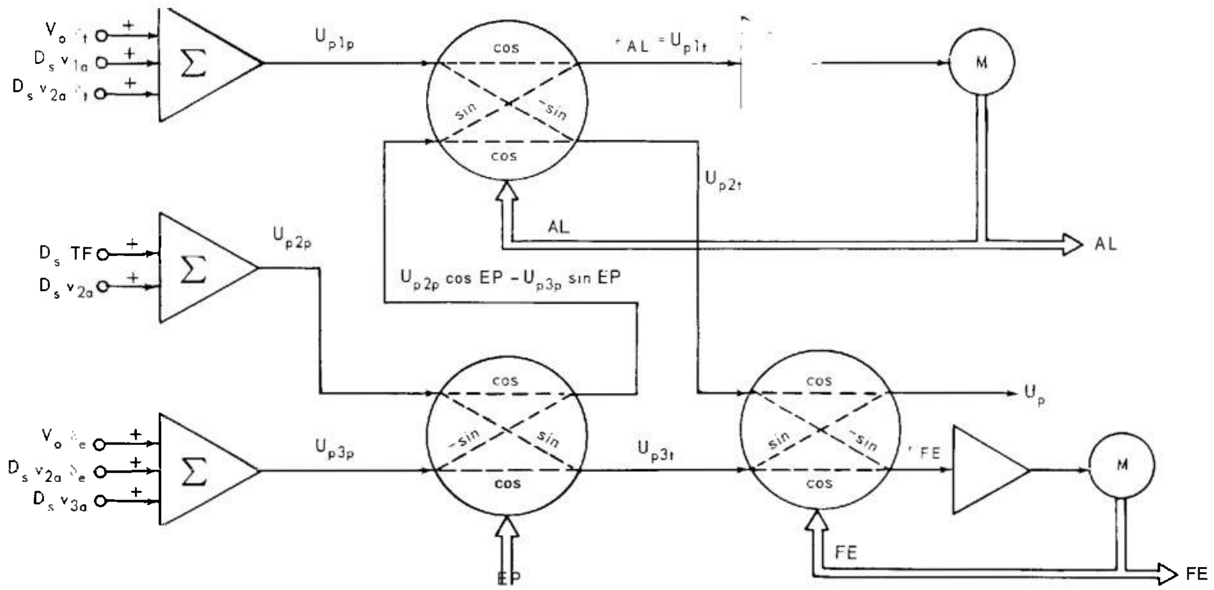
or

$$(TF)\, V_{oP} = D_p - (TF)\left[f_1(T) + f_2(RD) + f_3(MV) + f_4(FE)\right] \tag{4-337}$$

Based on Eq. 4-337, the equation solved by the time-of-flight computer is

$$\epsilon_{TF} = (TF)\, V_{oP} + (TF)\left[f_1(T) + f_2(RD) + f_3(MV) + f_4(FE)\right] - D_p \tag{4-338}$$

where ϵ_{TF} is the error of the servo employed in the computation. A block diagram of the time-of-flight computer is shown in Fig. 4-66. As indicated, TF is given by the position of the servo shaft after ϵ_{TF} has been driven to zero. As indicated in Fig. 4-66, the quantity



EQUATIONS SOLVED:

$$\left.\begin{aligned} U_{p1p} &= V_o \delta_t + D_s v_{1a} + D_s v_{2a} \delta_t \\ U_{p2p} &= \frac{D_s}{TF} + D_s v_{2a} \\ U_{p3p} &= V_o \delta_e + D_s v_{2a} \delta_e + D_s v_{3a} \end{aligned}\right\} \quad (4\text{-}329)$$

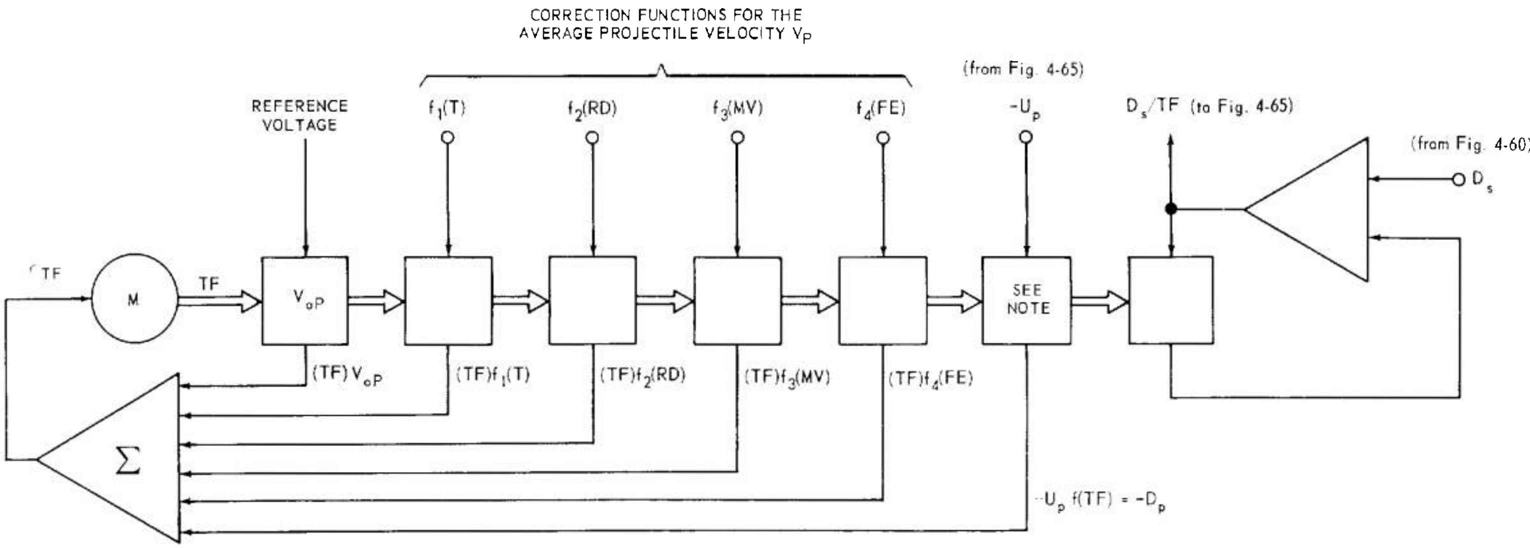
$$\left.\begin{aligned} \epsilon_{AL} = U_{p1t} &= U_{p1p} \cos AL + U_{p2p} \sin AL \cos EP - U_{p3p} \sin AL \sin EP \\ U_{p2t} &= -U_{p1p} \sin AL + U_{p2p} \cos AL \cos EP - U_{p3p} \cos AL \sin EP \\ U_{p3t} &= U_{p2p} \sin EP + U_{p3p} \cos EP \end{aligned}\right\} \quad (4\text{-}332)$$

$$\left.\begin{aligned} U_p &= U_{p2t} \cos FE + U_{p3t} \sin FE \\ \epsilon_{FE} &= U_{p3t} \cos FE - U_{p2t} \sin FE \end{aligned}\right\} \quad (4\text{-}333)$$

Figure 4-65. Block diagram of the transformation from platform coordinates to turret coordinates.

D_p — which is given by the product $U_p(TF)$ — is obtained by the use of a precision non-linear potentiometer. The resistance curve of this potentiometer is shaped to match tabulated slant-range time-of-flight data supplied by the Ballistics Research Laboratory, U.S. Army for the Vigilante round. When the servo error ϵ_{TF} is driven to zero, the sum of the voltage signals representing the product $V_P(TF)$ will be equal to the voltage signal representing D_p and the resulting angular displacement of the servo shaft will correctly represent TF.

The operation of the division circuit shown on the right-hand side of Fig. 4-66 is identical with that of the circuit analyzed in Fig. 4-60.



EQUATION SOLVED

$$\dot{T}F = (TF)V_{oP} + (TF)\left[f_1(T) + f_2(RD) + f_3(MV) + f_4(FE)\right] - D_p \qquad (4\text{-}338)$$

NOTE:

ALL POTENTIOMETERS ARE LINEAR POTENTIOMETERS EXCEPT THE ONE USED TO PRODUCE $-D_p$. THAT COMPONENT IS A PRECISION NONLINEAR POTENTIOMETER WHOSE RESISTANCE CURVE IS SHAPED TO MATCH THE SLANT-RANGE-VS.-TIME-OF-FLIGHT DATA FOR THE VIGILANTE ROUND.

Figure 4-66 Block diagram of the time-of-flight computer.

4-6.7 COMPLETE COMPUTER MECHANIZATION

The subsections of the computer that have been described in the preceding paragraphs are combined as shown in the complete functional schematic diagram of the computer given by Fig. 4-67. For reference purposes, this diagram also includes certain components that are not employed in the error analysis — for example, the radar tracking; system. This figure is obtained from a description of the final prototype system; hence, it differs in some details from the diagrams employed in the analytical phases of the system development — upon which the immediately preceding diagrams of this handbook are based.

The computer components that have been described are conventional electromechanical analog devices. The specific techniques employed in the mechanization are:

(a) Vector resolution, which is accomplished by electromagnetic resolvers mounted on gyro axes or driven by servos.

(b) Integration, which is accomplished by rate servos employing tachometric feedback or, alternatively, by torquers mounted on the gyro axes to precess tne gyro.

(c) Multiplication, which is accomplished by potentiometers driven by servos or operated by hand.

(d) Summation, in which electrical signals are summed by resistive networks and operational amplifiers.

The computer inputs are the tracking-radar range measurement D_o, the handgrip displacements Δ_{1p} and Δ_{3p}, and the various handset knobs that introduce parameters to the ballistic section. The computer outputs are the outputs of the hydraulic servos that position the gun in azimuth and elevation.

4-6.8 SOURCES AND PROPAGATION OF ERRORS

The accuracy analysis must begin with an appropriate model of the system. For this purpose, the block diagram of Fig. 4-2 is modified (see Fig. 4-68) by the addition of the target motion and the target-sight geometry, which form the input to the tracking system; and the gun-target geometry, which relates gun angles to the locations of the projectile bursts — the final output of the weapon system. The problem is then to determine a statistical measure of the error between target position and burst position, given an assumed target motion and assumed component errors,

Since the radar acquisition system and the alert alarm are not factors in the error analysis, they are not included in the block diagram of Fig. 4-68. Also, at the stage at which the accuracy analysis was made, experimental data were available on the tracking performance of human operators with the optical sight. Accordingly, the sight and the operator can be combined into a single block, labelled "tracker", of known characteristics. The resulting model for the accuracy study is as shown in Fig. 4-68. With the assumption that all errors are describable by an independent normal distribution or a combination of such distributions, so that the principle of superposition is applicable, the final error is the sum of three component errors that arise respectively in the tracker, in the computer, and in the gun.

The information available to the analysts at Frankford Arsenal included measurements of the tracking noise, which is the input error to the computer, and measurements of the errors in individual components of the computer, from which an overall computer error could be derived. Information concerning gun-tube vibrations and ammunition variations was not available; therefore, these sources of error, combined as the round-to-round error, were treated as a parametric variable in the presentation of the results.

A particular feature of the system design, mentioned earlier, makes necessary the consideration of an additional source of error. Since the computer is switched to regenerative tracking at the instant the firing button is pressed, the system is operating open-loop for an appreciable period of time while the salvo is being fired. Deviation of the aircraft from the predicted path as a result of flight roughness during this period would produce additional errors, and these were computed.

The following errors were intentionally neglected in the Frankford Arsenal analysis:(59)

(a) Errors caused by ammunition variations and gun-tube vibrations (as just mentioned)
(b) Dynamic errors in the computer, tracker, and gun-pointing servos
(c) Systematic tracking errors
(d) Errors in the introduction of ballistic correction data
(e) Errors caused by computer approximations to the true mathematical functions represented

Error sources (b) through (e) would produce increases in the computed values of the bias error, while error source (a) would increase the dispersion error.

The design of the Vigilante Antiaircraft Weapon System is such as to produce a rapid succession of bursts. In general, the burst pattern is a curved path relative to the target center, as represented in Fig. 4-69. This figure illustrates a typical salvo, shown in a moving coordinate system centered at the target center and in a plane perpendicular to the slant range through the target center. The dashed curves indicate a number of other possible salvo-burst patterns, each consisting of 48 rounds. For this type of firing pattern, the bias is defined as the distance of the mean center of impact of the bursts of a single salvo from the assumed target center; the dispersion is defined as the deviation of a particular round in the salvo from the center of impact of that salvo. The variance of the dispersion is the mean of the squared deviations, and the variance of the bias is the mean of the squared bias magnitudes obtained from a number of salvos.

The procedure followed in the analysis was to first compute the errors due to computer components and then extrapolate these through the firing interval. The component analysis was thus carried out in two steps:

1. The output errors due to component errors were determined for the normal tracking mode (see par 4-6.9).

2. These output errors were then extrapolated for the duration of the regenerative tracking mode, which covers the firing interval (see par 4-6.10).

To these extrapolated errors were added the errors computed for the flight-roughness effect (see par 4-6.11) and for the input-tracking-noise effect (see par 4-6.12). Errors were expressed in general as the variances of the bias and the dispersion. The results were used to determine the hit probability of the Vigilante Antiaircraft Weapon System (see par 4-6.13).

4-6.9 COMPONENT ERRORS ASSOCIATED WITH THE NORMAL TRACKING MODE

The computation associated with the normal tracking mode follows the transfer-function approach (whose use is described in par 4-4.4.3.2) and utilizes sets of equations of the form of Eqs. 4-194. However, since the input-noise errors are considered in par 4-6.12, only the second summation in Eqs. 4-194 is required here; i.e.,

$$\sum_{k=1}^{q} \int_{-\infty}^{\infty} |R_{y_i,\, m_k}(j\omega)|^2 \, \frac{d\,\sigma_{m_k}^2(j\omega)}{d\,(\log\omega)} \, d\,(\log\omega), \tag{4-339}$$

where the quantities are as defined in Eqs. 4-194, with

$$R_{i_k}(j\omega) = R_{y_i,\, m_k}(j\omega) \qquad \sigma_{\epsilon_{y_k}} = \sigma_{m_k}$$

This set of equations yields a set of variances for the errors at q outputs of a computer having q components.

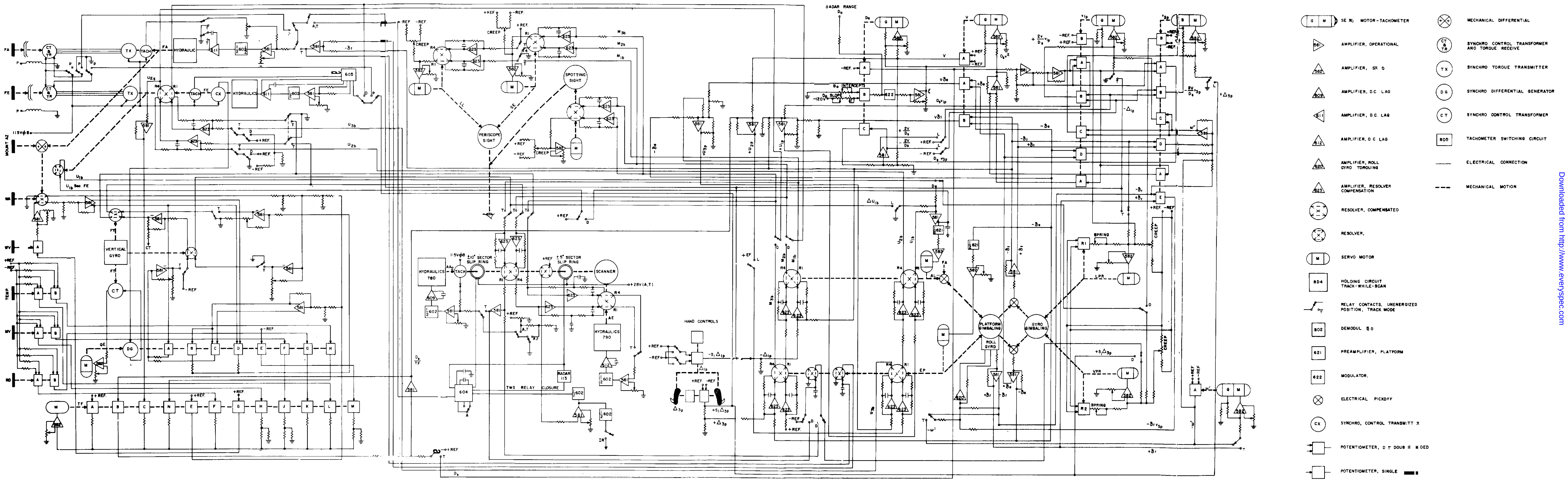


Figure 4-67 Functional schematic diagram of the computer used in the Vigilante Antiaircraft Weapon System

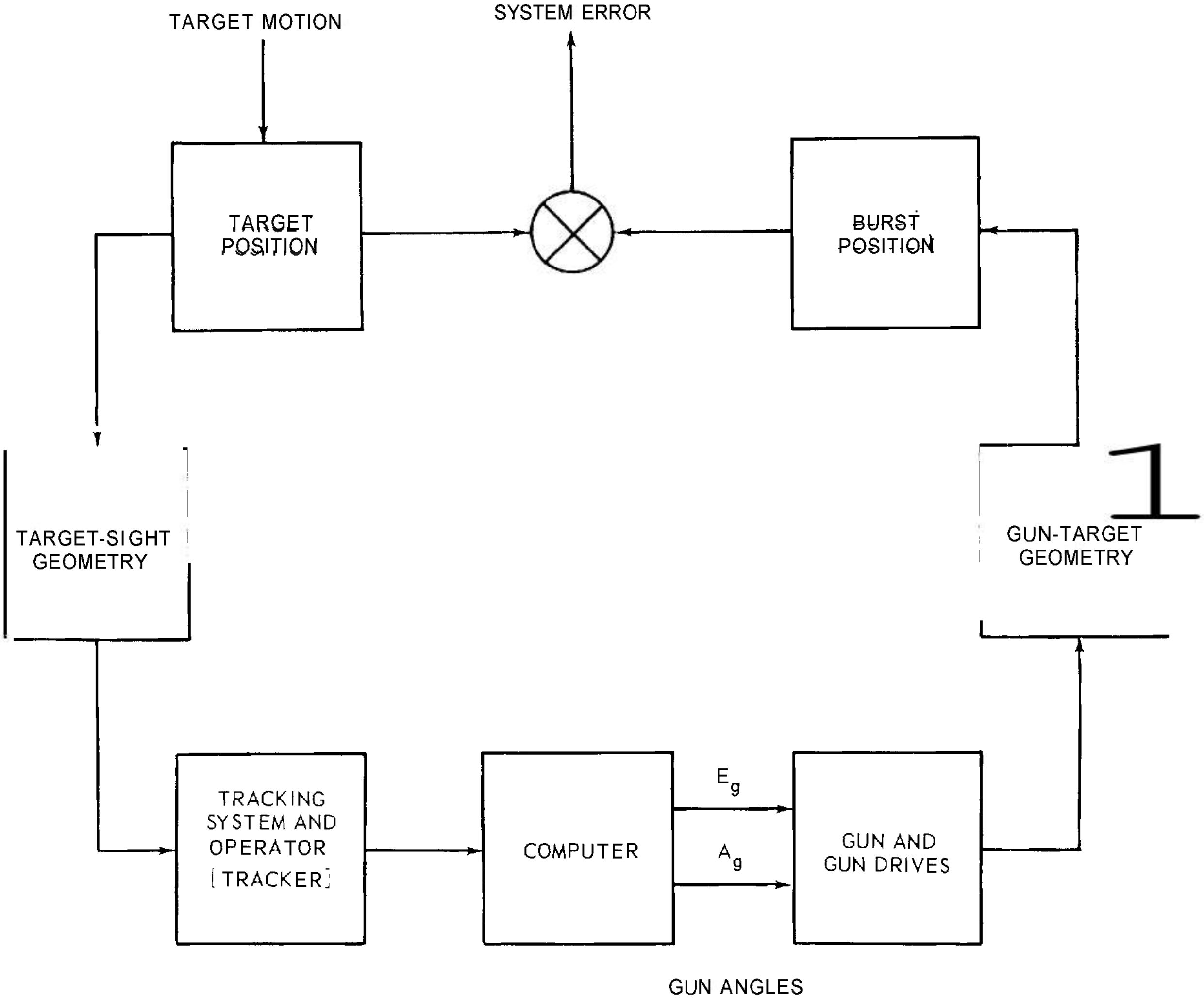


Figure 4-68. Functional block diagram of the Vigilante Antiaircraft Weapon System for error analysis.

Even if the measured power density spectra of the computer component errors were available, it would be desirable to approximate them by functions that are amenable to rapid computation. Several possibilities are available.

A common assumption is that all frequencies are equally likely, giving what is known as a white-noise spectrum (see Fig. 4-70(A)). Since a computer must have a maximum response frequency, however, an improvement in the approximation will be made if it is assumed that the white noise will be cut off at some frequency ω_c (see Fig. 4-70(B)). The cutoff frequency should be somewhat greater than the frequency that corresponds to the computer settling time.

Computer errors are commonly concentrated at the lower end of the frequency spectrum. A simple function that emphasizes the low frequencies is shown in Fig. 4-70(C). It is given by the expression

$$\Phi_{oo}(j\omega) = \frac{a^2 b^2}{\omega^2 + a^2} \tag{4-340}$$

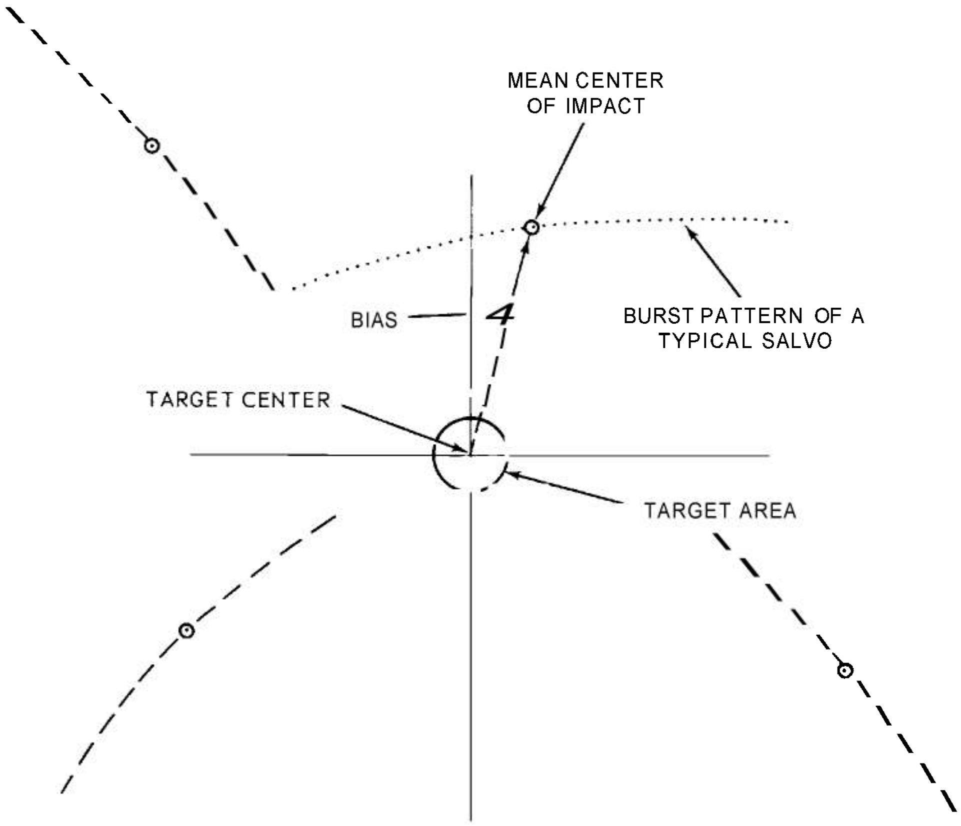


Figure 4-69. Typical burst patterns of the Vigilante Antiaircraft Weapon System.

where Φ_{OO} is a power spectral density function representing the errors of a component, and a and b are constants.

Frequently, the use of log ω as the frequency variable in place of ω is desirable in order to simplify the numerical integrations that must be carried out (see the discussion following Eq. 4-192 in par 4-4.4.3.2). Also, it eliminates the need to justify negative frequencies when considering $\int_{-\infty}^{\infty} [\quad] d\omega$; observe that log ω goes from $-\infty$ to ∞ as ω goes from 0 to ∞. A power spectral density per unit of logarithmic frequency, designated $\frac{d\sigma_o^2 (j\omega)}{d(\log \omega)}$, that is constant over a region of log w, as in Fig. 4-70(D), has the property that the power in a percentage frequency increment is constant; thus, the magnitude of the corresponding power spectral density per unit frequency, $\frac{d\sigma_o^2 (j\omega)}{d\omega}$ or $\Phi_{OO}(j\omega)$, decreases as the frequency increases, as shown in Fig. 4-70(E). The spectrum in Fig. 4-70(E) is zero for $0 < \omega < 0.001$ rad/sec; then jumps to 200; falls to quite small values in the vicinity of $\omega = 1$ rad/sec; and finally returns to zero at $\omega = 100$ rad/sec.

The settling time of the Vigilante computer is 3 seconds, corresponding to a frequency of about 0.5 cps. The power spectral density of Fig. 4-70(E) was employed, with the upper cutoff frequency at 100 rad/sec (well above the computer bandwidth). At low frequencies, the error components are varying sufficiently slowly to be included in the bias error; the

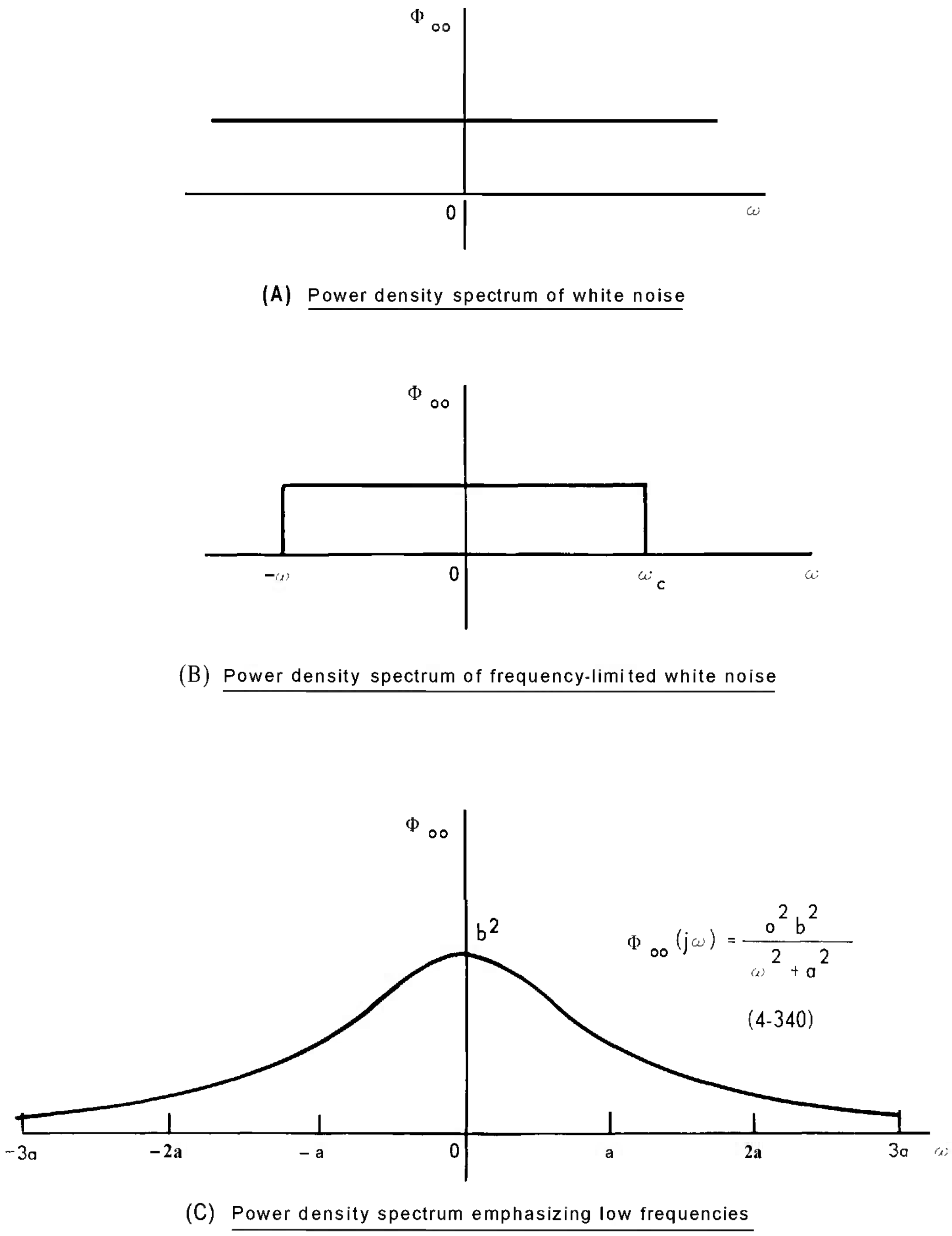
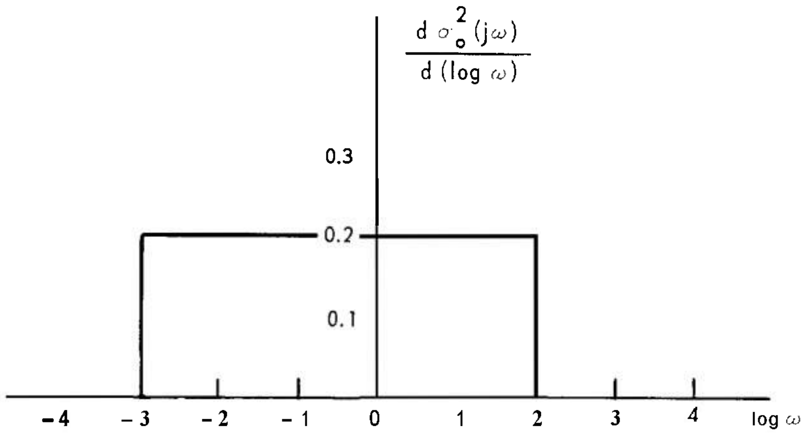


Figure 4-70, Characteristics associated with the power density spectrum.



$$\left.\begin{aligned}\frac{d\,\sigma_o^2(j\omega)}{d\,(\log\omega)} &= 0.2\ (-3 \le \log\omega \le 2)\\ &= 0 \ \text{elsewhere}\end{aligned}\right\} \qquad (4\text{-}341)$$

(D) Power spectral density function of computer component errors

(frequency variable log ω)

Figure 4-70. Characteristics associated with the power density spectrum (cont.).

lower cutoff frequency was accordingly set at 0.001 rad/sec. The logarithmic spectral density corresponding to Fig. 4-70(E) is rectangular, as shown in Fig. 4-70(D).

The power spectral densities of all the component errors were assumed to have the same frequency variation, but to differ by a multiplicative constant; thus

$$\frac{d\,\sigma_{m_k}^2(j\omega)}{d\,(\log\omega)} = C_{m_k}\,\frac{d\,\sigma_o^2(j\omega)}{d\,(\log\omega)} \qquad (4\text{-}343)$$

where $\frac{d\sigma_o^2(j\omega)}{d(\log\omega)}$ is the frequency-varying component and C_{m_k} is the set of q multiplicative constants. The application of Eqs. 4-341 and 4-343 to Eq. 4-339 yields

$$\sigma^{2} = 0.2 \int_{-3}^{-2} \sum_{k=1}^{q} {}_{k}\, |R_{y_i,\, m_k}(j\omega) \qquad \text{og}\ \omega) \qquad (4\text{-}344)$$

The R's may be evaluated from the differential equations of the components, assuming a sinusoidal component error function (this method was the one employed by Frankford Arsenal), or they may be obtained by the measurement or computation of the magnitude and

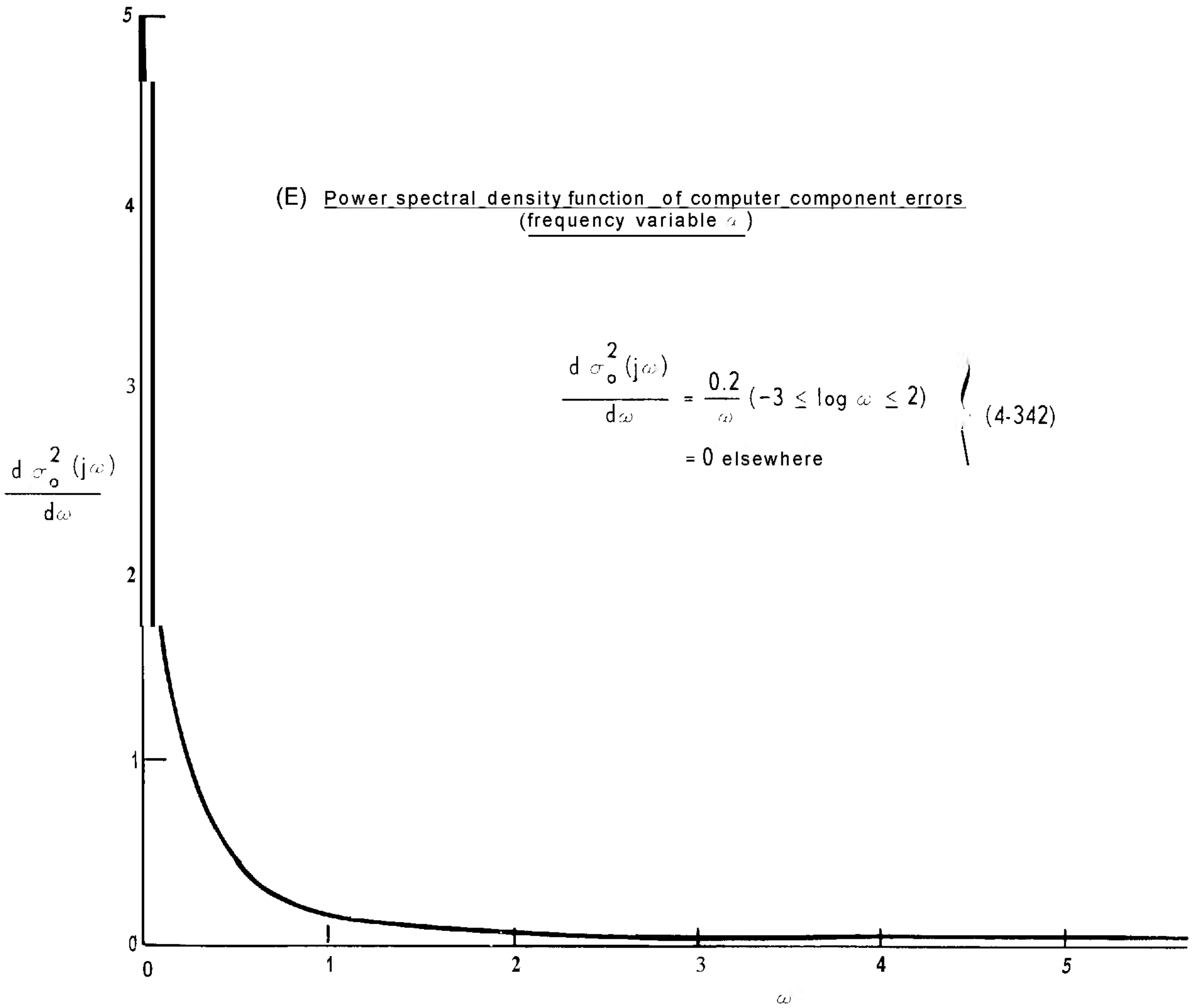
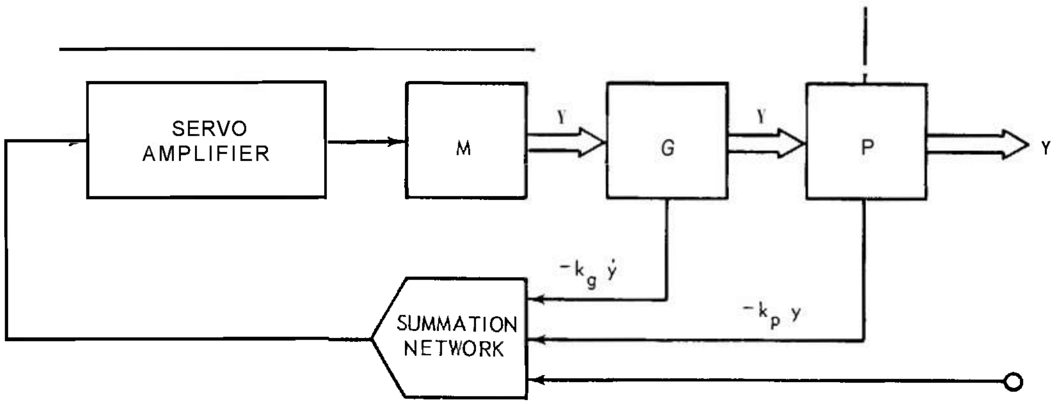


Figure 4-70. Characteristics associated with the power density spectrum (cont.).

phase angle of the transfer function of the component. Inthe case of the Vigilante computer, the servo lags were neglected, i.e., the servo elements were assumed to be dynamically ideal. Thus their error equations become quite simple as may be illustrated by considera- tion of the simple computing servo shown in Fig. 4-71.

The output of the servo —the shaft angular displacement y of an integrating motor M— is coupled to a tachometer generator G and a potentiometer P. The motor is driven by an amplifier that derives its signal fromasummation network whose inputs consist of the input voltage signal x and feedback voltage signals proportional to y and $\dot{y}$. The gain constant of the tachometer is designated k_g and that of the potentiometer, k_p. The combined gain of the summation network, amplifier, and motor is K. The output y is then related to the input voltage signal x by the relationships

$$\left.\begin{aligned} x - k_g \dot{y} - k_p y &= \epsilon \\ \text{and} \qquad \dot{y} &= K\epsilon \end{aligned}\right\} \tag{4-345}$$



$$\left.\begin{aligned} x - k_g \dot{y} - k_p y &= \epsilon \\ \dot{y} &= K\epsilon \end{aligned}\right\} \quad (4\text{-}345)$$

Figure 4-71. Functional block diagram of a simple computing servo.

where ϵ is the servo error at the output of the summation network.

The corresponding error equation of the computing servo can be obtained by the application of Eqs. 4-123 (see Derivation 4-9). The result is*

$$k_p \epsilon_y + \frac{1 + k_g K}{K} \dot{\epsilon}_y = \epsilon_x \tag{4-346}$$

*The procedure given in Derivation 4-9 to arrive at this result is provided to illustrate the application of the generalized differential error equations 4-123 to the determination of the error equation associated with given performance equations. It should be noted that in many cases the form of the performance equations may permit a shorter technique to be employed for determining the error equation. With regard to Eqs 4-345, for example, the relationship

$$x = k_p y + \frac{1 + k_g K}{K} \dot{y}$$

can be obtained by simply eliminating ϵ. This is the ideal performance equation that exists when both x and y are free from error. If x and y are assumed to have errors ϵ_x and ϵ_y, respectively, it is then readily evident from substituting $x + \epsilon_x$ for x and $y + \epsilon_y$ for y that the actual performance equation is

$$x + \epsilon_x = k_p(y + \epsilon_y) + \frac{1 + k_g K}{K} (\dot{y} + \dot{\epsilon}_y)$$

Subtraction of the ideal performance equation from the actual performance equation then shows that

$$\epsilon_x = k_p \epsilon_y + \frac{1 + k_g K}{K} \dot{\epsilon}_y$$

which is identical with Eq. 4-346.

Each computing servo may thus be represented by an error equation of the form

$$a\epsilon_y + b\dot{\epsilon}_y = c\epsilon_x \tag{4-347}$$

in which each term is, in general, a summation of terms. If the input error ϵ_x has the sinusoidal form $\epsilon_x = E_x \sin \omega t$, Eq. 4-347 has a steady-state solution in the form

$$\epsilon_y = [A \sin(\omega t + \phi) + B \cos(\omega t + \phi)]\ E_x \tag{4-348}$$

where

$$A = \frac{ac}{a^2 + b^2} \tag{4-349}$$

$$B = \frac{bc}{a^2 + b^2} \tag{4-350}$$

and ϕ = phase angle.

The parameters A and B are the real and imaginary parts, respectively, of the transfer function R of the computing servo.

The example of Fig. 4-71 may now be completed since $a = k_p$, $b = \frac{1 + k_g K}{K}$, and $c = 1$. For this example,

$$A = \frac{k_p K^2}{k_p^2 K^2 + (1 + k_g K)^2} \tag{4-351}$$

$$B = \frac{K(1 + k_g K)}{k_p^2 K^2 + (1 + k_g K)^2} \tag{4-352}$$

and

$$|R|^2 = A^2 + B^2 = \frac{K^2}{k_p^2 K^2 + (1 + k_g K)^2} \tag{4-353}$$

In a well-designed servo, $k_g K >> 1$. Therefore,

$$|R|^2 \cong \frac{1}{k_p^2 + k_g^2}. \tag{4-354}$$

The computer employed in the Vigilante Antiaircraft Weapon System (see Fig. 4-67) may be conveniently divided into sections, each having as its output a servo shaft rotation, and as inputs the signals derived from a number of potentiometers, resolvers, and the like. As an example of one such section, the servo that computes D_s, the smooth range vector or tracking vector (see Fig. 4-54), will be considered. The D_s, or range, servo is shown in Fig. 4-60,

This figure has been redrawn, with error terms included, in Fig. 4-72. The range servo is also shown in Fig. 4-73 — located in the upper left-hand corner of the figure. Figure 4-73 has been adapted from Fig. 4-67 by eliminating those computer subsections that do not appear in the error analysis. The symbols representing the component errors, the input and output errors, and the system variables are defined by Fig. 4-73. Ideal performance equations are provided where appropriate.

Figure 4-72 shows only those portions of the D_S and V (range-rate) servos that are of concern in the following error analysis. It should be noted that the constant S_1 that appears in Fig. 4-60 has been changed to S_4 in Fig. 4-72 in order to be consistent with the notation used in the Frankford Arsenal error analysis[59].

As shown by Eq. 4-319, the equation for the idealized (error-free) range servo is

$$\dot{D}_s = D_s v_{2a} + S_1 (D_o - D_s) \qquad (4\text{-}319)$$

[repeated]

This equation can be made consistent with the notation employed in the Frankford Arsenal error analysis by using the symbol S_4 in place of S_1 and by substituting V for $D_S v_{2a}$ (see par 4-6.5.3); i.e.,

$$D_s = S_4 (D_o - D_s) + V \qquad (4\text{-}355)$$

The corresponding equation with all error terms included is (see Fig. 4-72)

$$\epsilon_{s1} = -D_s - g_1 + S_4 (D_o - D_s - K_3) + V + K_{14} \qquad (4\text{-}356)$$

where

ϵ_{s1} = servo error
g_1 = the error of the range-servo tachometer generator
K_3 = the error in potentiometer No. 3
K14 = the error in potentiometer No. 14

and the other quantities have been defined previously. The ideal output of the range servo should be D_O and that of the range-rate servo should be $\dot{D}_O$. Accordingly, the output error of the range servo measured at its shaft is defined as

$$\epsilon_1 \triangleq D_s - D_o \qquad (4\text{-}357)$$

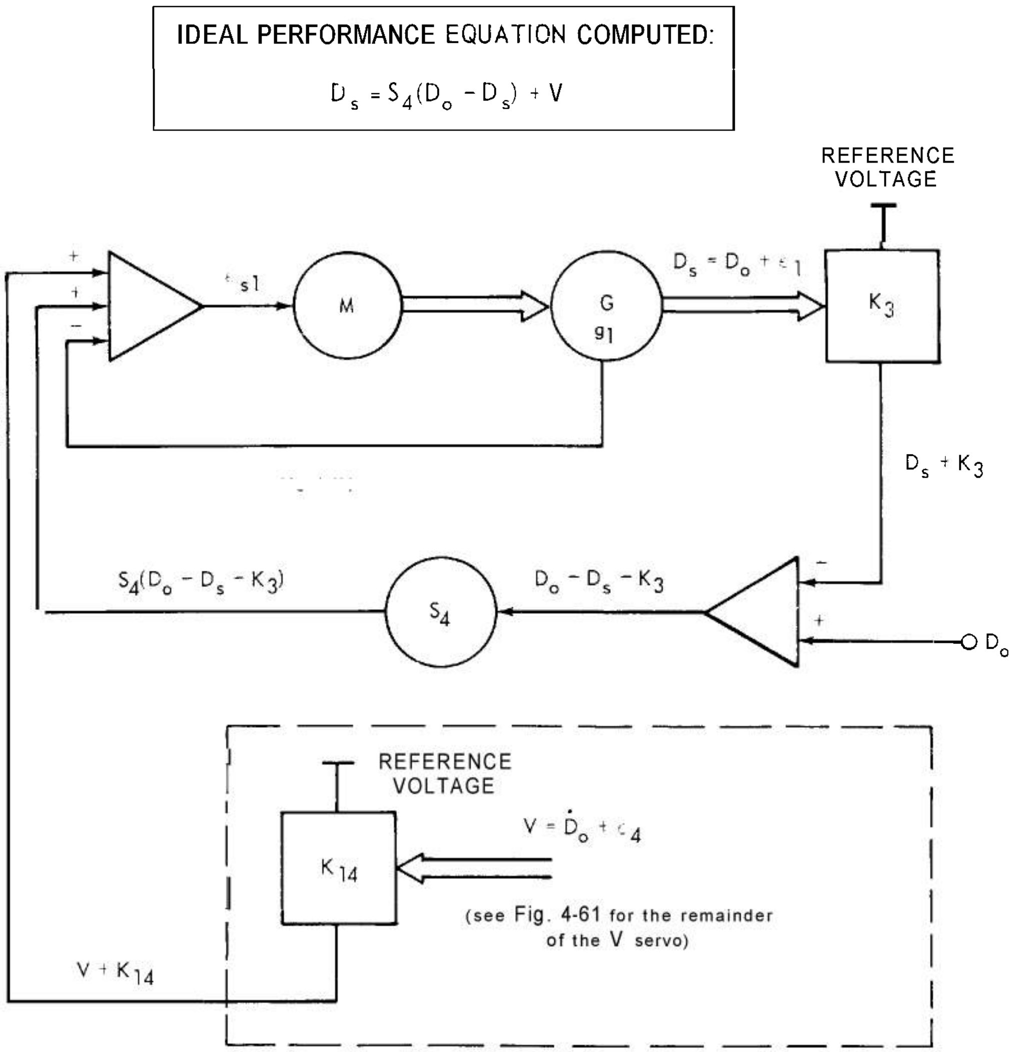
and the output error of the range-rate servo* measured at its shaft is defined as

$$\epsilon_4 \triangleq V - \dot{D}_o \qquad (4\text{-}358)$$

Substitution from Eqs. 4-357 and 4-358 into Eq. 4-356 shows — after a rearrangement of terms — that

$$\epsilon_{s1} = -\dot{D}_s - S_4 \epsilon_1 + \dot{D}_o + \epsilon_4 - g_1 - S_4 K_3 + K_{14} \qquad (4\text{-}359)$$

*The range-rate servo is represented in the functional diagram of Fig. 4-61.



ϵ_{s1} = servo error

ϵ_1 = the output error of the range servo measured ai its shaft

ϵ_4 = the output error of the range-rate servo measured at its shaft

g_1 = the error of the range-servo tachometer generator

K_3 = the error in potentiometer No. 3

K_{14} = the error in potentiometer No. 14

Figure 4-72. Functional block diagram of the range, or D_S, servo — a typical servo section of the Vigilante computer.

Also, since $D_S - \dot{D}_O = \epsilon_1$,

$$\epsilon_{s1} = -\dot{\epsilon}_1 - S_4 \epsilon_1 + \epsilon_4 - g_1 - S_4 K_3 + K_{14} \tag{4-360}$$

As the servo error ϵ_{S1} approaches zero,

$$S_4 \epsilon_1 - \epsilon_4 + \dot{\epsilon}_1 = -g_1 - S_4 K_3 + K_{14} \tag{4-361}$$

In Eq. 4-361, the input and output errors appear on the left-hand side of the equation and the component errors appear on the right-hand side. Equation 4-361 corresponds to the basic error equation, Eq. 4-115. Table 4-5, which sets forth the component error equations for the Vigilante computer, includes Eq. 4-361 as Eq. (8).

The other error equations of Table 4-5 are derived by similar techniques — or by the procedure of Derivation 4-9 for the more-complex error equations — for the other subsections of Fig. 4-73.* Errors in components other than potentiometers and tachometer-generators are defined as follows:

1. Fractional errors of resolver winding pairs are designated by the symbol β, with numerical subscripts used to identify the windings involved.
2. Fractional errors of gyro precession mechanisms are designated m, with numerical subscripts for component identification.
3. The errors of the summing networks are lumped with the potentiometer errors K.

The error equations can be solved for the output errors in the elevation and azimuth angles of the gun. These errors are designated ϵ_6 and ϵ_7, respectively.

Just prior to firing, the computer is switched from the normal tracking mode to the regenerative tracking mode, in which it continues to generate the last vector velocity that was received before switchover. The determination of the firing errors during the regenerative tracking mode, as derived from the manner in which the computer errors build up during the normal tracking mode, is described in par 4-6.10.

4-6.10 COMPONENT ERRORS ASSOCIATED WITH THE REGENERATIVE MODE

Two methods of computing the effect on the computer errors introduced by the switchover to regenerative tracking during the firing interval are available: a rigorous method and an approximate method. The rigorous method of error analysis would require that the system of error equations for the computer in its open-loop tracking mode be solved at the time of switchover. These equations have the form of Eq. 4-135, so that the solution would require the determination of the impulse responses of the computer components. (See par 4-4.4.3.1 for a description of the use of the impulse response method.) The impulse responses would then be convolved with the input error time functions to obtain the output errors at the time of switchover.

The laborious computation of all these convolution integrals was avoided by employing the approximate solution which requires only the variance of the output errors. This approach was possible because, at the time the detailed error analysis was carried out, firing tests on a prototype system had already been made by the contractor (see Reference 63). The data from these firing tests presented, in the form of a scatter diagram, the firing errors

*Note that the Frankford Arsenal error equations of Table 4-5 employ simplified notation of the form $\dot{\gamma}$ for v_{1a} and $\dot{\epsilon}_o$ for v_{3a}, for example.

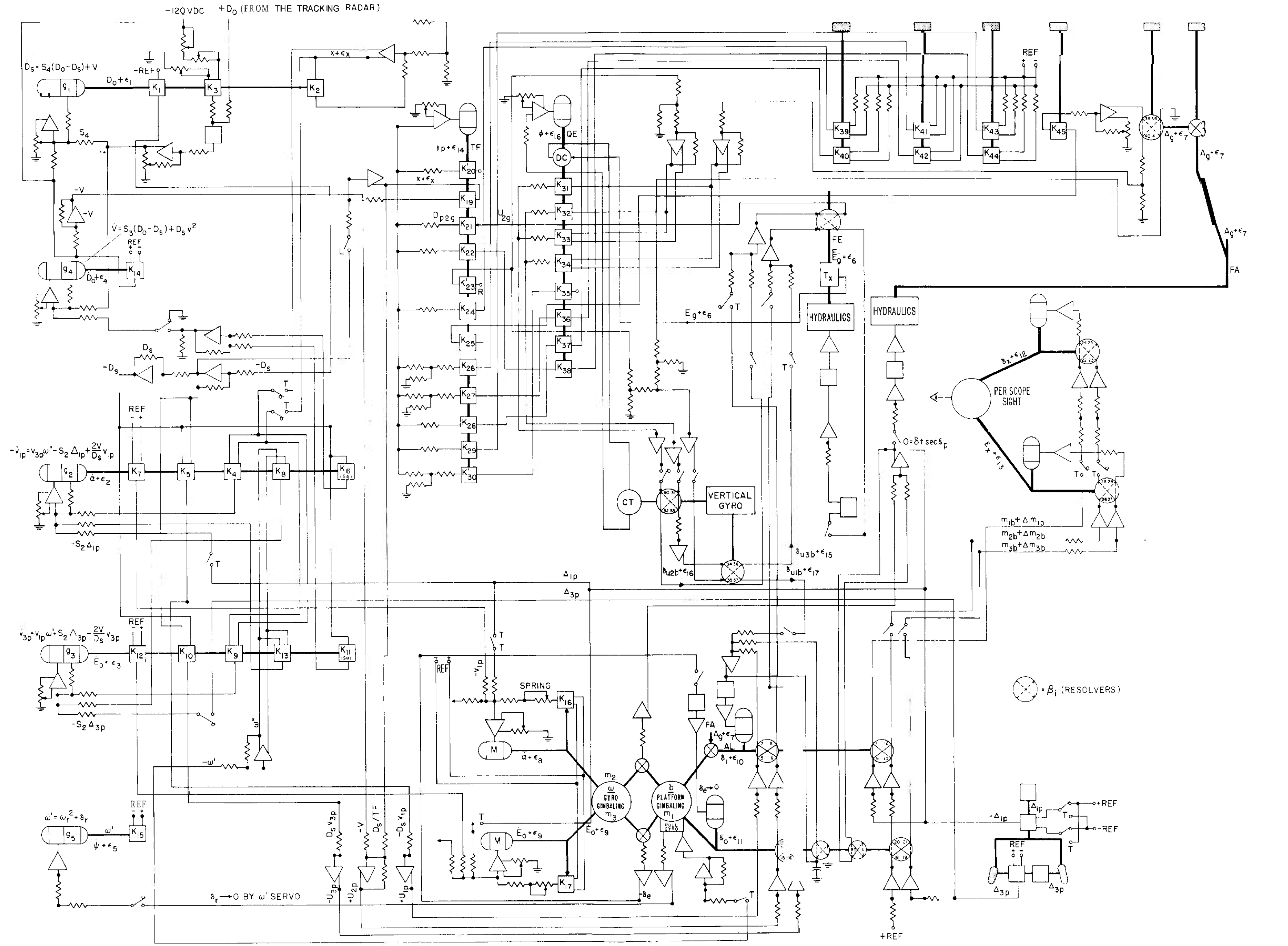


Figure 4-73. Schematic diagram of those parts of the Vigilante computer covered by the error analysis.

TABLE 4-5. TABLE OF COMPONENT ERROR EQUATIONS FOR THE VIGILANTE COMPUTER.

NOTES:

1. During the regenerative mode, the following equations are to be used with certain restrictions; specifically, Δ_{1p}, Δ_{3p}, S_3, and S_4 all equal zero. Also, equations concerning constraint (Eqs. 15 and 16) are to be deleted.

2. For convenience in presenting these equations, $\dot{\alpha}$ has been substituted for v_{1a} and $\dot{\mathcal{E}}_o$ for v_{3a}.

1. $\epsilon_8 + \dot{\psi}\,\epsilon_{11} - \cos\mathcal{E}_o\,\dot{\epsilon}_7 + \cos\mathcal{E}_o\,\dot{\epsilon}_{10} = -m_2$

2. $\epsilon_9 - \dot{\epsilon}_{11} = -m_3$

3. $\epsilon_2 - \epsilon_8 + S_1\Delta_{1p} = -K_7 + K_{16}$

4. $\epsilon_3 - \epsilon_9 + S_1\Delta_{3p} = -K_{12} + K_{17}$

5. $$\frac{-2\dot{D}_o\dot{\mathcal{E}}_o}{D_o^2}\epsilon_1 + \dot{\psi}\epsilon_2 + 2\frac{\dot{D}_o}{D_o}\epsilon_3 + 2\frac{\dot{\mathcal{E}}_o}{D_o}\epsilon_4 + \dot{\alpha}\epsilon_5 + \dot{\epsilon}_3 - S_2\Delta_{3p} = -g_3 + 2\frac{\dot{\mathcal{E}}_o\dot{D}_o}{D_o^2}K_2 - \dot{\psi}K_8 - 2\frac{\dot{D}_o}{D_o}K_9 - 2\frac{\dot{\mathcal{E}}_o}{D_o}K_{14} - \dot{\alpha}K_{15}$$

6. $$-2\frac{\dot{D}_o\dot{\alpha}}{D_o^2}\epsilon_1 + 2\frac{\dot{D}_o}{D_o}\epsilon_2 - \dot{\psi}\epsilon_3 + 2\frac{\dot{\alpha}}{D_o}\epsilon_4 - \dot{\mathcal{E}}_o\epsilon_5 + \dot{\epsilon}_2 - S_2\Delta_{1p} = -g_2 + 2\frac{\dot{\alpha}\dot{D}_o}{D_o^2}K_2 - 2\frac{\dot{D}_o}{D_o}K_4 + \dot{\psi}K_{13} - 2\frac{\dot{\alpha}}{D_o}K_{14} + \dot{\mathcal{E}}_o K_{15}$$

7. $(S_3 - \dot{\alpha}^2 - \dot{\mathcal{E}}_o^2\ \epsilon_1 - 2D_o\dot{\alpha}\epsilon_2 - 2D_o\dot{\mathcal{E}}_o\epsilon_3 + \dot{\epsilon}_4 = -g_4 + (\dot{\alpha}^2 + \dot{\mathcal{E}}_o^2)K_1 - S_3K_3 + D_oK_6 + D_oK_{11}$

8. $S_4\epsilon_1 - \epsilon_4 + \ \ 1 = -g_1 - S_4K_3 + K_{14}$

9. $\epsilon_5 - \tan\mathcal{E}_o\,\epsilon_8 - \dot{A}_o\sec\mathcal{E}_o\,\epsilon_{11} = -K_{15} + m_1 + \tan\mathcal{E}_o\,m_2$

$$\left|\begin{aligned}&\dot{\alpha}\cos\delta_1 - \cos\mathcal{E}_o\sin\delta_1\frac{1}{to} + \dot{\mathcal{E}}_o\sin\mathcal{E}_o\sin\delta_1\Big)\epsilon_1 + (D_o\cos\delta_1)\epsilon_2 + (D_o\sin\mathcal{E}_o\sin\delta_1)\epsilon_3 - (\cos\mathcal{E}_o\sin\delta_1)\epsilon_4 - \frac{R_P}{to}\epsilon_{10}\\&+\left(\sin\delta_1\frac{H_P}{to}\right)\epsilon_{11} + \left(\frac{D_o}{2}\cos\mathcal{E}_o\sin\delta_1 + f'_{23}\right)\mathcal{E}_{14}\end{aligned}\right|$$

$$=\left\{\begin{aligned}&-(\dot{\alpha}D_o)\beta_5 - \left[\left(\dot{D}_o + \frac{D_o}{tp}\right)\cos\mathcal{E}_o - \dot{\mathcal{E}}_oD_o\sin\mathcal{E}_o\right]\beta_7 + \left(\dot{D}_o + \frac{D_o}{tp}\right)\sin\delta_1\beta_{13} + (\dot{\mathcal{E}}_oD_o\sin\delta_1)\beta_{15} - f_{23}\beta_{35} - f_{32}f_{23}\beta_{37}\\&-\left(\sin\delta_1\dot{\mathcal{E}}_o\sin\mathcal{E}_o + \dot{\alpha}\cos\delta_1 - \frac{1}{tp}\cos\mathcal{E}_o\sin\delta_1\right)K_1 - D_o\cos\delta_1K_5 - D_o\sin\mathcal{E}_o\sin\delta_1K_{10} + \cos\mathcal{E}_o\sin\delta_1K_{14}\\&-\frac{D_o}{tp^2}\cos\mathcal{E}_o\sin\delta_1K_{19} - K_{23} - f_{25}\sin AK_{45}\end{aligned}\right\}$$

11. $$\left\{\begin{aligned}&\left[\left(\frac{1}{tp}\sin\mathcal{E}_o + \dot{\mathcal{E}}_o\cos\mathcal{E}_o\right)\cos\mathcal{E}_g - \left(\dot{\alpha}\sin\delta_1 + \cos\mathcal{E}_o\cos\delta_1\frac{1}{tp} - \dot{\mathcal{E}}_o\sin\mathcal{E}_o\cos\delta_1\right)\sin\mathcal{E}_g\right]\epsilon_1 - (\sin\mathcal{E}_g\sin\delta_1D_o)\epsilon_2\\&+(\cos\mathcal{E}_g\cos\mathcal{E}_oD_o + \sin\mathcal{E}_g\sin\mathcal{E}_o\cos\delta_1D_o)\epsilon_3 - (\sin\mathcal{E}_o\cos\mathcal{E}_g - \cos\mathcal{E}_o\cos\delta_1\sin\mathcal{E}_g)\epsilon_4 + \left(\cos\mathcal{E}_gf_{23}f'_{32} + \sin\mathcal{E}_gf_{23}f'_{33} - \frac{D_p}{tp}\right)\epsilon_6\\&+\left[\cos\mathcal{E}_g\left(\dot{D}_o + \frac{D_o}{tp}\right)\cos\mathcal{E}_o - \dot{\mathcal{E}}_oD_o\sin\mathcal{E}_o\cos\mathcal{E}_g + \left(\dot{D}_o + \frac{D_o}{tp}\right)\sin\mathcal{E}_o\cos\delta_1\sin\mathcal{E}_g + \dot{\mathcal{E}}_oD_o\cos\mathcal{E}_o\cos\delta_1\sin\mathcal{E}_g\right]\epsilon_{11}\\&+\left[\left(\cos\mathcal{E}_o\cos\delta_1\frac{D_o}{tp^2}\sin\mathcal{E}_g - \frac{D_o}{tp^2}\cos\mathcal{E}_g\sin\mathcal{E}_o\right) + f'_{23}f_{32}\cos\mathcal{E}_g + f_{33}f'_{23}\sin\mathcal{E}_g\right]\epsilon_{14}\end{aligned}\right.$$

$$=\left|\begin{aligned}&-\frac{H_{tp}^{\ b}}{t_p}\beta_1 - \frac{R_{pb}}{tp}\beta_3 + \left[\left(\dot{D}_o - \frac{D_o}{tp}\right)\cos\mathcal{E}_o\sin\mathcal{E}_g - \dot{\mathcal{E}}_oD_o\sin\mathcal{E}_o\sin\mathcal{E}_g\right]\beta_6 + (\dot{\alpha}D_o\sin\mathcal{E}_g)\beta_8 + \sin\mathcal{E}_g\cos\delta_1\left(\dot{D}_o + \frac{D_o}{tp}\right)\beta_{13}\\&-(\dot{\mathcal{E}}_oD_o\cos\mathcal{E}_g)\beta_{14} + (\dot{\mathcal{E}}_oD_o\sin\mathcal{E}_g\cos\delta_1)\beta_{15} - \cos\mathcal{E}_g\left(\dot{D}_o + \frac{D_o}{tp}\right)\beta_{16} - f_{33}f_{23}\sin\mathcal{E}_g\beta_{30} - f_{32}f_{23}\cos\mathcal{E}_g\beta_{31}\\&+f_{32}f_{23}\sin\mathcal{E}_g\beta_{32} + f_{33}f_{23}\cos\mathcal{E}_g\beta_{33} - f_{32}f_{23}\cos\mathcal{E}_g\beta_{34} - f_{23}\cos\mathcal{E}_g\beta_{36}\\&-\left(\frac{1}{tp}\sin\mathcal{E}_o\cos\mathcal{E}_g + \dot{\mathcal{E}}_o\cos\mathcal{E}_o\cos\mathcal{E}_g - \sin\mathcal{E}_g\sin\delta_1\dot{\alpha} - \sin\mathcal{E}_g\cos\mathcal{E}_o\cos\delta_1\frac{1}{tp} + \sin\mathcal{E}_g\sin\mathcal{E}_o\cos\delta_1\dot{\mathcal{E}}_o\right)K_1\\&+(D_o\sin\mathcal{E}_g\sin\delta_1)K_5 - (D_o\cos\mathcal{E}_g\cos\mathcal{E}_o + D_o\sin\mathcal{E}_g\sin\mathcal{E}_o\cos\delta_1)K_{10} - (\sin\mathcal{E}_o\cos\mathcal{E}_g - \cos\mathcal{E}_o\cos\delta_1\sin\mathcal{E}_g)K_{14}\\&-\left(\frac{D_o}{tp^2}\sin\mathcal{E}_g\cos\dot{\mathcal{E}}_o\cos\delta_1 - \frac{D_o}{tp^2}\cos\mathcal{E}_g\sin\mathcal{E}_o\right)K_{19} - (\cos\mathcal{E}_gf_{32} + \sin\mathcal{E}_gf_{33})K_{23} - \cos\mathcal{E}_gf_{23}K_{32} - \sin\mathcal{E}_gf_{23}K_{33}\\&+(\cos A\cos\mathcal{E}_gf_{34}f_{25} + \cos A\sin\mathcal{E}_gf_{31}f_{25})K_{45}\end{aligned}\right|$$

12. $$\sin\mathcal{E}_x\sin\delta_x\epsilon_{10} + (\cos\mathcal{E}_o\cos\mathcal{E}_x + \sin\mathcal{E}_o\sin\mathcal{E}_x\cos\delta_1)\epsilon_{11} - \cos\delta_x\epsilon_{13} - \sin\delta_1\sin\mathcal{E}_x\Delta_{1p} + (\sin\mathcal{E}_o\sin\mathcal{E}_x\cos\delta_1 + \cos\mathcal{E}_o\cos\mathcal{E}_x)\Delta_{3p}$$

$$= \cos\mathcal{E}_o\sin\mathcal{E}_x\beta_{10} + \sin\mathcal{E}_x\cos\delta_1\beta_{18} - \cos\mathcal{E}_x\beta_{21} - \sin\mathcal{E}_o\beta_{27} - \cos\delta_1\cos\mathcal{E}_o\beta_{29}$$

13. $$\left\{\begin{aligned}&\cos\mathcal{E}_x\epsilon_{10} + (\sin\delta_x\cos\mathcal{E}_x\sin\mathcal{E}_o\cos\delta_1 - \cos\delta_x\sin\mathcal{E}_o\sin\delta_1 - \sin\mathcal{E}_x\sin\delta_x\cos\mathcal{E}_o)\epsilon_{11} - \epsilon_{12} - (\cos\delta_x\cos\delta_1 + \sin\delta_x\cos\mathcal{E}_x\sin\delta_1)\Delta_{1p}\\&-(\cos\delta_x\sin\mathcal{E}_o\sin\delta_1 + \cos\mathcal{E}_o\sin\mathcal{E}_x\sin\delta_x - \sin\mathcal{E}_o\cos\delta_1\cos\mathcal{E}_x\sin\delta_x)\Delta_{3p}\end{aligned}\right\}$$

$$=\left\{\begin{aligned}&\cos\mathcal{E}_o\cos\mathcal{E}_x\sin\delta_x\beta_{10} - \cos\mathcal{E}_o\cos\delta_x\beta_{11} - (\cos\delta_x\sin\delta_1 - \sin\delta_x\cos\mathcal{E}_x\cos\delta_1)\beta_{18} + \sin\mathcal{E}_x\sin\delta_x\beta_{21} + \sin\delta_x\beta_{22}\\&-\cos\delta_x\beta_{24} + \cos\mathcal{E}_o\cos\delta_1\sin\delta_x\beta_{26} + \sin\delta_x\sin\mathcal{E}_o\beta_{28}\end{aligned}\right\}$$

14. $$\left\{\begin{aligned}&-\left[tp\cos\mathcal{E}_g(\dot{\alpha}\sin\delta_1 - \sin\mathcal{E}_o\cos\delta_1\dot{\mathcal{E}}_o) + \cos\mathcal{E}_o\cos\delta_1\cos\mathcal{E}_g + \sin\mathcal{E}_g\sin\mathcal{E}_o + \sin\mathcal{E}_g\cos\mathcal{E}_o\dot{\mathcal{E}}_o tp\right]\epsilon_1 - tp(D_o\sin\delta_1\cos\mathcal{E}_g)\epsilon_2\\&+tp(\sin\mathcal{E}_o\cos\delta_1\cos\mathcal{E}_gD_o - \sin\mathcal{E}_g\cos\mathcal{E}_oD_o)\epsilon_3 - tp(\cos\mathcal{E}_o\cos\delta_1\cos\mathcal{E}_g + \sin\mathcal{E}_g\sin\mathcal{E}_o)\epsilon_4 + \left[(f_{30}f'_{35}) - (\sin\mathcal{E}_gf_{23}f'_{32} - \cos\mathcal{E}_gf_{23}f'_{33})tp\right]\epsilon_6\\&-\left[\sin\mathcal{E}_g\cos\mathcal{E}_o(\dot{D}_otp + D_o) - \sin\mathcal{E}_g\dot{\mathcal{E}}_oD_o\sin\mathcal{E}_otp - \cos\mathcal{E}_g\sin\mathcal{E}_o\cos\delta_1(\dot{D}_otp + D_o) - \cos\mathcal{E}_g\cos\mathcal{E}_o\cos\delta_1\dot{\mathcal{E}}_oD_otp\right]\epsilon_{11}\\&+\left\{\left[\frac{D_o}{tp}(\cos\delta_1\cos\mathcal{E}_o\cos\mathcal{E}_g + \sin\mathcal{E}_g\sin\mathcal{E}_o)\right] - tp\,[\sin\mathcal{E}_gf'_{23}f_{32} - \cos\mathcal{E}_gf_{33}f'_{23}] - \frac{D_{pb}}{tp} + f'_{20} + f_{35}f'_{30}\right\}\epsilon_{14}\end{aligned}\right\}$$

$$=\left\{\begin{aligned}&R_{pb}\beta_2 + H_{pb}\beta_4 + \left[(\dot{D}_otp + D_o)\cos\mathcal{E}_o - tp\dot{\mathcal{E}}_oD_o\sin\mathcal{E}_o\right]\cos\mathcal{E}_g\beta_6 + (\dot{\alpha}tpD_o\cos\mathcal{E}_g)\beta_8 + (\dot{D}_otp + D_o)\cos\delta_1\cos\mathcal{E}_g\beta_{13} + \sin\mathcal{E}_g\dot{\mathcal{E}}_oD_otp\beta_{14}\\&+tp\dot{\mathcal{E}}_oD_o\cos\delta_1\cos\mathcal{E}_g\beta_{15} + \sin\mathcal{E}_g(\dot{D}_otp + D_o)\beta_{16} - tp\cos\mathcal{E}_gf_{33}f_{23}\beta_{30} + tp\sin\mathcal{E}_gf_{32}f_{23}\beta_{31} + tp\cos\mathcal{E}_gf_{32}f_{23}\beta_{32} - tp\sin\mathcal{E}_gf_{33}f_{23}\beta_{33}\\&+tp\sin\mathcal{E}_gf_{32}f_{23}\beta_{34} + tp\sin\mathcal{E}_gf_{23}\beta_{36} + (\dot{\alpha}tp\sin\delta_1\cos\mathcal{E}_g + \cos\mathcal{E}_g\cos\delta_1\cos\mathcal{E}_o - \sin\mathcal{E}_o\cos\delta_1\cos\mathcal{E}_g\dot{\mathcal{E}}_otp + \sin\mathcal{E}_g\sin\mathcal{E}_o + tp\dot{\mathcal{E}}_o\sin\mathcal{E}_g\cos\mathcal{E}_o)K_1\\&+tpD_o\sin\delta_1\cos\mathcal{E}_gK_5 + (\sin\mathcal{E}_g\cos\mathcal{E}_o - \sin\mathcal{E}_o\cos\delta_1\cos\mathcal{E}_g)D_otpK_{10} + (\cos\mathcal{E}_g\cos\delta_1\cos\mathcal{E}_o + \sin\mathcal{E}_g\sin\mathcal{E}_o)tpK_{14}\\&-\frac{D_o}{tp}(\cos\mathcal{E}_g\cos\delta_1\cos\mathcal{E}_o + \sin\mathcal{E}_g\sin\mathcal{E}_o)K_{19} - K_{20} - \frac{D_{pb}}{tp}K_{21} + tp(\sin\mathcal{E}_gf_{32} - \cos\mathcal{E}_gf_{33})K_{23} - f_{35}K_{30} + tp\sin\mathcal{E}_gf_{23}K_{32}\\&+tp\cos\mathcal{E}_gf_{23}K_{33} - f_{30}K_{35} - f_{24}K_{39} - f_{37}f_{28}K_{40} - f_{26}K_{41} - f_{27}f_{36}K_{42} - f_{29}K_{43} - f_{22}f_{38}K_{44} + tp(\cos\mathcal{E}_g\cos A f_{31}f_{25} - \sin\mathcal{E}_g\cos A f_{34}f_{25})K_{45}\end{aligned}\right.$$

15. no error in sight position $\}\ \tan\mathcal{E}_x\tan\delta_x\epsilon_{12} - \epsilon_{13} = 0\ \{$

16. $\epsilon_7 - \sec\mathcal{E}_o(\cos\mathcal{E}_x\sec\mathcal{E}_o + \sin\mathcal{E}_o\sin\mathcal{E}_x\sin\delta_1\tan\delta_1)\epsilon_{12} = 0$ (no error in sight position)

resultingfrom random computer errors. Sets of computer component errors were randomly selected, and the resulting vertical and lateral components of the firing errors were plotted as functions of time. By this means, it was demonstrated that the firing errors vary linearly with time.

The vertical and horizontal components of the firing error are designated $V_i(t)$ and $L_i(t)$, respectively, with i having values from 1 to n. These components are measured in target-centered coordinates, and would normally be measured in feet or yards. In order to keep all computations in angular measure, however, the equivalent angle in milliradians measured at the gun is used. Therefore, values of $V_i(t)$ and $L_i(t)$ may be converted to miss distances through multiplication by the range D_p. Since the errors vary linearly with time during the regenerative tracking interval, values of $V_i(t_o)$, $L_i(t_o)$, $V_i(t_2)$, and $L_i(t_2)$ explicitly define the regenerative tracking period, where t_o is the starting time and t_2 the stopping time of regenerative tracking.

The variance of the firing errors at the start of regeneration may be determined from the set of n errors by restating the definition of the variance (given by Eq. 4-37) in terms of ensemble averages. So stated, the variance is the difference between the ensemble average of the squares of n equally probable errors, and the square of the ensemble average of these errors, or

$$\sigma^2_{V_i}(t_o) = \frac{1}{n} \sum^{n} \qquad \left| \quad \sum V_i(t_o) \right| \tag{4-362}$$

and

$$\sigma^2_{L_i}(t_o) = \frac{1}{n} \sum_{i=1}^{n} L_i^2(t_o) - \left[\frac{1}{n} \sum_{i=1}^{n} L_i(t_o) \right]^2 \tag{4-363}$$

where

$\sigma^2_{V_i}(t_o)$ = the variance of the vertical components of the set of n firing errors at time t_o

$\sigma^2_{L_i}(t_o)$ = the corresponding horizontal variance

Reference to Fig. 4-69 shows that the bias of a particular salvo may 'be computed by time-averaging the errors over the firing interval. The dispersion is given by the time average of the squared errors over the same interval, minus the bias squared.

The time averages of the firing errors are

$$\widetilde{V_i(t)} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} V_i(t)\, dt \tag{4-364}$$

and

$$\overline{L_i(t)} = \frac{}{t_2 - t_1} \int_{t_1}^{t_2} L_i(t)\, dt \qquad 1 \tag{4-365}$$

where $\widetilde{V_i(t)}$ and $\widetilde{L_i(t)}$ are the time averages* of $V_i(t)$ and $L_i(t)$, respectively, t_1 is the time at which firing starts, and t_2 is the time that regenerative tracking and firing ceases.

*As discussed in par 4-4. 2. 3, a wiggly line over the random variable is employed to represent a time average, while a straight line represents an ensemble average; for example, $\tilde{x}$ or $\bar{x}$.

In the Vigilante system, firing starts 1/3 second after t_o, the start of regenerative tracking, and persists for one second. Thus,

$$t_1 = t_o + 1/3 \text{ second} \tag{4-366}$$

and

$$t_2 = t_o + 4/3 \text{ second.} \tag{4-367}$$

Since it has been established that the firing errors vary linearly with time, Eqs. 4-364 and 4-365 may be evaluated in terms of $V_i(t_o)$ and $V_i(t_2)$, as shown in Derivation 4-9. The results are

$$\widetilde{V_i(t)} = \frac{5}{8} V_i(t_2) + \frac{3}{8} V_i(t_o) \tag{4-368}$$

and

$$\overline{L_i(t)} = \frac{5}{8} L_i(t_2) + \frac{3}{8} L_i(t_o). \tag{4-369}$$

Similarly, the time averages of the squared firing errors are

$$\widetilde{V_i^2(t)} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} V_i^2(t)\, dt = \frac{4}{9\left[V_i(t_2) - V_i(t_o)\right]} \left\{ V_i^3(t_2) - \left[\frac{1}{4} V_i(t_2) + \frac{3}{4} V_i(t_o)\right]^3 \right\} \tag{4-370}$$

and

$$\widetilde{L_i^2(t)} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} L_i^2(t)\, dt = \frac{4}{9\left[L_i(t_2) - L_i(t_o)\right]} \left\{ L_i^3(t_2) - \left[\frac{1}{4} L_i(t_2) + \frac{3}{4} L_i(t_o)\right]^3 \right\} \tag{4-371}$$

The set of n values of the firing-error components $V_i(t_o)$, $L_i(t_o)$, $V_i(t_2)$, and $L_i(t_2)$ given in References 59 and 63 was entered in Eqs. 4-368 and 4-369 to give a set of n values of the bias error (in vertical and horizontal components). The variances of these sets of component values of the bias were then determined from ensemble averages of the squares of the bias component values. The components of the variance of the set of bias errors that result from this procedure are given by the relationships

$$\sigma_{bV}^2 = \overline{\widetilde{V_i(t)}^2} = \frac{1}{n} \sum_{i=1}^{n} \left[\frac{5}{8} V_i(t_2) + \frac{3}{8} V_i(t_o)\right]^2 \tag{4-372}$$

and

$$\sigma_{bL}^2 = \overline{\widetilde{L_i(t)}^2} = \frac{1}{n} \sum_{i=1}^{n} \left[\frac{5}{8} L_i(t_2) + \frac{3}{8} L_i(t_o)\right]^2 \tag{4-373}$$

where σ_{bV}^2 and σ_{bL}^2 are the vertical and lateral components of thevariance of the bias, respectively, and $\overline{\widetilde{V}_i(t)}^2$ and $\overline{\widetilde{L}_i(t)}^2$ are the ensemble averages of the squares of the vertical and lateral component time averages $\widetilde{V}_i(t)$ and $\widetilde{L}_i(t)$, respectively.

The dispersion of the vertical errors is given by $\widetilde{V_i^2(t)} - \widetilde{V}_i(t)^2$. The variance of the dispersion for the set of data is the ensemble average; therefore,

$$\sigma_{dV}^2 = \overline{\widetilde{V_i^2(t)} - \widetilde{V}_i(t)^2} = \overline{\widetilde{V_i^2(t)}} - \overline{\widetilde{V}_i(t)^2} = \overline{\widetilde{V_i^2(t)}} - \sigma_{bV}^2 \tag{4-374}$$

'where σ_{dV}^2 is the vertical component of the variance of the dispersion and, as before, each bar indicates the ensemble average of the quantity beneath it. Application of Eqs. 4-370 and 4-372 shows that

$$\sigma_{dV}^2 = \frac{1}{n} \sum_{i=1}^{n} \left\{ \frac{4}{9} \, \frac{V_i^3(t_2) - \left[\frac{1}{4} V_i(t_2) + \frac{3}{4} V_i(t_o)\right]}{V_i(t_2) - V_i(t_o)} \quad \Big| \quad {}_i(t_2) + \frac{3}{8} V_i(t_o) \right] \tag{4-375}$$

Similarly, the expression for σ_{dL}, the lateral component of the variance of the dispersion, is

$$\sigma_{dL}^2 = \overline{\widetilde{L_i^2(t)}} - \overline{\widetilde{L}_i(t)^2} = \overline{L_i^2(t)} - \sigma_{bL}^2 \tag{4-376}$$

Application of Eqs. 4-371 and 4-373 shows that

$$\sigma_{dL}^2 = \frac{1}{n} \sum_{i=1}^{n} \left\{ \frac{4}{9} \, \frac{L_i^3(t_2) - \left[\frac{1}{4} L_i(t_2) + \frac{3}{4} L_i(t_o)\right]^3}{L_i(t_2) - L_i(t_o)} - \left[\frac{5}{8} L_i(t_2) + \frac{3}{8} L_i(t_o)\right]^2 \right\} \tag{4-377}$$

The set of n known values of $V_i(t_o)$, and also of $V_i(t_2)$, $L_i(t_o)$, and $L_i(t_2)$, permits the evaluation of the variances from Eqs. 4-372 through 4-377.

The error analysis of the computer has produced values of the variances $\sigma_{\epsilon_6}^2$ and $\sigma_{\epsilon_7}^2$ of the angular errors in the elevation and azimuth angles of the gun, which are designated ϵ_6 and ϵ_7, respectively. These errors may be transformed to a coordinate system centered at the target as shown in Fig. 4-74. The linear errors (miss distances) are, as the figure indicates, $\epsilon_6 D_p$ in the vertical axis and $\epsilon_7 D_p \cos E_g$ in the horizontal axis, where D_p is the range to the target at impact and E_g is the elevation angle of the gun. Again using'angular measure, the vertical component of the error is ϵ_6, while the lateral component is $\epsilon_7 \cos E_g$. The vertical variance is $\sigma_{\epsilon_6}^2$, while the lateral variance is $\sigma_{\epsilon_7}^2 \cos^2 E_g$.

If, at time t_o, the ratios of the variances of the computer errors to the variances of the measured set of firing errors are defined as

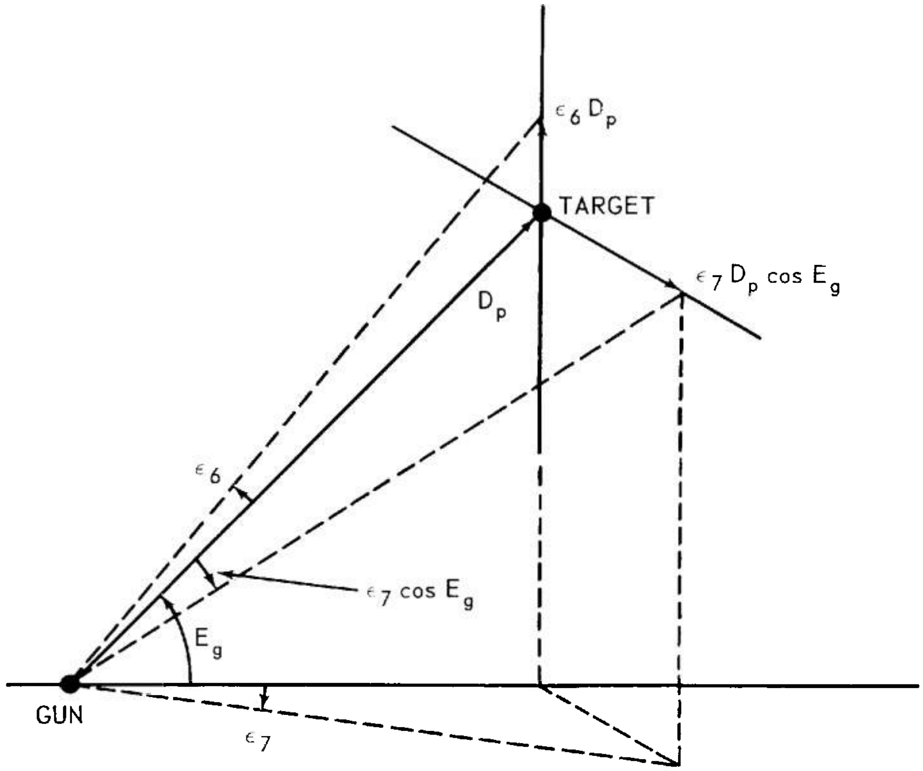


Figure 4-74. Relationship of the computer errors ϵ_6 and ϵ_7 to the gun-target geometry.

$$K_V^2 = \sigma_{\epsilon_6}^2 (t_o) \Big/ \sigma_{V_i}^2 (t_o) \tag{4-378}$$

$$K_L^2 = \sigma_{\epsilon_7}^2 (t_o) \cos^2 E_g \Big/ \sigma_{L_i}^2 (t_o) \tag{4-379}$$

and if it is assumed that the ratios K_V^2 and K_L^2 are invariant during the regenerative period, then the total bias error caused by computer errors may be obtained from the relationship

$$\sigma_b^2 = \sigma_{b\epsilon_6}^2 + \sigma_{b\epsilon_7}^2 \cos^2 E_g = K_V^2 \sigma_{bV}^2 + K_L^2 \sigma_{bL}^2 \tag{4-380}$$

where

σ_b^2 = variance of the total angular bias error

$\sigma_{b\epsilon_6}^2$ and $\sigma_{b\epsilon_7}^2$ = variances of the bias components, respectively, of the elevation and azimuth output errors of the computer.

The corresponding expression for the total dispersion is

$$\sigma_d^2 = \sigma_{d\epsilon_6}^2 + \sigma_{d\epsilon_7}^2 \cos^2 E_g = K_V^2 \sigma_{dV}^2 + K_L^2 \sigma_{dL}^2 \tag{4-381}$$

where

σ_d^2 = variance of the total angular dispersion error

$\sigma_{d\epsilon_6}^2$ and $\sigma_{d\epsilon_7}^2$ = variances of the dispersion components, respectively, of the elevation and azimuth output errors of the computer

The results computed by Frankford Arsenal for five points representing five typical sets of firing conditions* are tabulated in Table 4-6. As may be seen from the table, the variances of the elevation and azimuth computer errors at time t_o are tabulated for each point. The corresponding values of $\sigma_{V_i}^2(t_o)$ and $\sigma_{L_i}^2(t_o)$ from the firing tests are also entered. The ratios K_V^2 and K_L^2 can then be computed from Eqs. 4-378 and 4-379, and the total variances, σ_b^2 and σ_d^2, can be obtained from Eqs. 4-380 and 4-381. As might be expected, the errors are about the same in the vertical and lateral directions. The computer errors contribute principally to the bias, with a very minor dispersion component.

The variances of the total bias and dispersion of the miss distance can be computed by multiplying both σ_b^2 and σ_d^2 by the square of D_p, the predicted slant range at the midpoint of the firing interval (see Table 4-6).

4-6. 11 FLIGHT-ROUGHNESS ERRORS

Even though the pilot of a target aircraft may be attempting to hold a uniform course for firing or bombing purposes, atmospheric effects (generally classed as gusts) will induce random motions of the aircraft. During the tracking period, these effects are lumped with the tracker-generated noise. During the firing interval, however, the miss distance caused by the flight roughness must be computed separately.

To do this, it is necessary to derive expressions similar to those derived in par 4-4. 4. 3. 2 (which describes the transfer-function approach) for samples of a stationary random variable having a known power density spectrum, for a given time interval [64]. Consider a sinusoidal error ϵ of the form

$$\epsilon = |\epsilon| \sin \omega t. \tag{4-382}$$

The mean value of ϵ in an interval $t_o < t < t_o + \tau$ is

$$\overline{\epsilon(j\omega, \tau, t_o)} = \frac{1}{\tau} \int_{t_o}^{t_o+\tau} |\epsilon| \sin \omega t \, dt \tag{4-383}$$

$$\frac{|\epsilon|}{\omega\tau} \left[\quad \omega t_o - \cos \omega (t_o + \tau) \right]. \tag{4-384}$$

If all values of t_o are equally likely, the variance of this bias or mean error is

*These conditions are defined in Table 4-11 and the associated text; see par 4-6. 13.

$$
\begin{aligned}
\sigma_b^2(j\omega,\tau) &= \lim_{T\to\infty}\left\{\frac{1}{2T}\int_{-T}^{T}\bar{\epsilon}^2\,dt_o\right\} \\
&= \lim_{T\to\infty}\left\{\frac{1}{2T}\int_{-T}^{T}\frac{|\epsilon|^2}{\omega^2\tau^2}\left[\cos\omega t_o - \cos\omega(t_o + \tau)\right]^2 dt,\right\} \qquad (4\text{-}385) \\
&= \frac{|\epsilon|^2}{\omega^2\tau^2}(1 - \cos\omega\tau).
\end{aligned}
$$

Similarly, the variance of the dispersion about the mean is

$$
\begin{aligned}
\sigma_d^2(j\omega,\tau) &= \lim_{T\to\infty}\left\{\frac{1}{2T}\int_{-T}^{T}\left[\overline{\epsilon^2} - \bar{\epsilon}^2\right]dt_o\right\} \\
&= \lim_{T\to\infty}\left\{\frac{1}{2T}\int_{-T}^{T}\left[\left(\frac{1}{\tau}\int_{t_o}^{t_o+\tau}|\epsilon|^2\sin^2\omega t\,dt\right) - \left(\frac{1}{\tau}\int_{t_o}^{t_o+\tau}|\epsilon|\sin\omega t\,dt\right)^2\right]dt_o\right\} \qquad (4\text{-}386)
\end{aligned}
$$

In Eqs. 4-385 and 4-386, the superscript bar denotes an average of the variable concerned over the interval $t_o < t < t_o + \tau$. The inner integrals of Eq. 4-386 are

$$
|\epsilon|^2\left\{\frac{1}{2} + \frac{1}{4\omega\tau}\left[\sin 2\omega t_o - \sin 2\omega(t_o + \tau)\right] - \frac{1}{\omega^2\tau^2}\left[\cos\omega t_o - \cos\omega(t_o + \tau)\right]^2\right\}
$$

and the result of the second integration is

$$
\sigma_d^2(j\omega,\tau) = \frac{1}{2}|\epsilon|^2 - \frac{|\epsilon|^2}{\omega^2\tau^2}(1 - \cos\omega\tau). \qquad (4\text{-}387)
$$

The quantities $\sigma_b^2(j\omega,\tau)$ and $\sigma_d^2(j\omega,\tau)$ are the contributions of a particular frequency w to the total variances of the mean and dispersion. To find the total variances, the ratios of the variances of the desired parameters to the variance of the error will be defined as

$$
|R_b(j\omega,\tau)|^2 = \frac{\sigma_b^2(j\omega,\tau)}{\sigma_\epsilon^2} \qquad (4\text{-}388)
$$

and

$$
|R_d(j\omega,\tau)|^2 = \frac{\sigma_d^2(j\omega,\tau)}{\sigma_\epsilon^2} \qquad (4\text{-}389)
$$

TABLE 4-6. BIAS AND DISPERSION CAUSED BY COMPUTER ERRORS.

[These data are based on Fig. AI in Vol. II of Reference 59]

Point	Midpoint Predicted Range D_p (yd)	Variance of Elevation Error $\sigma^2_{\epsilon_6}(t_o)$	Variance of Azimuth Error $\sigma^2_{\epsilon_7}(t_o)$	Variance of Vertical Error $\sigma^2_{V_i}(t_o)$	Variance of Lateral Error $\sigma^2_{L_i}(t_o)$	K^2_V	L^2_V
I	4000	49.71	37.19	23. 90	26. 20	2, 08	1, 29
II	3000	13.65	15.09	11.20	10. 40	1, 22	1, 27
III	2000	4. 84	7.74	4. 50	7. 40	1, 08	0, 80
IV	1500	2.95	12.73	3. 60	7.20	0, 82	1, 05
V	2000	19.18	6. 86	6. 60	8. 00	2, 91	0, 57

Point	Vertical Component of Bias Variance σ^2_{bV}	Lateral Component of Bias Variance σ^2_{bL}	Variance of Total Bias σ^2_b	Variance of Total Bias of the Miss Distance $D_p^2\sigma_b^2$ (yd^2)	Standard Deviation of Total Bias of the Miss Distance $D_p\sigma_b$ (yd)	Vertical Component of Dispersion Variance σ^2_{dV}	Lateral Component of Dispersion Variance σ^2_{dL}	Variance of Total Dispersion σ^2_d	Variance of the Total Dispersion of the Miss Distance $D_p^2\sigma_d^2$ (yd^2)	Standard Deviation of the Total Dispersion of the Miss Distance $D_p\sigma_d$ (yd)
I	53.30	48.60	173.56	2776. 96	52.70	0. 960	0. 365	2. 47	39.52	6.29
II	24.00	24. 30	60.14	541. 26	23. 27	0.435	0. 297	0.91	8.19	2.86
III	13. 80	17.00	28. 50	114.00	10'.68	0.200	0.315	0.47	1.88	1.37
IV	10.00	15.50	24. 48	55.08	7.42	0. 167	0. 340	0.49	1.10	1.05
V	6. 00	14.50	25. 73	102. 92	10. 14	0, 290	0.390	1. 07	4.28	2.07

Notes: (1) All variances are expressed in (mils)^2 except as specifically noted.
(2) $\sigma^2_{\epsilon_6}$ and $\sigma^2_{\epsilon_7}$ were computed by Frankford Arsenal.
(3) $\sigma^2_{V_i}$ and $\sigma^2_{L_i}$ were measured in firing tests by the contractor.

As shown by Derivation 4-10,

$$\sigma_\epsilon^2 = \frac{1}{2} |\epsilon|^2 \tag{4-390}$$

Therefore

$$|R_b(j\omega,\tau)|^2 = \frac{2}{\omega^2\tau^2}(1 - \cos\omega\tau) \tag{4-391}$$

$$|R_d(j\omega,\tau)|^2 = 1 - \frac{2}{\omega^2\tau^2}(1 - \cos\omega\tau) = 1 - |R_b(j\omega,\tau)|^2 \tag{4-392}$$

As indicated by the notation, the ratios $|R_b(j\omega\tau)|^2$ and $|R_d(j\omega\tau)|^2$ are transfer functions (see Fig. 4-34) and the total variance may be found in each case by integrating the product of the ratio and the power spectral density of the error over all ω, as in Eq. 4-189. Thus,

$$\sigma_b^2 = \int_{-\infty}^{\infty} \frac{2}{\omega^2\tau^2}(1 - \cos\omega\tau)\frac{d(\sigma^2}{dw}\,du \tag{4-393}$$

and

$$\sigma_d^2 = \int_{-\infty} \left[\frac{-}{2\ 2}(1 - \cos\omega\tau) \right| \frac{}{d\omega}\,d\omega \tag{4-394}$$

where the previous notation for the bias, or mean h, has been substituted for $\bar{\epsilon}$, and

$$\frac{d(\sigma_\epsilon^2)}{d\omega} = \frac{\Phi_{\epsilon\epsilon}(j\omega)}{2\pi} = \text{the power spectral density of the error.} \tag{4-395}$$

Values of the power spectral density of the miss distance caused by flight roughness are given by Harries[65] for typical aircraft. These data are given per unit variance of miss distance, which is given in turn by.

$$\sigma_\epsilon^2 = 9(t_p + t_m)^2 \tag{4-396}$$

where

σ_ϵ^2 = the variance of the miss distance in (yards)^2

t_p = the time of flight of the projectile in seconds

t_m = the time from t_o, the start of regenerative tracking to the midpoint of the firing interval in seconds

As shown in Fig. 4-75, the sum $t_p + t_m$ is the prediction time.

The results of integrating Eqs. 4-393 and 4-394 with the spectral densities given in Ref. 65 are presented in Table 4-7 for the five points previously chosen for calculation (see Table 4-6 and the associated text).

Note that an appreciable bias error, but negligible dispersion, is induced by the flight-roughness effect.

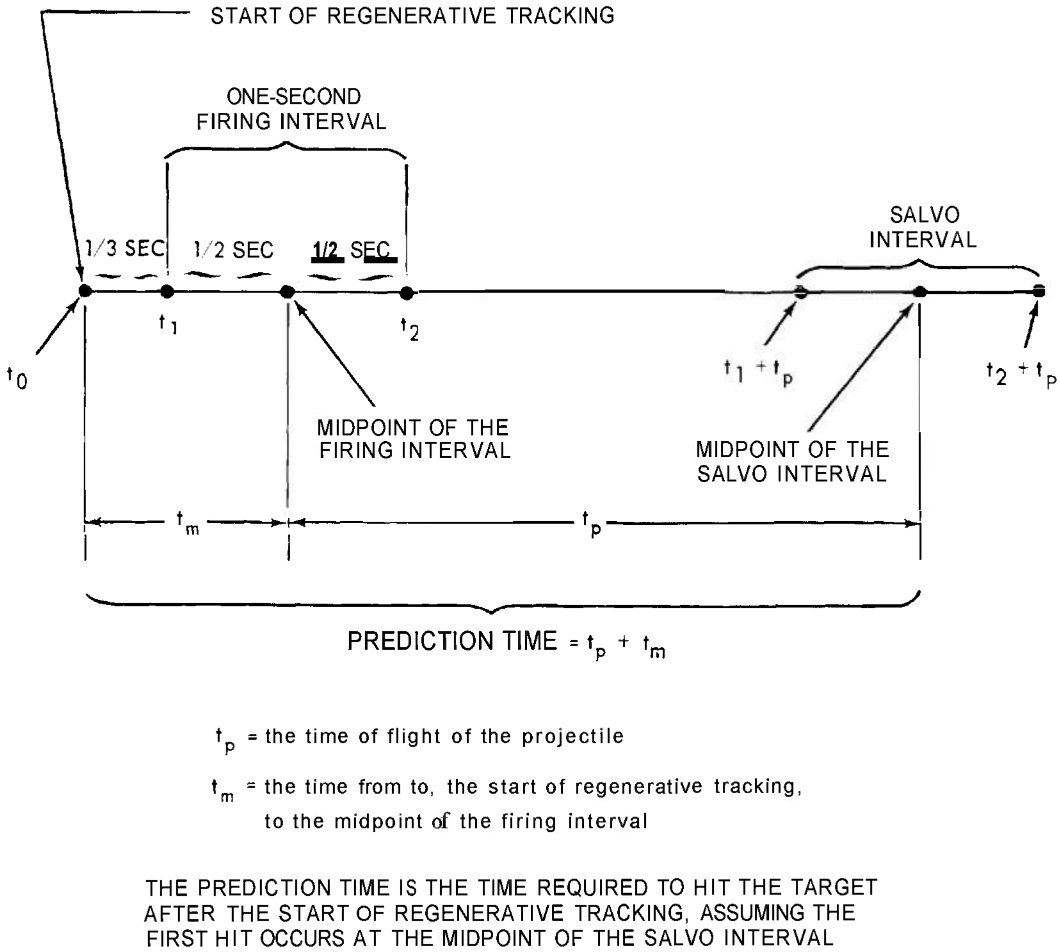


Figure 4-75. Time relationships associated with tracking and firing.

4-6.12 INPUT-TRACKING-NOISE ERRORS

The bias and dispersion caused by the input tracking noise, i. e., noise arising in the optical tracking system, can be determined by methods analogous to those discussed in par 4-6. 11. The variances of the bias and the dispersion of the input tracking noise are each made up of the sum of the variances of their vertical and lateral components. These components are each in turn dependent on the three tracking variables of elevation, azimuth, and range. Anyone of these six components of bias and dispersion has the general form derived in the paragraphs which follow.

Let one of these component output errors be designated by ϵ_0, and the corresponding input error be designated by ϵ_1. It has been found that a sufficiently good approximation of the transfer function between these error components is given by the parabola

$$\frac{\epsilon_o(j\omega,\tau)}{\epsilon_i(j\omega,\tau)} = \sum_m a_m (d_m + e_m t + f_m t^2) \qquad (4\text{-}397)$$

TABLE 4-7. BIAS AND DISPERSION OF THE MISS DISTANCE CAUSED BY FLIGHT ROUGHNESS.

Point	Prediction Time (sec) $t_p + t_m$	Standard Deviation of Bias (yd) σ_b	Standard Deviation of Dispersion (yd) σ_d
I	8.467	25.34	1.80
II	5.588	16.72	1.19
III	3.490	10.44	0.74
IV	2.676	8.01	0.57
V	3.490	10.44	0.74

where a, d, e, and f_m are sets of constants.

The bias and dispersion are determined as before: first, by averaging over the firing interval $t_o < t < t_o + \tau$, and second by averaging over all time — assuming all values of t_o to be equally likely. The variances of the bias and dispersion are then given by

$$\sigma_b^2(j\omega, \tau) = \lim_{T\to\infty} \left\{ \frac{1}{2T} \int_{-T}^{T} [\overline{\epsilon_o(j\omega, t_o, \tau)}]^2 \, dt_o \right\} \tag{4-398}$$

and

$$\sigma_d^2(j\omega, \tau) = \lim_{T\to\infty} \left\{ \frac{1}{2T} \int_{-T}^{T} \left[\overline{\epsilon_o^2(j\omega, t_o, \tau)} - \overline{\epsilon_o(j\omega, t_o, \tau)}^{\,2} \right] dt_o \right\} \tag{4-399}$$

where the bar superscript indicates an average over the firing interval τ; i.e.,

$$\overline{\epsilon_o(j\omega, t_o, \tau)} = \frac{1}{\tau} \int_{t_o}^{t_o+\tau} \epsilon_o(j\omega, t) \, dt \tag{4-400}$$

and

$$\overline{\epsilon_o^2(j\omega, t_o, \tau)} = \frac{1}{\tau} \int_{t_o}^{t_o+\tau} \epsilon_o^2(j\omega, t) \, dt. \tag{4-401}$$

If the input error has the sinusoidal form

$$\epsilon_i(j\omega, t) = |\epsilon_i| \sin \omega t, \qquad -\infty < t < \infty \tag{4-402}$$

then (see Derivation 4-11)

$$\sigma^2_{\epsilon_i} = \frac{|\epsilon_i|^2}{2} \tag{4-403}$$

and the applicable transfer functions may be defined as

$$|R_b(j\omega,\tau)|^2 = \frac{\sigma^2_b(j\omega,\tau)}{\sigma^2_{\epsilon_i}} \tag{4-404}$$

and

$$|R_d(j\omega,\tau)|^2 = \frac{\sigma^2_d(j\omega,\tau)}{\sigma^2_{\epsilon_i}} \tag{4-405}$$

Note that Eq. 4-403 was obtained by averaging over all time and not over the firing interval τ exclusively.

The derivation thus far applies for any frequency ω and a particular firing interval τ. Thus, the variance of the bias and the variance of the dispersion are

$$\sigma^2_b(\tau) = \int_{-\infty}^{\infty} |R_b(j\omega,\tau)|^2 \frac{d\sigma^2_{\epsilon_i}(j\omega,\tau)}{d(\log\omega)}\, d(\log\omega) \tag{4-406}$$

and

$$\sigma^2_d(\tau) = \int_{-\infty}^{\infty} |R_d(j\omega,\tau)|^2 \frac{d\sigma^2_{\epsilon_i}(j\omega,\tau)}{d(\log\omega)}\, d(\log\omega) \tag{4-407}$$

where

$$|R_b(j\omega,\tau)|^2 = \frac{2}{|\epsilon_i|^2} \lim_{T\to\infty} \left\{ \frac{1}{2T} \int_{-T}^{T} \left[\frac{1}{\tau} \int_{t_o}^{t_o+\tau} \epsilon_o(j\omega,t)\, dt \right]^2 dt_o \right\} \tag{4-408}$$

and

$$|R_d(j\omega,\tau)|^2 = \frac{2}{|\epsilon_i|^2} \lim_{T\to\infty} \left\{ \frac{1}{2T} \int_{-T}^{T} \left[\frac{1}{\tau} \int_{t_o}^{t_o+\tau} \epsilon_o^2(j\omega,t)\, dt \right] dt_o \right\} - |R_b(j\omega,\tau)|^2 \tag{4-409}$$

In Eqs. 4-408 and 4-409 $\epsilon_o(j\omega,\tau)$ is given by

$$\epsilon_o(j\omega,t) = \sum_m a_m(d_m + e_m t + f_m t^2)\, \epsilon_i(j\omega,t) \tag{4-410}$$

and

$$\epsilon_i(j\omega, t) = |e_i| \sin \omega t \ , \qquad -\infty < t < \infty \tag{4-411}$$

The integrals of Eqs. 4-408 and 4-409 were evaluated numerically by using values of the constants d_m, em, and f_m at three points within the time interval. The bias and dispersion were computed for the same five points previously employed (see Table 4-6 and associated text), and the RMS miss distance at the midpoint of the firing interval was also computed. The results are tabulated in Table 4-8, where it is shown that the bias caused by input noise is much more significant than the associated dispersion. The bias coincides with the value of the miss distance at the midpoint of the firing interval to a very close degree.

4-6.13 HIT PROBABILITY OF THE VIGILANTE ANTIAIRCRAFT WEAPON SYSTEM

The preceding paragraphs have derived expressions and numerical values for the component errors of the Vigilante Antiaircraft Weapon System. The present paragraph presents the procedures whereby the overall miss distances, the engagement hit probabilities, and the single-shot hit probabilities were determined for various conditions — using the results derived in the preceding paragraphs.

As a first step in carrying out these procedures, the overall errors of the Vigilante Antiaircraft Weapon System were determined by appropriate summations of the component errors just described that are due to (1) the computer, (2) flight roughness, and (3) input noise. Operations were then performed on these component errors as follows:

1. Computer errors were computed in two stages. First, the bias and dispersion of E_6 and E_7, the errors in the elevation and azimuth angles of the gun, were found from the computer-error analysis. Next, the bias and dispersion at the end of regeneration were computed from Eqs. 4-380 and 4-381; see Table 4-6.
2. Flight-roughness errors were computed from Eqs. 4-394 and 4-395, with numerical values obtained from Eq. 4-396 and Ref. 65; see Table 4-7.
3. Errors caused by input noise were obtained by integrating Eqs. 4-406 and 4-407, using values obtained from Eqs. 4-408 and 4-409; see Table 4-8.

These component errors are given in Tables 4-6, 4-7 and 4-8 as standard deviations of the bias and dispersion of the miss distance, in yards. These values were squared to give the variances, and entered in Tables 4-9 and 4-10 — the variances of the bias in Table 4-9 and the variances of the dispersion in Table 4-10. The variances of the component errors in columns A, B, and C of Table 4-9 were then added to give the variance of the bias of the total miss distance. The same operation was performed with columns A, B, and C of Table 4-10 to yield the variance of the dispersion of the total miss distance. 'The sum-of-the-variances method just described is valid for both bias errors and dispersion errors since all errors are independent of one another.

Inspection of the errors tabulated in Tables 4-6, 4-7, and 4-8 shows that in all cases the bias is much greater than the dispersion. Since the bias caused by computer errors is consistently greater than the total of the other bias errors, it can be concluded that the performance could be improved by increasing the accuracy of the computer components. Also, since the errors are predominately bias errors, the introduction of additional dispersion will improve the hit probability. Further discussion of the effects of additional dispersion is given subsequently, in connection with Fig. 4-76.

Next, the values of the variance of the total bias and the variance of the total dispersion were combined with an appropriate expression (as discussed in the subsequent paragraphs) in order to compute the engagement hit probability by solution of Tappert's expression for the engagement hit probability with statistical bias and dispersion; see Eq. D4-5.11 of Derivation 4-5 in the Appendix to Chapter 4. Certain simplifying assumptions required to make Eq. D4-5.11 valid can be stated as follows:

TABLE 4-8. BIAS AND DISPERSION OF THE MISS DISTANCE CAUSED BY INPUT NOISE.

Point	Standard Deviation of Miss Distance (yd)	Standard Deviation of Bias σ_b (yd)	Standard Deviation of Dispersion σ_d (yd)
I	47.78	47.76	0.79
II	27.72	27.72	0.77
III	15.52	15.40	0.85
IV	11.38	11.37	1.18
V	12.87	12.94	1.39

TARGET:

Minimum Horizontal Range = 1000 yd Constant Altitude = 1000 yd

Constant Target Speed = 300 yd/sec (533 knots)

TABLE 4-9. VARIANCE OF THE BIAS ERRORS, A RECAPITULATION AND SUMMATION.

(from Fig. I, p. 1 of "Hit Probability of the Vigilante System", Vol. II; see Ref. 59)

Point	Midpoint Predicted Range (yd)	A Variance Due to Computer Component Errors (yd^2)	B Variance Due to Tracking Noise (yd^2)	C Variance Due to Flight Roughness (yd^2)	Total Variance σ_b^2 (yd^2)	Standard Deviation of the Bias σ_b (yd)
I	4000	2776.96	2281.02	641.98	5699.96	75.5
II	3000	541.26	768.40	279.57	1589.23	39.9
III	2000	114.00	237.16	109.07	460.23	21.5
IV	1500	55.08	129.28	64.12	248.48	15.8
V	2000	102.92	167.44	109.07	379.43	19.5

1. The target is assumed to be two-dimensional, which can be justified as follows. The normal employment of the Vigilante Antiaircraft Weapon System is for defense against low-flying attack-type or fighter-type aircraft. Either contact-fuzed ammunition or proximity-fuzed ammunition can be employed. As a part of this tactical situation, a short, direct trajectory is employed and the bursts occur at, or near, the first contact of the target that is normal to the path of the projectile. Therefore, the area of that aspect of the target that is normal to the range vector is of principal interest for the problem at hand. (The possibility of employing a three-dimensional model should be considered, however. Such a model could be obtained by (1) re-deriving Eq. D4-5. 11 of Derivation 4-5 in three dimensions, (2) employing a

TABLE 4-10. VARIANCE OF THE DISPERSION ERRORS, A RECAPITULATION AND SUMMATION.

(from Fig. I, p. 1 of "Hit Probability of the Vigilante System", Vol. II; see Ref, 59)

Point	Midpoint Predicted Range (yd)	A Variance Due to Computer Component Errors (yd^2)	B Variance Due to Tracking Noise (yd^2)	C Variance Due to Flight Roughness (yd^2)	Total Variance σ_d^2 (yd^2)	Standard Deviation of the Bias σ_d (yd)
I	4000	39.52	0.62	3.25	43.39	6.6
II	3000	8.19	0.59	1.41	10.19	3.2
III	2000	1.88	0.72	0.55	3.15	1.8
IV	1500	1.10	1.39	0.32	2.81	1.7
V	2000	4.28	1.93	0.55	6.76	2.6

three-dimensional target model, and (3) including the errors along the range axis — i. e., the time-of-flight errors — as well as the vertical and lateral errors. However, in view of the great computational complexity that would be introduced and the small improvement in the accuracy of the error analysis that would be obtained in the case of the Vigilante Antiaircraft Weapon System, the three-dimensional model was not employed.)

2. The target area A is relatively small compared with the dispersion of the bursts. This assumption is justified by the relatively small size of the target and the form of the burst pattern (see Fig. 4-69).

3. The bias and the dispersion are each made up of independent normal distributions along the x and y axes, which is equivalent to an independent radial distribution. In the case of the dispersion, the total miss-distance distribution is made up of a large number of individual random errors; by the central limit theorem (see par 4-4. 2. 4), the total miss distance will be normally distributed, regardless of the distributions of the individual errors. By definition, the dispersion has zero mean; the mean of the burst pattern is the bias, Similarly, the total bias component of the miss distance is made up of a large number of individual bias errors; therefore, it must also be normally distributed. The mean of the bias distribution is the invariant calibration error. It is assumed that this error is removed by spotting corrections made during a calibration procedure.

Although the target was assumed to be two-dimensional, it is important to note that the effects of various target aspects were considered. The areas of the target in its principle aspects were assumed to be the following:

Front-View Area:	6.5 square yards
Side-View Area:	26.6 square yards
Top-View Area:	50.9 square yards

Areas for other aspects of the target were obtained by use of the ellipsoid

$$0.023669\,x^2 + 0.0014133\,y^2 + 0.00038598\,z^2 = 1 \qquad (4\text{-}412)$$

where x, y, and z represent the roll, pitch, and yaw axes of the aircraft, respectively. At any particular aspect of the aircraft, the target area A, in square yards, was taken to be numerically equal to the radius vector of the ellipsoid of Eq. 4-412 that is in the direction of the line of sight.

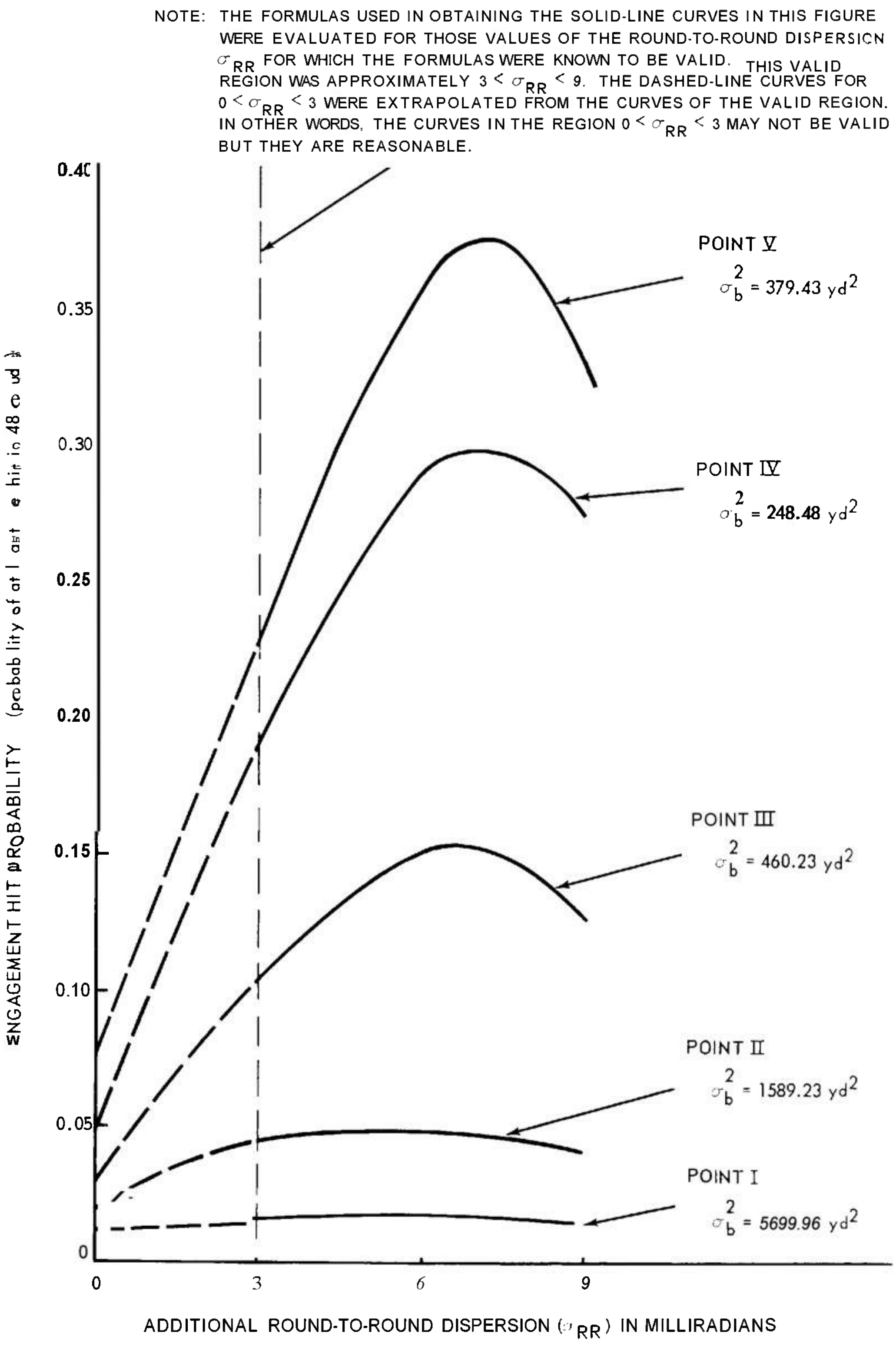


Figure 4-76. Engagement hit probability versus the additional round-to-round dispersion.

Since the Vigilante Antiaircraft Weapon System normally fires 48 rounds in quick succesion, the engagement hit probability is the probability that there will be at least one hit in 48 rounds. The engagement hit probability was computed from Eq. D4-5. 11 of Derivation 4-5 with n = 48. The computation was carried out at five points representing typical sets of conditions on a straight-line target course at constant speed and altitude. Table 4-11 gives the parameters for points I through V. The ranges are those existing at the midpoint of the one-second firing interval. The solution of Eq. D4-5. 11 of Derivation 4-5 was carried out at each of the five points, with values of σ_b^2 and σ_d^2 obtained from Tables 4-9 and 4-10, respectively, and values of A obtained from Table 4-11. In performing these computations, it was necessary to include an additional round-to-round dispersion caused by ammunition variations and gun-tube vibration. This additional round-to-round dispersion σ_{RR} was treated as an independent variable. The resulting engagement hit probability, which in this case is the probability of at least one hit in 48 rounds, is plotted against σ_{RR} in Fig. 4-76 for points I through V.

As one would expect for a problem in which bias errors are much larger than dispersion errors (see par 4-4. 1. 1, par 4-4. 1. 2, and Fig. 4-3), the engagement hit probability is improved by an increase in the additional round-to-round dispersion, up to the point at which the dispersion errors become predominant. This fact is evident from Fig. 4-76, in which it can also be seen that little or no improvement is evident in the case of points I and II. These two points have large bias errors relative to the target dimensions. At point III, on the other hand, considerable improvement is obtained if σ_{RR} is increased to 6 to 8 milliradians. The standard deviation of the bias at point III is 21. 5 yards or 10. 1 milliradians, and is more nearly comparable with the target dimensions. At points IV and V, the improvement is still more marked.

In case the gun and ammunition variations should be found to provide less than the optimum dispersion, a programmed pattern generator could be provided in order to optimize the system[66].

It is also of interest to determine the single-shot hit probability. In order to do this, refer to Tappert's development of the engagement hit probability with statistical bias and dispersion that results in Eq. D4-5. 11 of Derivation 4-5. An expression employed in the derivation — Eq. D4-5. 8 — is repeated here for reference purposes:

$$(Pr)_{eh} = 1 - \int_0^1 \left[1 - \frac{A}{2\pi\sigma_d^2} \quad x^{\sigma_b^2/\sigma_d^2} \right]^n dx \tag{4-413}$$

where

$(Pr)_{eh}$ = the engagement hit probability

A = the target area

n = the number of shots fired during the course of the engagement

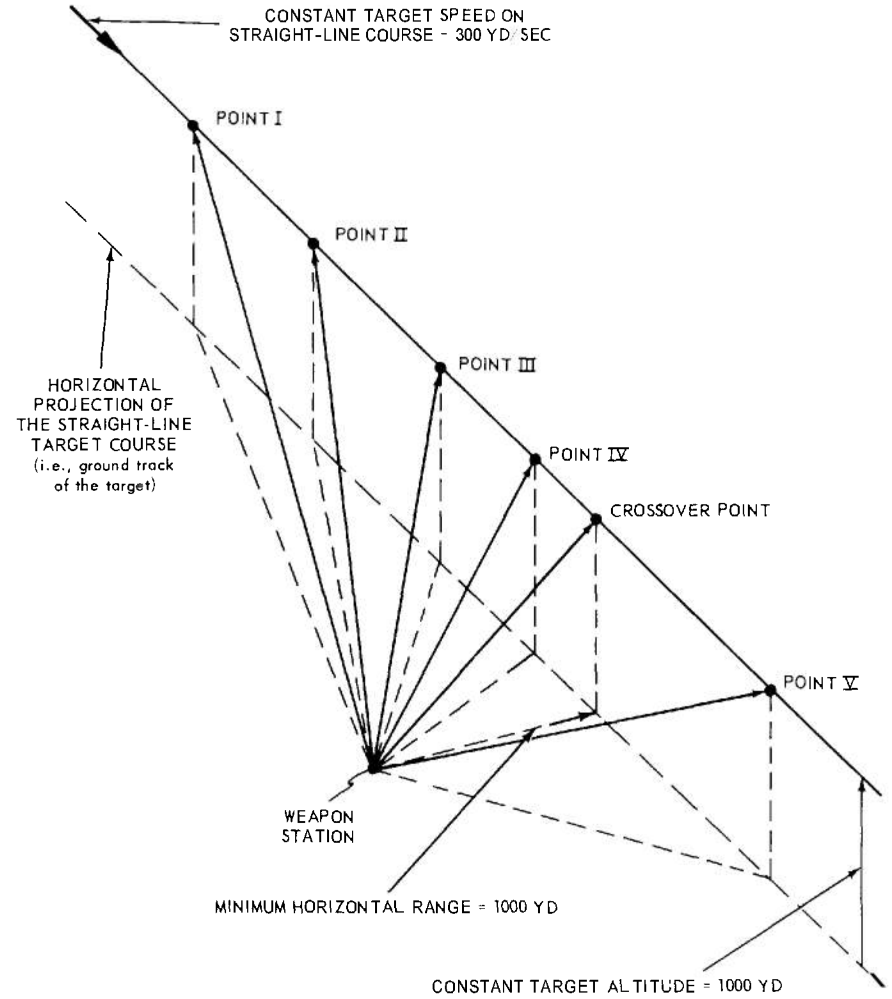
$$x = \exp\left[-\frac{r^2}{2\sigma_b^2} \right] \tag{4-414}$$

r = the radial distance from the target center

The engagement hit probability given by Eq. 4-413 may be converted to the single-shot hit probability $(Pr)_{ssh}$ by setting n = 1, i. e.,

$$(Pr)_{ssh} = 1 - \int_0^1 \left\{ 1 - \frac{A}{2\pi\sigma_d^2} \quad x^{\sigma_b^2/\sigma_d^2} \right\} dx \tag{4-415}$$

TABLE 4-11. DEFINITION OF POINTS ON THE ASSUMED TARGET COURSE SELECTED FOR COMPUTATION.



Point No.	Midpoint Predicted Range* (yd)	Area of Target† A (yd)^2
I	4000	6.5698
II	3000	6.7217
III	2000	7.4119
IV	1500	9.8611
V	2000	20.2411

*This is the predicted slant range to the target that exists at the midpoint of the firing interval (see Fig. 4-75).

†It is important to note that the derivation of the data given in this column accounts for many more factors than the elliposoid equation 4-412. For the circumstances under consideration here, the use of Eq. 4-412 alone would oversimplify the situation.

Use of the integration formula given by Eq. D4-5.9 of Derivation 4-5 then provides the solution to Eq. 4-415 in the form

$$(Pr)_{ssh} = \frac{A}{2\pi(\sigma_b^2 + \sigma_d^2)} \tag{4-416}$$

Values of the single-shot hit probability $(Pr)_{ssh}$, together with values of the engagement hit probability $(Pr)_{eh}$, are tabulated in Table 4-12 as functions of the additional round-to-round dispersion σ_{RR}. As shown by Table 4-12, the single-shot hit probability is much less than the engagement hit probability.

TABLE 4-12. SINGLE-SHOT AND ENGAGEMENT HIT PROBABILITIES FOR A 48-ROUND SALVO

Point	Round-to-Round Dispersion σ_{RR} (Millirad)	Single-Shot Hit Probability $(Pr)_{ssh}$	Engagement Hit Probability* $(Pr)_{eh}$
I	0	0.0004	0.0110
	2.51	0,0004	0.0146
	5.77	0.0003	0.0153
	8.85	0.0003	0.0141
II	0	0.0013	0.0182
	2.80	0.0013	0.0463
	5.90	0.0011	0.0494
	8.94	0.0009	0.0425
III	0	0.0051	0.0281
	2.87	0.0048	0.1233
	5.93	0.0039	0.1527
	8.96	0.0030	0.1300
IV	0	0,0125	0.0504
	2.78	0.0117	0.1849
	5.89	0.0095	0.2928
	8.93	0.0073	0.2750
V	0	0.0167	0.0758
	2.70	0.0155	0.2159
	5.86	0.0123	0.3540
	8.91	0.0092	0.3317

*Probability of at least one hit in 48 rounds.

The determination of the hit probability of the Vigilante Antiaircraft Weapon System has been described because it provides an example of a highly complex system — requiring a high order of analytical ability to achieve a solution, and also requiring a lengthy computational procedure. The computations, too lengthy to reproduce here, may be found in Reference 59. Many fire control systems have characteristics that make the hit probability much easier to compute, but the basic elements of the analysis are the same.

It should be noted that, for an antiaircraft fire control system, it is the engagement hit probability that is of primary importance since there is little likelihood of obtaining a hit with any particular individual shot. For certain other tactical situations, however —such as tank fire, for example — the engagement hit probability is usually of less significance than the single-shot hit probability. This stems from the fact that unless the particular opponent involved is put out of action by either the first round fired (preferably) or a rapidly corrected subsequent round, if the first projectile misses the target, there may be no opportunity to fire further shots before the firing weapon is itself destroyed or disabled. Therefore, for this type of tactical situation, the first-round hit probability and the subsequent-round hit probability are of prime importance. For a discussion that applies to this type of circumstance, see Reference 67. That document, which was published in support of the U. S. Armor Policy Statements on "Method of Expressing Chance of a Hit" and "Accuracy Requirements for Tank Weapons", describes the various mathematical models to be used in predicting the hit capabilities of various tank weapon systems.

APPENDIX TO CHAPTER 4
DERIVATIONS AND EXAMPLES

This appendix contains the following derivations and examples that are referred to in the text of Chapter 4 but are included here in order to avoid interfering with the continuity of the presentation in the body of Chapter 4:

Derivation 4-1. Derivation of the kill probability for the diffuse target.

If a set of orthogonal axes x, y, z is located at the target center and it is desired to determine the kill probability for bursts occurring on the x-axis, a reasonable assumption would be that the kill probability is unity for a burst occurring at the target center ($x = 0$), and is zero at burst points infinitely remote from the target center ($x = \pm\infty$). It can be further reasonably assumed that along the entire x-axis the kill probability as a function of the burst point x has the general form of a Gaussian probability curve symmetrical about the target center, i.e., (see par 4-4.2.4),

$$(Pr)_{(kill)}(x) = C_1 \exp\left[-\frac{x^2}{2\sigma_c^2}\right] \tag{D4-1.1}$$

where

x = distance of the burst point from the target center

C_1 = constant of proportionality

The quantity σ_c is a constant whose magnitude depends on (1) the size and armoring ("hardness") of the particular target under consideration, and (2) the characteristics of the ammunition to be employed; it is called the vulnerable radius* of the target for the particular ammunition used and has the same units of length as x.

The constant of proportionality C_1 is evaluated by setting x equal to zero in Eq. D4-1.1 which gives

$$(Pr)_{(kill)}(0) = C_1 e^0 = C_1 \tag{D4-1.2}$$

Since it has been assumed that $(Pr)_{(kill)}(0)$ is unity, Eq. D4-1.2 shows that C_1 is unity also. Therefore

$$(Pr)_{(kill)}(x) = \exp\left[-\frac{x^2}{2\sigma_c^2}\right] \tag{D4-1.3}$$

If it is assumed that the probabilities along the three axes are independent and that the characteristics of the target are the same in all axes,† then

$$(Pr)_{(kill)}(y) = \exp\left[-\frac{y^2}{2\sigma_c^2}\right] \tag{D4-1.4}$$

and

$$(Pr)_{(kill)}(z) = \exp\left[-\frac{z^2}{2\sigma_c^2}\right]. \tag{D4-1.5}$$

By extension of Eq. 4-23 to three variables, the joint probability of kill $(Pr)_{(kill)}(x,y,z)$ can be written as

$$(Pr)_{(kill)}(x,y,z) = (Pr)_{(kill)}(x)\,(Pr)_{(kill)}(y)\,(Pr)_{(kill)}(z) \tag{D4-1.6}$$

*In Reference 27, the quantity σ_c is termed the lethal radius.

†See par 4-6.13 concerning the hit probability of the Vigilante Antiaircraft Weapon System for an example of how a nonuniform target can be treated.

Derivation 4- 1. (Continued)

Accordingly, substitution from Eqs. D4- 1.3, D4- 1.4 and D4- 1.5 into Eq. D4-1.6 shows that

$$(Pr)_{(kill)}(x,y,z) = \exp\left[-\frac{x^2+y^2+z^2}{2\sigma_c^2}\right] \quad \text{(D4- 1.7)}$$

This kill-probability relationship can be converted to spherical-polar-coordinate form — in terms of r, θ, and ϕ — by means of the geometrical relationship (see Fig. D4- 1.1)

$$r^2 = x^2 + y^2 + z^2 \quad \text{(D4- 1.8)}$$

where

r = radial distance of burst point from target center.

Substitution from Eq. D4- 1.8 into Eq. D4- 1.7 gives

$$(Pr)_{(kill)}(r,\theta,\phi) = \exp\left[-\frac{r^2}{2\sigma_c^2}\right]. \quad \text{(D4- 1.9)}$$

The quantity $(Pr)_{(kill)}(r,\theta,\phi)$ is known as the kill probability for the diffuse target and is henceforth represented accordingly by the symbol $(Pr)_{d(kill)}$, i.e.,

$$(Pr)_{d(kill)} = (Pr)_{(kill)}(r,\theta,\phi) = \exp\left[-\frac{r^2}{2\sigma_c^2}\right]. \quad \text{(D4-1.10)}$$

As would be expected for the uniform target characteristics assumed, Eq. D4- 1.10 shows that $(Pr)_{d(kill)}$ is dependent only on the radial distance r of the burst point from the center of the target, i.e., it is independent of θ and ϕ.

When interest is restricted to the x, y-plane, then, for a given point located by polar coordinates

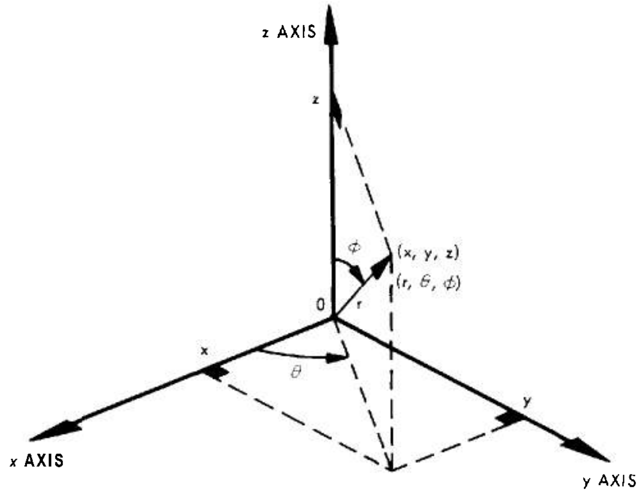
$$(Pr)_{d(kill)}(r,\theta) = \exp\left[-\frac{r^2}{2\sigma_c^2}\right]. \quad \text{(D4- 1.11)}$$

For the same point located by rectangular coordinates

$$(Pr)_{d(kill)}(x,y) = \exp\left[-\frac{x^2+y^2}{2\sigma_c^2}\right] \quad \text{(D4- 1.12)}$$

since $r^2 = x^2 + y^2$ in the x, y-plane (see Fig. D4- 1.1). Figure D4- 1.2 shows plots of the kill probability for a diffuse target, $(Pr)_{d(kill)}(r,\theta)$, and the probability of not killing the same target, $Q_{d(kill)}(r,\theta)$, both as functions of the distance r of the burst point from the target center. The kill-probability curve is actually a section through the bivariate probability

Derivation 4-1. (Continued)



$$r^2 = x^2 + y^2 + z^2$$

$$\theta = \tan^{-1} \frac{y}{x}$$

$$\phi = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{z}$$

Figure D4-1.1. Geometrical relationships between three-dimensional rectangular and polar coordinates.

$(Pr)_{d(kill)}(r,\theta)$ at some particular value of φ. Note that the plot for $Q_{d(kill)}(r,6)$ is governed by the relationship

$$Q_{d(kill)}(r,\theta) = 1 - (Pr)_{d(kill)}(r,\theta) \qquad \text{(D4-1.13)}$$

inasmuch as the sum of the probability of killing the target and not killing the target is always unity. Figure D4-1.2 also illustrates the vulnerable radius, defined as σ_c, and the vulnerable area a defined as the area of the circle with radius $\sqrt{2}\sigma_c$, i.e.,

$$a = 2\pi\sigma_c^2. \qquad \text{(D4-1.14)}$$

Note that the vulnerable area as defined by Eq. D4-1.14 does not represent an area within which a kill is certain. Rather, since it is the area with a radius $\sqrt{2}\sigma_c$, Eq. D4-1.11 shows that all bursts occurring within this area have a kill probability, $(Pr)_{d(kill)}(r,\theta)$, greater than e^{-1} or 0.368 (see Fig. D4-1.3). Examination of Fig. D4-1.3 shows that the kill probsbility in the area outside the vulnerable area is relatively slight.

Derivation 4-1. (Continued)

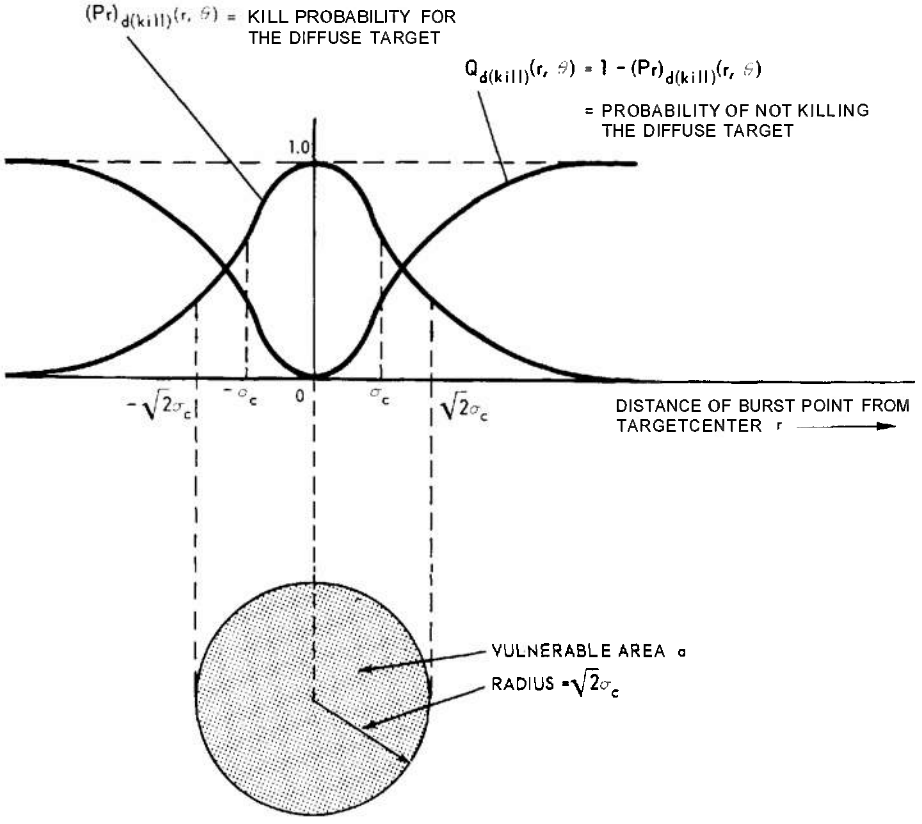


Figure D4-1.2. Plots of the probabilities $(Pr)_{d(kill)}(r,\theta)$ and $Q_{d(kill)}(r,\theta)$ and the vulnerable area a.

Derivation 4-1. (Continued)

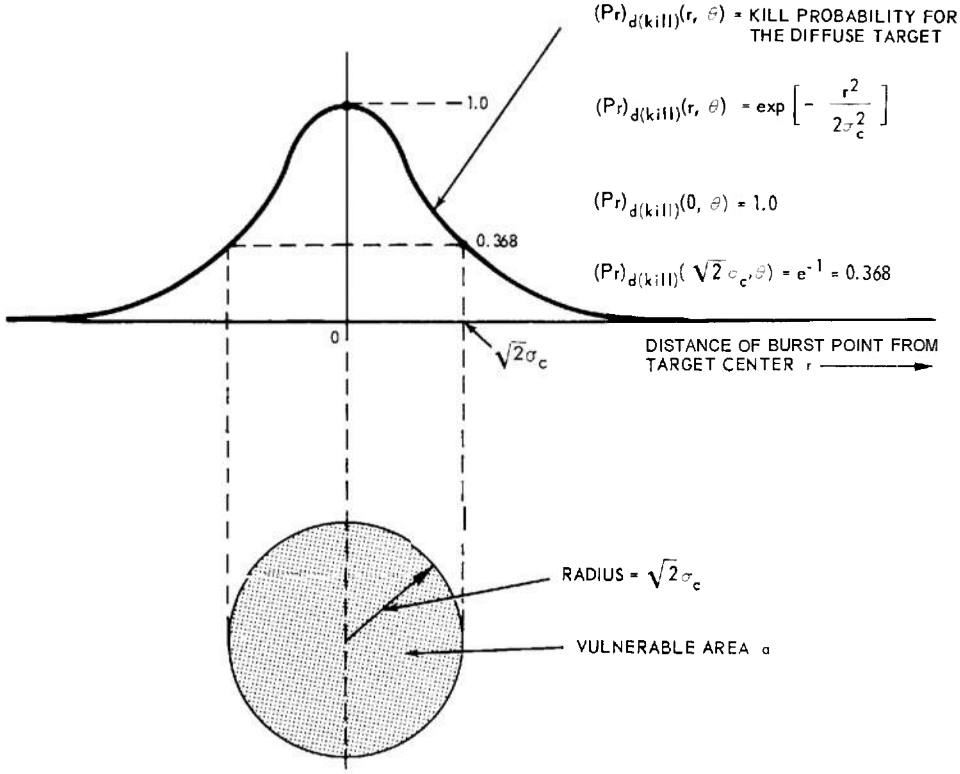


Figure D4-1.3. Plot of the kill probability for the diffuse target.

Derivation 4-2. Derivation of the single-shot kill probability in two dimensions, for fixed bias and normal radial dispersion.

This derivation depends on the following four assumptions:

1. A diffuse target, as previously defined (see par 4-4.3.1)
2. Restriction to two dimensions, x and y, in a plane perpendicular to the path of the projectile at the target
3. A fixed bias h located along the x-axis
4. A Gaussian distribution of the dispersion along each axis, with the same variance σ_d^2 along each axis.

With these assumptions, the probability density functions of the burst pattern, along the x- and y-axes, are, from Eq. 4-38, given by the expressions

$$p(x) = \frac{1}{\sigma_d\sqrt{2\pi}} \exp\left[-\frac{(x-h)^2}{2\sigma_d^2}\right] \tag{D4-2.1}$$

$$p(y) = \frac{1}{\sigma_d\sqrt{2\pi}} \exp\left[-\frac{y^2}{2\sigma_d^2}\right] \tag{D4-2.2}$$

where

h = bias of the burst pattern

σ_d^2 = variance of the burst pattern

and both h and σ are expressed in a consistent set of linear units. As shown by Fig. D4-2.1(A) and (B), these expressions are simply normal distributions along the x- and y-axes, with mean values at x = h and y = 0.

The probability density functions p(x) and p(y) are measures of the relative likelihood of occurrence of a burst at a given point on the x-axis, (x, 0), or at a given point on the y-axis, (0, y), respectively. Since the distributions have been assumed to be independent, they may be combined in accordance with the procedure followed in par 4-4.2.4 to give the bivariate distribution

$$p(x,y) = p(x)\,p(y) = \frac{1}{2\pi\sigma_d^2} \exp\left[-\frac{(x-h)^2+y^2}{2\sigma_d^2}\right] \tag{D-4-2.3}$$

as can be seen from Eq. 4-18 with a substitution of variables

$$p(x,y) = \lim_{\substack{\Delta x\to 0\\ \Delta y\to 0}} \left[\frac{\Pr[x,y]}{\Delta x\,\Delta y}\right] = \frac{\Pr[x,y]}{dxdy} . \tag{D4-2.4}$$

Thus, for a normally distributed burst pattern, Pr[x,y] is the probability that a burst will occur in a differential area, dxdy, that is located at a given point (x,y). The probability density function p(x,y) is, therefore, a measure of the relative likelihood of occurrence of a burst at a given point (x,y). For example (see Fig. D4-2.2(A)), if x=h and y=0, then p(x, y) has its maximum value which is $1/2\pi\sigma_d^2$. For any other set of x,y coordinates — e.g., $x = x_1$ and $y = y_1$ — p(x,y) is determined as in Fig. D4-2.2(B).

The probability of a kill at any particular point in the x,y-plane with a single shot is the

Derivation 4-2 (Continued)

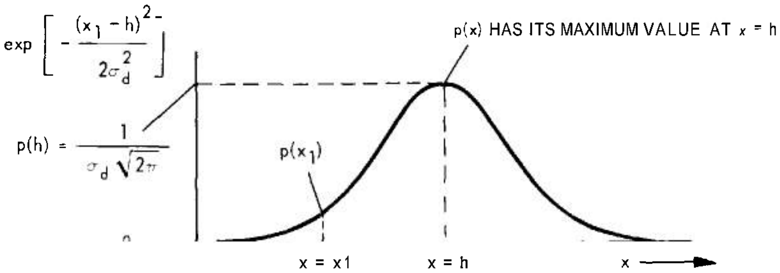
$$p(x) = \frac{1}{\sigma_d\sqrt{2\pi}} \exp\left[-\frac{(x-h)^2}{2\sigma_d^2}\right]$$

FOR $x = h$, $p(x)$ HAS ITS MAXIMUM VALUE; i.e.,

$$p(h) = \frac{1}{\sigma_d\sqrt{2\pi}} \exp\left[-\frac{(h-h)^2}{2\sigma_d^2}\right] = \frac{1}{\sigma_d\sqrt{2\pi}} e^{-0} = \frac{1}{\sigma_d\sqrt{2\pi}}$$

FOR ANY OTHER VALUE OF x, SAY x_1, $p(x)$ IS GIVEN BY

$$p(x_1) = \frac{1}{\sigma_d\sqrt{2\pi}} \exp\left[-\frac{(x_1-h)^2}{2\sigma_d^2}\right]$$



(A) Normal distribution along the x-axis

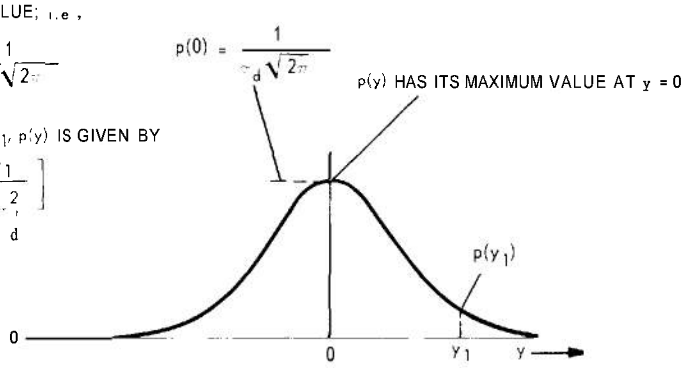
$$p(y) = \frac{1}{\sigma_d\sqrt{2\pi}} \exp\left[-\frac{y^2}{2\sigma_d^2}\right]$$

FOR $y = 0$, $p(y)$ HAS ITS MAXIMUM VALUE; i.e.,

$$p(0) = \frac{1}{\sigma_d\sqrt{2\pi}} e^{-0} = \frac{1}{\sigma_d\sqrt{2\pi}}$$

FOR ANY OTHER VALUE OF y, SAY y_1, $p(y)$ IS GIVEN BY

$$p(y_1) = \frac{1}{\sigma_d\sqrt{2\pi}} \exp\left[-\frac{y_1^2}{2\sigma_d^2}\right]$$



(B) Normal distribution along the y-axis

Figure D4-2.1. Gaussian probability density functions employed in connection with single-shot kill probability.

oint probability of the occurrence of the following two events:

1. The event that the burst occurs at the point concerned. (The probability of this event s $Pr[x,y]$.)

2. The event that a target kill is achieved if any burst occurs at that point. (The probability of this event is $(Pr)_{d(kill)}(x,y)$, which — as defined in connection with Eq. 4-55 — is he probability of killing a diffuse target when the burst occurs at point (x,y).)

Since discrete events (in the form of individual shots) are involved, the application of Eq. 4-3 shows that the single-shot kill probability on a diffuse target at a given point (x,y)

Derivation 4-2 (Continued)

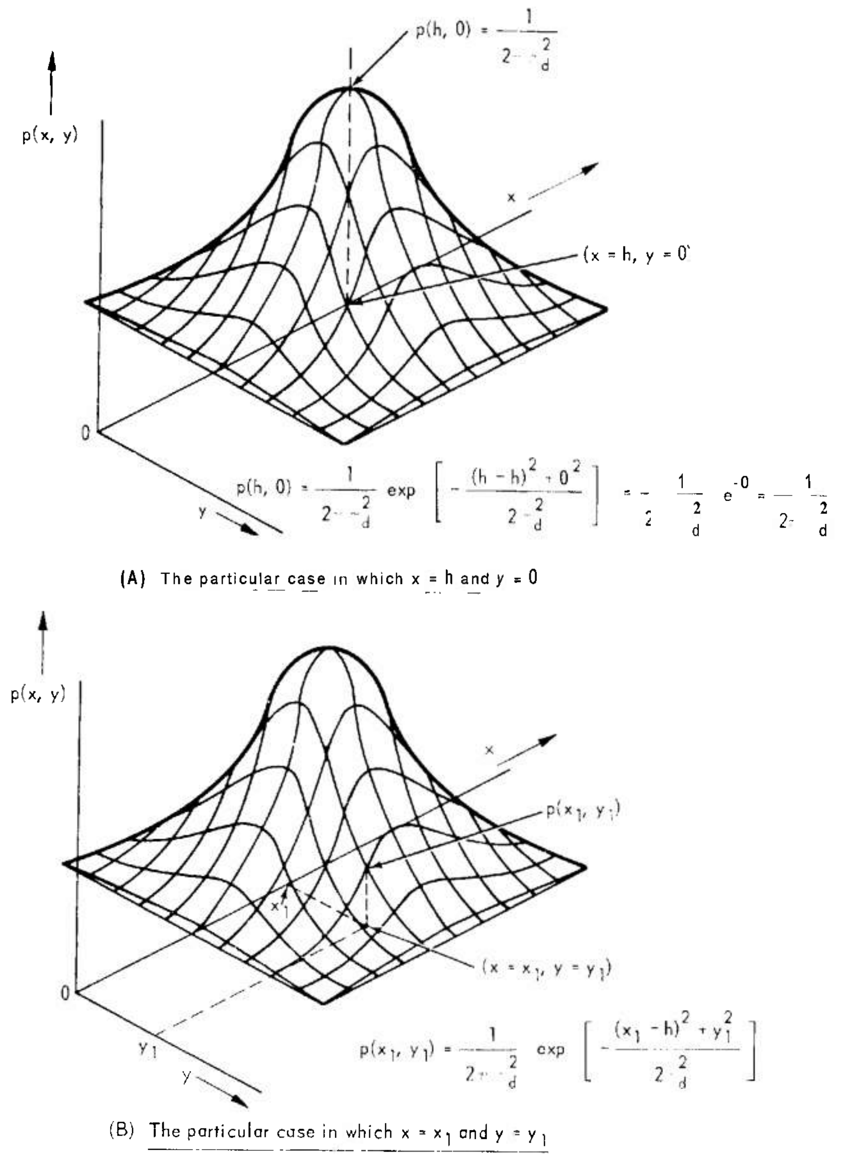


Figure D4-2.2. Bivariate normal distribution for selected values of x and y.

s given by the relationship

$$(Pr)_{(kill)ss}(x,y) = Pr[x,y]\,(Pr)_{d(kill)}(x,y) \qquad (D4\text{-}2.5)$$

There

- $(Pr)_{(kill)ss}(x,y)$ = single-shot kill probability on a diffuse target at a particular point (x,y)
- $Pr[x,y]$ = probability of a burst occurring in a differential area dxdy that is located at the point (x,y)
- $(Pr)_{(kill)d}(x,y)$ = probability of killing a diffuse target (centered at $x = 0$, $y = 0$) when the burst occurs at point (x,y).

Derivation 4-2 (Continued)

For a large number of shots, the probabilities in Eq. D4-2.5 can be expressed in terms of probability density functions. Dividing both sides of Eq. D4-2.5 by the incremental area $\Delta x \Delta y$ gives

$$\frac{(Pr)_{(kill)ss}(x,y)}{\Delta x \Delta y} = \frac{Pr[x,y]}{\Delta x \Delta y} (Pr)_{d(kill)}(x,y) \tag{D4-2.6}$$

Equation 4-18 shows that in the limit, as Δx and Δy approach zero,

$$p_{(kill)ss}(x,y) = p(x,y)\,(Pr)_{d(kill)}(x,y) \tag{D4-2.7}$$

where

$p_{(kill)ss}(x,y)$ = probability density function of a kill on a diffuse target at a particular point (x,y) with a single shot

$p(x,y)$ = probability density function of a burst at the particular point (x,y).

The single-shot kill probability $(Pr)_{ssk}$ for the entire x, y-plane — i.e., the probability that a given individual shot fired on a target will achieve a kill — can now be determined by integrating the probability density function $p_{(kill)ss}(x,y)$ over the infinite surface of the x, y-plane, i.e.,

$$(Pr)_{ssk} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_{(kill)ss}(x,y)\,dxdy \tag{D4-2.8}$$

= single shot kill probability.

Substitution from Eq. D4-2.7 into Eq. D4-2.8 shows that

$$(Pr)_{ssk} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x,y)\,(Pr)_{d(kill)}(x,y)\,dxdy. \tag{D4-2.9}$$

For the case of zero bias — i.e., for $h = 0$ — the integration indicated in Eq. D4-2.9 is accomplished very simply by converting from the rectangular-coordinate form of p(x, y) that is given in Eq. D4-2.3 to the corresponding equation in polar-coordinate form. The necessary relationships for performing this conversion follow. Since, from Eq. D4-2.4, $Pr[x,y]$ is the probability of a burst occurring in a differential area dxdy located at a given point (x,y), and since this probability is the same for the equivalent differential area $rdrd\theta$ in polar coordinates (all differential areas being equal), it follows that

$$Pr[x,y] = p(x,y)\,dxdy = Pr[r,\theta] = p(r,\theta)\,rdrd\theta \tag{D4-2.10}$$

where

$p(r,\theta)$ = probability density function of the normally distributed burst pattern expressed in the polar coordinates r and θ

$Pr[r,\theta]$ = probability of a burst occurring in a differential area $rdrd\theta$ that is located at a given point (r,θ).

Derivation 4-2. (Continued)

From geometrical considerations (see Fig. D4-2.3(B))

$$dxdy = rdrd\theta. \qquad \text{(D4-2.11)}$$

Inasmuch as $r^2 = x^2 + y^2$ (see Fig. D4-2.3(A)) and h = 0 for the case of zero bias, Eqs D4-2.3, D4-2.10 and D4-2.11 show that

$$\Pr[r,\theta] = p(r,\theta)rdrd\theta = \frac{1}{2\pi\sigma_d^2}\exp\left[-\frac{r^2}{2\sigma_d^2}\right]rdrd\theta \qquad \text{(D4-2.12)}$$

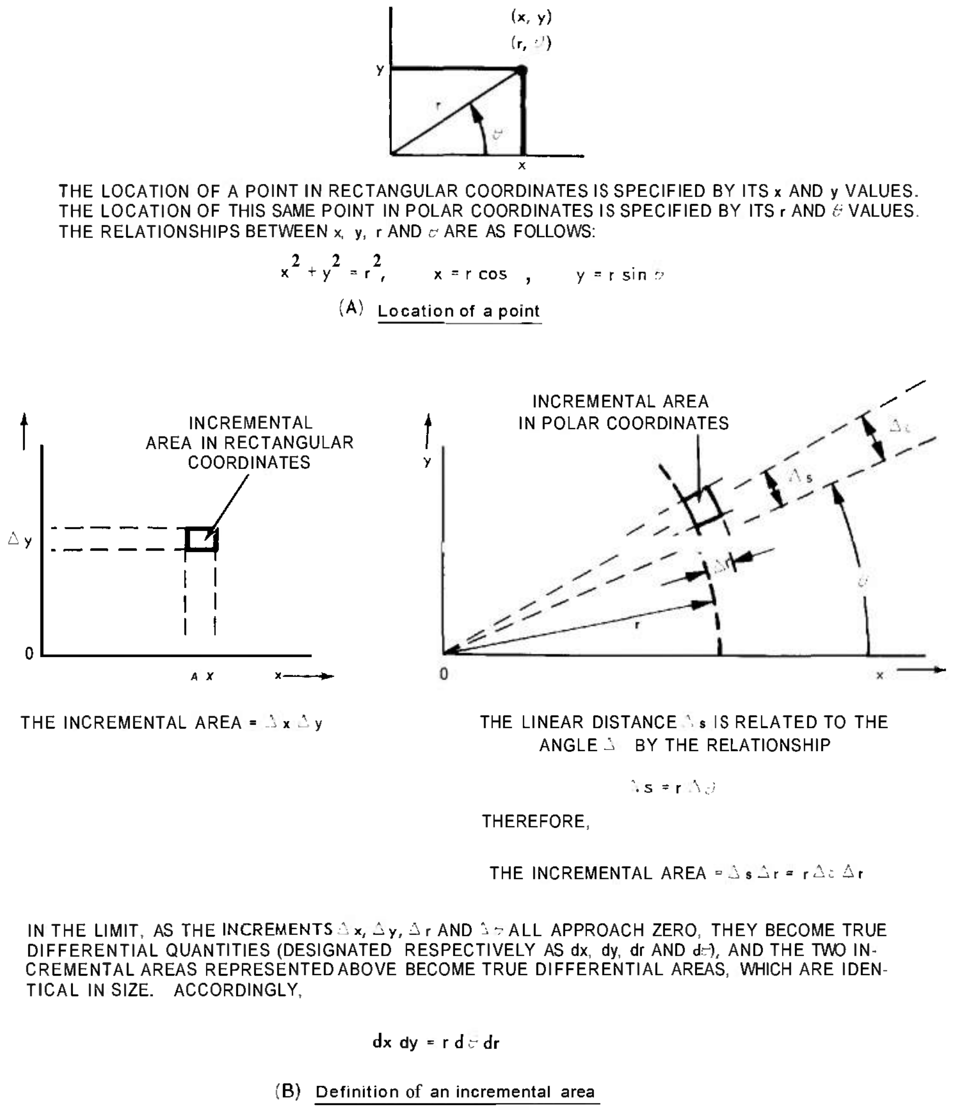


Figure D4-2.3. Relationship between rectangular (Cartesian) coordinates and polar coordinates.

Derivation 4-2. (Continued)

Because the probability of an event occurring in the differential area dxdy is the same as the probability of that same event occurring in the differential area $rdrd\theta$, Eq. D4-2.9 can be written in the alternative form

$$(Pr)_{ssk} = \int_0^{2\pi} \int_0^{\infty} p(r,\theta)\,(Pr)_{d(kill)}(r,\theta)\,rdrd\theta. \tag{D4-2.13}$$

It is now possible to develop the relationship for the single-shot kill probability $(Pr)_{ssk}$ by substituting from Eqs. 4-55 and D4-2.12 into Eq. D4-2.13 and integrating the resulting relationship over all values of r and θ, as follows

$$(Pr)_{ssk} = \frac{1}{2\pi\sigma_d^2} \int_0^{2\pi} \int_0^{\infty} \exp\left[-\frac{r^2}{2\sigma_c^2}\right] \exp\left[-\frac{r^2}{2\sigma_d^2}\right] rdrd\theta$$

$$= \frac{1}{2\pi\sigma_d^2} \int_0^{2\pi} \int_0^{\infty} \exp\left[-\left(\qquad \frac{1}{\sigma_d^2}\right)\right] rdrd\theta. \tag{D4-2.14}$$

By making the arbitrary substitution

$$\frac{1}{\sigma_1^2} = \frac{1}{\sigma_c^2} + \frac{1}{\sigma_d^2} \tag{D4-2.15}$$

for the sake of simplification, Eq. D4-2.14 can be rewritten in the form

$$(Pr)_{ssk} = \frac{1}{2\pi\sigma_d^2} \int_0^{2\pi} \int_0^{\infty} \exp\left[-\frac{r^2}{2\sigma_1^2}\right] rdrd\theta. \tag{D4-2.16}$$

Performance of the first integration yields

$$(Pr)_{ssk} = \frac{1}{\sigma_d^2} \int_0^{\infty} \exp\left[-\frac{r^2}{2\sigma_1^2}\right] rdr \tag{D4-2.17}$$

The integral of Eq. D4-2.17 can be found by means of standard tables of integrals.* The result is

$$(Pr)_{ssk} = \frac{1}{\sigma_d^2}\left[-\sigma_1^2 \exp\left[-\frac{r^2}{2\sigma_1^2}\right]\right]_0^{\infty} = \frac{\sigma_1^2}{\sigma_d^2}. \tag{D4-2.18}$$

* See page 200, Eq. 861.4 of Reference 23, with the substitution $x^2 = r^2/2\sigma_1^2$.

Derivation 4-2. (Continued)

$$(Pr)_{ssk} = \frac{1}{\sigma_d^2}\left[\frac{1}{\frac{1}{\sigma_c^2} + \frac{1}{\sigma_d^2}}\right] = \frac{\sigma_c^2}{\sigma_c^2 + \sigma_d^2} \qquad \text{(D4-2.19)}$$

where, as already noted,

σ_c = vulnerable radius

σ_d^2 = variance of the burst pattern.

Equation D4-2.14 shows that **Eq.** D4-2.19 can be rewritten in terms of the vulnerable area a as

$$(Pr)_{ssk} = \frac{a}{a + 2\pi\sigma_d^2}. \qquad \text{(D4-2.20)}$$

Derivation 4-3. Derivation of the single-shot kill probability in three dimensions, for fixed bias and normal radial dispersion.

The expansion of the expression for the single-shot kill probability to include all points in the infinite volume x,y,z is accomplished by the inclusion of the variable z in Eq. D4-2.9, yielding

$$(Pr)_{ssk} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x,y,z)\,(Pr)_{d(kill)}(x,y,z)\,dx\,dy\,dz \qquad \text{(D4-3.1)}$$

where

$(Pr)_{ssk}$ = single-shot kill probability in three dimensions, i.e., the probability that a given individual shot fired on a diffuse target centered at $x = y = z = 0$ will achieve a kill for a burst occurring at any point (x,y,z) in the entire x,y,z volume

$p(x,y,z)$ = probability density function of the presence of a burst at the particular point (x,y,z)

$(Pr)_{d(kill)}(x,y,z)$ = probability of achieving a kill on a diffuse target centered at $x=y=z=0$ when the burst occurs at point (x,y,z).

The three-dimensional kill probability for a diffuse target is given by Eq. D4-1.7 while the three-dimensional density function of the burst pattern can be formed from independent Gaussian distributions lying along each of the three axes, each having the form of Eq. D4-2.1. These distributions are

$$p(x) = \frac{1}{\sigma_d\sqrt{2\pi}} \exp\left[-\frac{(x-h)^2}{2\sigma_d^2}\right] \qquad \text{(D4-3.2)}$$

$$p(y) = \frac{1}{\sigma_d\sqrt{2\pi}} \exp\left[-\frac{(y-k)^2}{2\sigma_d^2}\right] \qquad \text{(D4-3.3)}$$

$$p(z) = \frac{1}{\sigma_d\sqrt{2\pi}} \exp\left[-\frac{(z-\ell)^2}{2\sigma_d^2}\right] \qquad \text{(D4-3.4)}$$

where

$p(x)$, $p(y)$, and $p(z)$ = independent Gaussian probability density functions of the burst-pattern dispersion, along the x-, y-, and z-axes, respectively

h, k, and ℓ = components of the bias along the x-, y-, and z-axes, respectively

σ_d^2 = variance of the burst pattern.

The trivariate distribution $p(x,y,z)$ is formed, similarly to the bivariate distribution discussed in par 4-4.2.4, by multiplication of the independent distributions $p(x)$, $p(y)$, and $p(z)$, yielding

$$p(x,y,z) = p(x)\,p(y)\,p(z) = \frac{1}{(2\pi)^{3/2}\sigma_d^3} \exp\left[-\frac{(x-h)^2+(y-k)^2+(z-\ell)^2}{2\sigma_d^2}\right] \qquad \text{(D4-3.5)}$$

Lacking a fourth dimension, one cannot form a simple physical model or graphical representation of a trivariate distribution. The most useful visualization is afforded by the analogy

Derivation 4-3. (Continued)

of a cloud or nebula of microscopic particles in which at any point the particle density represents the probability density. The cloud is most dense at the point (h,k,ℓ), and the density decreases in all directions from this point. The density along any radial axis has the form of a normal distribution, for example, the form of Fig. D4-2.1(A) for the x-axis.

The integrals indicated in Eq. D4-3.1 may be performed, with some manipulation, after substitution of Eqs. D4-1.5 and D4-1.7. With the indicated substitutions, Eq. D4-3.1 becomes:

$$(Pr)_{ssk} = \frac{1}{(2\pi)^{3/2}\sigma_d^3} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[-\frac{(x-h)^2+(y-k)^2+(z-\ell)^2}{2\sigma_d^2}\right] \exp\left[-\frac{x^2+y^2+z^2}{2\sigma_c^2}\right] dxdydz. \tag{D4-3.6}$$

This expression states that the single-shot kill probability is obtained by integration of the elemental probability that a kill is secured in the element of volume dxdydz, located at point (x,y,z).

In the remainder of this derivation, simpler derivations are obtained if the probability density functions are transformed to spherical coordinates. It will be necessary to transform not only the exponential functions of x, y, and z into the spherical coordinates r, ϕ, and θ, but also the element of volume dxdydz which will be designated dV.

The required coordinate transformation can be readily obtained by use of simple relationships of vector analysis as shown in Derivation 4-4.*

---

* The geometrical explanation of the coordinate transformation is employed in this summary in order to make clear the physical basis of the operations. However, a general rule that could be used for the transformation of a probability density function is the expression

$$p(u_1, u_2, u_3) = p(x,y,z)\,\frac{\partial(x,y,z)}{\partial(u_1,u_2,u_3)} \tag{D4-3.7}$$

where $\frac{\partial(x,y,z)}{\partial(u_1,u_2,u_3)}$ (the Jacobian of the transformation) is

$$\frac{\partial(x,y,z)}{\partial(u_1,u_2,u_3)} = \begin{vmatrix} \frac{\partial f_x}{\partial u_1} & \frac{\partial f_x}{\partial u_2} & \frac{\partial f_x}{\partial u_3} \\ \frac{\partial f_y}{\partial u_1} & \frac{\partial f_y}{\partial u_2} & \frac{\partial f_y}{\partial u_3} \\ \frac{\partial f_z}{\partial u_1} & \frac{\partial f_z}{\partial u_2} & \frac{\partial f_z}{\partial u_3} \end{vmatrix} \tag{D4-3.8}$$

Definitions of the symbols are given in Derivation 4-4.

Derivation 4-3. (Continued)

Returning to Eq. D4-3.6, let $\sigma_d^2 = m\sigma_c^2$ for convenience in algebraic manipulation in the exponent only. Then combine exponents, expand, and complete the square in x, y, and z. The result is

$$(Pr)_{ssk} = \frac{1}{(2\pi)^{3/2}\sigma_d^3} \exp\left[-\frac{h^2+k^2+\ell^2}{2(m+1)\sigma_c^2}\right] \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[-\frac{m+1}{2m\sigma_c^2}\left[\left(x-\frac{h}{m+1}\right)^2 + \left(y-\frac{k}{m+1}\right)^2 + \left(z-\frac{\ell}{m+1}\right)^2\right]\right] dxdydz. \tag{D4-3.9}$$

Next, make the following change of variables

$$\begin{aligned} x' &= x - \frac{h}{m+1} \\ y' &= y - \frac{k}{m+1} \\ z' &= z - \frac{\ell}{m+1} \end{aligned} \tag{D4-3.10}$$

which is a linear translation of the center of the coordinate system. Then

$$(Pr)_{ssk} = \frac{1}{(2\pi)^{3/2}\sigma_d^3} \exp\left[-\frac{h^2+k^2+\ell^2}{2(m+1)\sigma_c^2}\right] \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[-\frac{m+1}{2m\sigma_c^2}\left(x'^2+y'^2+z'^2\right)\right] dx'dy'dz'. \tag{D4-3.11}$$

Obviously, dx' = dx, dy' = dy, and dz' = dz, so that Eq. D4-3.11 can be formed into polar coordinates by making use of Eq. D4-4.13 and the geometrical relationship $r^2 = x'^2 + y'^2 + z'^2$. The transformed equation is

$$(Pr)_{ssk} = \frac{1}{(2\pi)^{3/2}\sigma_d^3} \exp\left[-\frac{h^2+k^2+\ell^2}{2(m+1)\sigma_c^2}\right] \int_0^{\pi}\int_0^{2\pi}\int_0^{\infty} \exp\left[-\left(\frac{m+1}{2m\sigma_c^2}\right)r^2\right] r^2 \sin\phi \, dr d\theta d\phi \tag{D4-3.12}$$

The \$-integral would be zero if the entire space were considered (0 to 2π). There is, however, a large class of engagements where the model of the target can include only the half-space ($0<\phi<\pi$, $0<\theta<2\pi$, $0<r<\infty$). Most ground targetsfall in this class, and also any engagement in which contact- or proximity-fuzed ammunition is employed. Therefore, this class of engagement will be assumed here and the limits of integration in Eq. D4-3.12 will

Derivation 4-3. (Continued)

be set accordingly. For other classes, the remaining half-space can be included by doubling the integral, because of symmetry.

Since the integrals exist, the order of integration can be changed, yielding

$$(Pr)_{ssk} = \frac{1}{(2\pi)^{3/2}\sigma_d^3} \exp\left[-\frac{h^2+k^2+\ell^2}{2(m+1)\sigma_c^2}\right] \int_0^{\infty}\int_0^{2\pi}\int_0^{\pi} \exp\left[-\left(\frac{m+1}{2m\sigma_c^2}\right) r^2\right] r^2 \sin\phi \, d\phi d\theta dr. \tag{D4-3.13}$$

Integrating first with respect to φ and then with respect to θ, yields

$$(Pr)_{ssk} = \frac{4\pi}{(2\pi)^{3/2}\sigma_d^3} \exp\left[-\frac{h^2+k^2+\ell^2}{2(m+1)\sigma_c^2}\right] \int_0^{\infty} \exp\left[-\left(\frac{m+1}{2m\sigma_c^2}\right) r^2\right] r^2 dr. \tag{D4-3.14}$$

The definite integral in Eq. D4-3.14 is a standard form, and is evaluated* to give

$$(Pr)_{ssk} = \frac{1}{(m+1)^{3/2}} \exp\left[-\frac{h^2+k^2+\ell^2}{2(m+1)\sigma_c^2}\right] = \left(\frac{\sigma_c^2}{\sigma_c^2+\sigma_d^2}\right)^{3/2} \exp\left[-\frac{h^2+k^2+\ell^2}{2(\sigma_c^2+\sigma_d^2)}\right] \tag{D4-3.15}$$

In two dimensions, the factor p(z) is dropped from Eq. D4-3.5 leaving

$$p(x,y) = \frac{1}{2\pi\sigma_d^2} \exp\left[-\frac{(x-h)^2+(y-k)^2}{2\sigma_d^2}\right]. \tag{D4-3.16}$$

Equation D4-3.6 then becomes

$$(Pr)_{ssk} = \frac{1}{2\pi\sigma_d^2} \int\int_{-\infty}^{\infty} \exp\left[-\frac{(x-h)^2+(y-k)^2}{2\sigma_d^2}\right] \exp\left[-\frac{x^2+y^2}{2\sigma_c^2}\right] xdy \tag{D4-3.17}$$

After translation of the axes as in Eq. D4-3.11, the conversion to polar coordinates is per-

* By using Eq. 861.7 on page 201 of Reference 23.

Derivation 4-3. (Continued)

formed, making use of Eq. D4-4.14 for the element of area dxdy. These steps yield the relationship

$$(Pr)_{ssk} = \frac{1}{2\pi\sigma_d^2}\exp\left[-\frac{h^2+k^2}{2(m+1)\sigma_c^2}\right]\int_0^{2\pi}\int_0^{\infty}\exp\left[-\frac{m+1}{2m\sigma_c^2}r^2\right] r\,dr\,d\theta. \tag{D4-3.18}$$

Performing the indicated integrations* gives

$$(Pr)_{ssk} = \frac{1}{\sigma_d^2}\exp\left[-\frac{h^2+k^2}{2(m+1)\sigma_c^2}\right]\left[\frac{m\sigma_c^2}{m+1}\right]$$

or

$$\boxed{(Pr)_{ssk} = \frac{\sigma_c^2}{\sigma_c^2+\sigma_d^2}\exp\left[-\frac{h^2+k^2}{2\left(\sigma_c^2+\sigma_d^2\right)}\right]} \tag{D4-3.19}$$

If, in the two-dimensional case, the bias is considered to be on the x-axis (this s equivalent to a rotation of the x,y coordinate system, so that the x-axis passes through the mean of the burst-pattern dispersion), then k = 0 and Eq. D4-3.19 reduces to

$$\boxed{(Pr)_{ssk} = \frac{\sigma_c^2}{\sigma_c^2+\sigma_d^2}\exp\left[-\frac{h^2}{2\left(\sigma_c^2+\sigma_d^2\right)}\right]}\ . \tag{D4-3.20}$$

In many practical cases, the vulnerable area a (which, from Eq. D4-1.14, is equal to $2\pi\sigma_c^2$) is much less than $2\pi\sigma_d^2$. Equation D4-3.20 then reduces to

$$\boxed{(Pr)_{ssk} \approx \frac{\quad}{2\pi\sigma_d^2}\exp\left[-\frac{\quad}{\sigma_d}\right]} \tag{D4-3.21}$$

If no bias exists, then h = k = ℓ = 0, and Eqs. D4-3.15 and D4-3.19 show that

* Using Eq. 861.4 on page 200 of Reference 23.

Derivation 4-3. (Continued)

$$\boxed{(Pr)_{ssk} = \left(\frac{\sigma_c^2}{\sigma_d^2 + \sigma_c^2}\right)^{3/2}} \tag{D4-3.22}$$

$$\boxed{(Pr)_{ssk} = \frac{\sigma_c^2}{\sigma_d^2 + \sigma_c^2}} \tag{D4-3.23}$$

for the three-dimensional and two-dimensional cases, respectively.

Derivation 4-4. Transformation of three-dimensional coordinates.

Consider a vector $\vec{r}$ whose components in an orthogonal rectangular coordinate system (see Fig. D4-4.1) are given by the relationship

$$\vec{r} = x\vec{u}_x + y\vec{u}_y + z\vec{u}_z \tag{D4-4.1}$$

where

$\vec{u}_x, \vec{u}_y, \vec{u}_z$ = unit vectors along the x-, y-, and z-axes, respectively

x, y, z = the components of $\vec{r}$ along these respective axes.

Assume that it is desired to express this vector in terms of another orthogonal coordinate system which, for the sake of generality, has curvilinear axes,* u_1, u_2, and u_3, that are functionally related to x, y, z by the relationships

$$\begin{aligned} x &= f_x(u_1, u_2, u_3) \\ y &= f_y(u_1, u_2, u_3) \\ z &= f_z(u_1, u_2, u_3) . \end{aligned} \tag{D4-4.2}$$

An incremental change $d\vec{r}$ can be expressed in the original coordinate system by the expression

$$d\vec{r} = dx\vec{u}_x + dy\vec{u}_y + dz\vec{u}_z . \tag{D4-4.3}$$

The derivative of the vector to a curve, with respect to arc length along the curve, is the unit vector tangent to the curve. Therefore

$$\frac{\partial \vec{r}}{\partial s_1} = \vec{u}_1 ; \quad \frac{\partial \vec{r}}{\partial s_2} = \vec{u}_2 ; \quad \frac{\partial \vec{r}}{\partial s_3} = \vec{u}_3 \tag{D4-4.4}$$

where

s_1, s_2, s_3 = the respective arc lengths along the curvilinear u_1, u_2, and u_3 axes

$\vec{u}_1, \vec{u}_2, \vec{u}_3$ = unit vectors tangent to these axes,

A set of scale factors C_1, C_2, C_3 that relate arc length to distance along a unit vector can be defined as follows

$$C_1 = \frac{ds_1}{du_1} , \quad C_2 = \frac{ds_2}{du_2} , \quad C_3 = \frac{ds_3}{du_3} . \tag{D4-4.5}$$

Multiplying Eqs. D4-4.4 by Eqs. D4-4.5 gives expressions of the form

$$\frac{d\vec{r}}{ds_1} \cdot \frac{ds_1}{du_1} = C_1\vec{u}_1 . \tag{D4-4.6}$$

* See Section 6.16 of Reference 31 for a discussion of orthogonal curvilinear coordinate *systems*.

Derivation 4-4. (Continued)

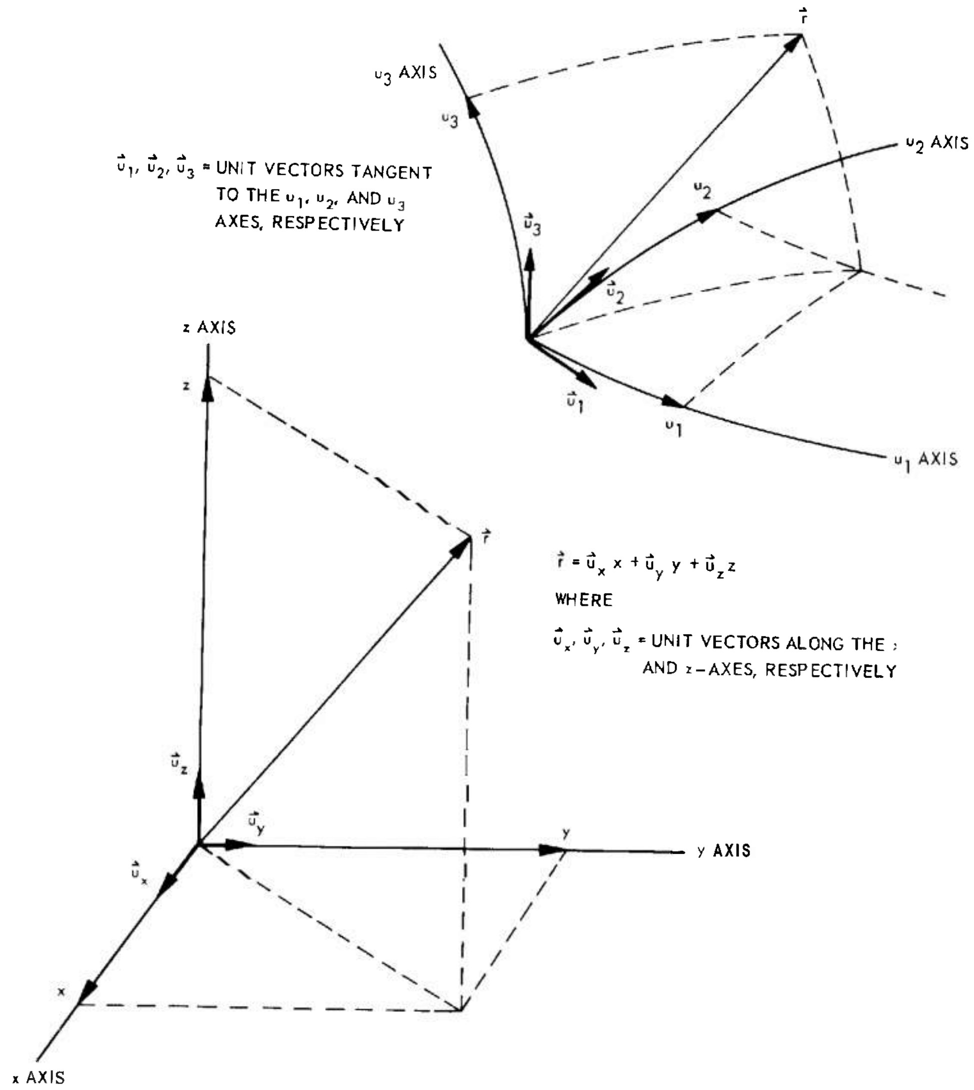


Figure D4-4.1 Components of the vector $\vec{r}$ in an orthogonal rectangular coordinate system and in an orthogonal curvilinear coordinate system,

Since the magnitude of a unit vector is unity, the scalar portion of Eq. D4-4.6 is

$$C_1 = \left| \frac{\partial \vec{r}}{\partial u_1} \right| \tag{D4-4.7a}$$

Similarly,

Derivation 4-4. (Continued)

$$C_2 = \left| \frac{\partial \vec{r}}{\partial u_2} \right| \tag{D4-4.7b}$$

and

$$C_3 = \left| \frac{\partial \vec{r}}{\partial u_3} \right| \tag{D4-4.7c}$$

Equations D4-4.7 provide a means of evaluating C_1, C_2, and C_3 when the functional relationships of Eqs. D4-4.2 are known.

The element of volume dV in the (u_1, u_2, u_3) axis system is the rectangular parallelepiped formed by the tangent unit vectors $\vec{u}_1$, $\vec{u}_2$, $\vec{u}_3$ — the lengths of the sides being ds_1, ds_2, and ds_3 (see Fig. D4-4.2). Thus, the element of volume is

$$dV = ds_1\, ds_2\, ds_3 = C_1 C_2 C_3\, du_1\, du_2\, du_3 . \tag{D4-4.8}$$

In spherical coordinates, (r, ϕ, θ), as shown in Fig. D4-4.3, the coordinates are related to x, y and z by the functional relationships

$$\begin{aligned} x &= r \cos\theta \sin\phi \\ y &= r \sin\theta \sin\phi \\ z &= r \cos\phi . \end{aligned} \tag{D4-4.9}$$

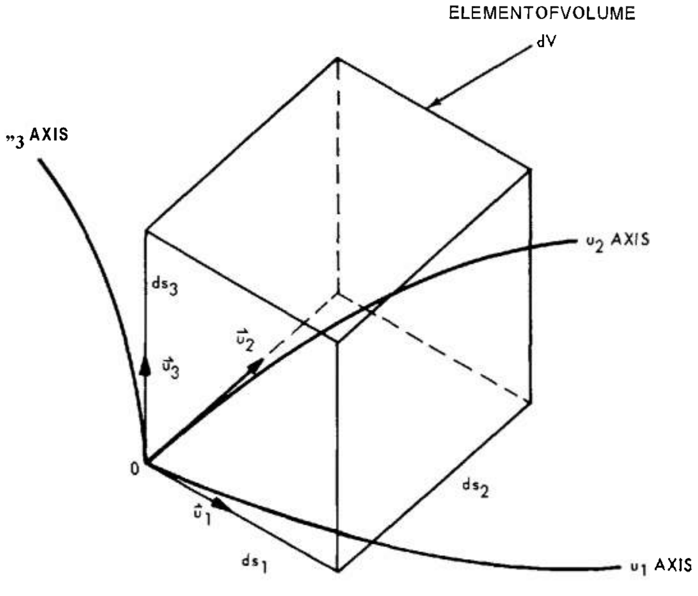


Figure D4-4.2. Element of volume dV in u_1, u_2, u_3 coordinates.

Derivation 4-4. (Continued)

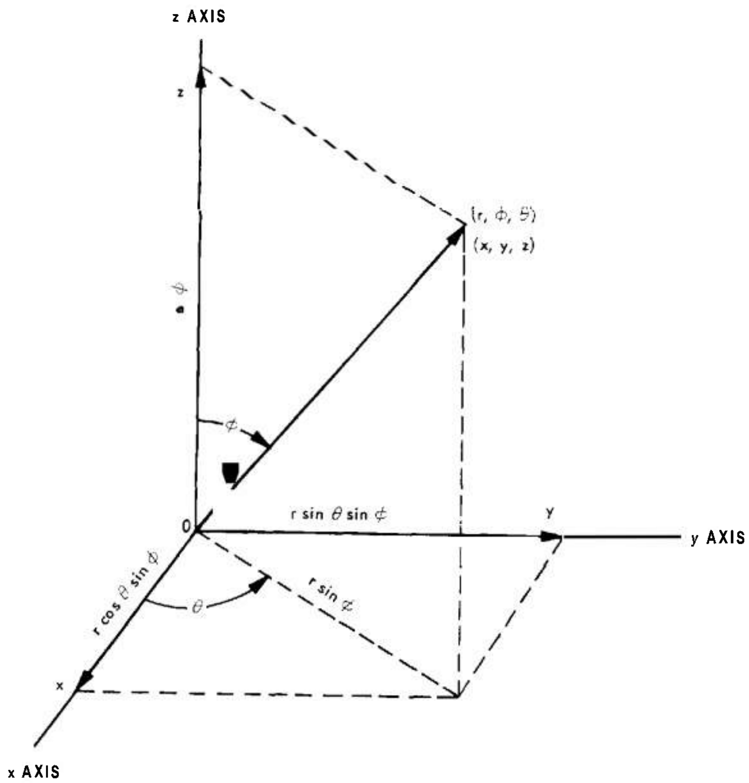


Figure D4-4.3. Geometrical relationships between three-dimensional rectangular and polar coordinates.

For the spherical coordinate system, which is a particular case of the curvilinear coordinate system, let the coordinates r, ϕ and θ substitute, respectively, for the coordinates u_1, u_2, and u_3. Correspondingly, the subscripts 1, 2 and 3 relating to u_1, u_2, and u_3 are replaced by subscripts r, ϕ and θ.

Then, in accordance with Eqs. D4-4.7, each scale factor (C_r, C_ϕ, C_θ) is found by taking a partial derivative of Eq. D4-4.1 — after substituting the functional relationships of Eqs. D4-4.9 — and determining the magnitude of the resulting vector. For example

$$
\begin{aligned}
C_r = \left|\frac{\partial \vec{r}}{\partial r}\right| &= \left|\cos\theta \sin\phi\, \vec{u}_x + \sin\theta \sin\phi\, \vec{u}_y + \cos\phi\, \vec{u}_z\right| \\
&= (\cos^2\theta \sin^2\phi + \sin^2\theta \sin^2\phi + \cos^2\phi)^{1/2} \\
&= 1.
\end{aligned}
\qquad \text{(D4-4.10a)}
$$

Similarly,

Derivation 4-4. (Continued)

$$C_\phi = \left|\frac{\partial \vec{r}}{\partial \phi}\right| = \left| r\cos\theta\cos\phi\,\vec{u}_x + r\sin\theta\cos\phi\,\vec{u}_y - r\sin\phi\,\vec{u}_z \right|$$

$$= (r^2\cos^2\theta\cos^2\phi + r^2\sin^2\theta\cos^2\phi + r^2\sin^2\phi)^{1/2} \qquad \text{(D4-4.10b)}$$

$$= r$$

and

$$C_\theta = \left|\frac{\partial \vec{r}}{\partial \theta}\right| = \left| -r\sin\theta\sin\phi\,\vec{u}_x + r\cos\theta\sin\phi\,\vec{u}_y \right|$$

$$= (r^2\sin^2\theta\sin^2\phi + r^2\cos^2\theta\sin^2\phi)^{1/2} \qquad \text{(D4-4.10c)}$$

$$= r\sin\phi.$$

By rearrangement of Eq. D4-4.6, the unit tangent vectors are given by expressions of the form

$$\vec{u}_r = \frac{1}{C_r}\frac{\partial \vec{r}}{\partial r}. \qquad \text{(D4-4.11)}$$

Since $C_r = 1$ and $\partial\vec{r}/\partial r$ is the expression within the magnitude symbols of Eq. D4-4.10a, it follows that

$$\vec{u}_r = \cos\theta\sin\phi\,\vec{u}_x + \sin\theta\sin\phi\,\vec{u}_y + \cos\phi\,\vec{u}_z. \qquad \text{(D4-4.12a)}$$

Similarly,

$$\vec{u}_\phi = \cos\theta\cos\phi\,\vec{u}_x + \sin\theta\cos\phi\,\vec{u}_y - \sin\phi\,\hat{u}_z \qquad \text{(D4-4.12b)}$$

and

$$\vec{u}_\theta = -\sin\theta\,\hat{u}_x + \cos\theta\,\vec{u}_y. \qquad \text{(D4-4.12c)}$$

The element of volume is, from Eq. D4-4.8,

$$dV = C_r C_\phi C_\theta\, dr\, d\phi\, d\theta = r^2\sin\phi\, dr\, d\phi\, d\theta. \qquad \text{(D4-4.13)}$$

In certain cases, the only concern is with errors lying within a plane that passes through the center of the target. If the z-axis is chosen to be normal to this surface, there will be no component of the z-axis in the surface. An element of area, $dA = dxdy$, is transformed into the area of one side of the volume element in Fig. D4-4.2 and is bounded by the sides ds_1

Derivation 4-4. (Continued)

(or ds_r) and ds_3 (or ds_θ). Then, from Eq. D4-4.5

$$
\begin{aligned}
dA &= ds_r ds_\theta = C_r C_\theta \, dr d\theta \\
&= r \sin\phi \, dr d\theta \\
&= r dr d\theta
\end{aligned}
\tag{D4-4.14}
$$

since $\phi = 90°$.

Derivation 4-5. Derivation of the engagement hit probability for the case when the bias is variable.

An expression for the engagement hit probability can be derived from Eq. 4-78 by substitution from Eqs. 4-76 and 4-77. The result is

$$(Pr)_{eh} = \int_0^{2\pi}\int_0^{\infty}\left\{1 - \left[1 - \frac{A}{2\pi\sigma_d^2}\exp\left(-\frac{r^2}{2\sigma_d^2}\right)\right]^n\right\}\frac{r}{2\pi\sigma_b^2}\exp\left[-\frac{r^2}{2\sigma_b^2}\right]dr\,d\theta. \tag{D4-5.1}$$

Since the integrand of Eq. D4-5.1 is independent of 8, the integration on 8 yields a constant, 2π. Multiplying through the integrand then yields

$$(Pr)_{eh} = \frac{1}{\sigma_b^2}\int_0^{\infty} r\exp\left(-\frac{r^2}{2\sigma_b^2}\right)dr - \int_0^{\infty}\frac{r}{\sigma_b^2}\exp\left(-\frac{r^2}{2\sigma_b^2}\right)\left[1 - \frac{a}{2\pi\sigma_d^2}\exp\left(-\frac{r^2}{2\sigma_d^2}\right)\right]^n dr \tag{D4-5.2}$$

The first integral evaluates to σ_b^2, as can be seen from Eqs. D4-2.17 and D4-2.18. Therefore, the first term of Eq. D4-5.2 is unity. The second integral is evaluated by a change of variable, in which

$$x = \exp\left[-\frac{r^2}{2\sigma_b^2}\right] \tag{D4-5.3}$$

Then

$$dx = -\frac{r}{\sigma_b^2}\exp\left[-\frac{r^2}{2\sigma_b^2}\right]dr \tag{D4-5.4}$$

and

$$\exp\left[-\frac{r^2}{2\sigma_d^2}\right] = \exp\left[-\frac{r^2\sigma_b^2}{2\sigma_b^2\sigma_d^2}\right] = \left[\exp\left(-\frac{r^2}{2\sigma_b^2}\right)\right]^{\frac{\sigma_b^2}{\sigma_d^2}} = x^{\frac{\sigma_b^2}{\sigma_d^2}}. \tag{D4-5.5}$$

Substitution of the expressions just derived in Eqs. D4-5.3 through D4-5.5 into Eq. D4-5.2 gives

$$(Pr)_{eh} = 1 - \int_{x_1}^{x_2}\left\{-\left[1 - \frac{A}{2\pi\sigma_d^2}x^{\frac{\sigma_b^2}{\sigma_d^2}}\right]^n\right\}dx. \tag{D4-5.6}$$

The limits of integration, x_1 and x_2, are evaluated by setting $x = x_1$ in Eq. D4-5.3 when $r = 0$,

Derivation 4-5. (Continued)

and $x = x_2$ when $r = \infty$. This procedure yields

$$x_1 = e^0 = 1; \quad x_2 = e^{-\infty} = 0. \tag{D4-5.7}$$

The reversal of the limits in Eq. D4-5.6 yields the integrable expression

$$(Pr)_{eh} = 1 - \int_0^1 \left[1 - \frac{A}{2\pi\sigma_d^2} x^{\frac{\sigma_b^2}{\sigma_d^2}}\right]^n dx \tag{D4-5.8}$$

Equation D4-5.8 can be integrated by means of the reduction formula*:

$$\int (1 + cx^p)^n\, dx = \frac{1}{1 + np}\left[x(1 + cx^p)^n + np\int (1 + cx^p)^{n-1}\, dx\right]. \tag{D4-5.9}$$

Letting $c = -A/2\pi\sigma_d^2$ and $p = \sigma_b^2/\sigma_d^2$, gives

$$(Pr)_{eh} = 1 - \frac{1}{1 + n\frac{\sigma_b^2}{\sigma_d^2}}\left| x\left(1 - \frac{A}{2\pi\sigma_d^2} x^{\sigma_b^2/\sigma_d^2}\right)^n \right|_0^1 - \frac{n\frac{\sigma_b^2}{\sigma_d^2}}{1 + n\frac{\sigma_b^2}{\sigma_d^2}} \int_0^1 \left(1 - \frac{A}{2\pi\sigma_d^2} x^{\sigma_b^2/\sigma_d^2}\right)^{n-1} dx$$

$$= 1 - \frac{\left(1 - \frac{A}{2\pi\sigma_d^2}\right)^n}{1 + n\frac{\sigma_b^2}{\sigma_d^2}} - \frac{n\frac{\sigma_b^2}{\sigma_d^2}}{1 + n\frac{\sigma_b^2}{\sigma_d^2}} \int_0^1 \left(1 - \frac{A}{2\pi\sigma_d^2} x^{\sigma_b^2/\sigma_d^2}\right)^{n-1} dx. \tag{D4-5.10}$$

*Adapted from item 370 on page 69 of Reference 23.

Derivation 4-5. (Continued)

Carrying out the repeated integrations yields the series

$$(Pr)_{eh} = 1 - \frac{\left(1 - \frac{A}{2\pi\sigma_d^2}\right)^n}{1 + n\frac{\sigma_b^2}{\sigma_d^2}} - \frac{n\frac{\sigma_b^2}{\sigma_d^2}\left(1 - \frac{A}{2\pi\sigma_d^2}\right)^{n-1}}{\left[1 + n\frac{\sigma_b^2}{\sigma_d^2}\right]\left[1 + (n-1)\frac{\sigma_b^2}{\sigma_d^2}\right]}$$

$$\frac{n(n-1)\left(\frac{\sigma_b^2}{\sigma_d^2}\right)^2\left(1 - \frac{A}{2\pi\sigma_d^2}\right)^{n-2}}{\left[1 + n\frac{\sigma_b^2}{\sigma_d^2}\right]\left[1 + (n-1)\frac{\sigma_b^2}{\sigma_d^2}\right]\left[1 + (n-2)\frac{\sigma_b^2}{\sigma_d^2}\right]}$$

$$- \frac{n!\left(\frac{\sigma_b^2}{\sigma_d^2}\right)^n}{\left[1 + n\frac{\sigma_b^2}{\sigma_d^2}\right]\left[1 + (n-1)\frac{\sigma_b^2}{\sigma_d^2}\right]\cdots[1]} \tag{D4-5.11}$$

This series may be written in the more compact form'

$$(Pr)_{eh} = 1 - \sum_{k=0}^{n} T_k \tag{D4-5.12}$$

where

$$T_o = \frac{\left(1 - \frac{A}{2\pi\sigma_d^2}\right)^n}{1 + n\left(\frac{\sigma_b^2}{\sigma_d^2}\right)} \tag{D4-5.13}$$

$$T_k = \frac{(n-k+1)\left(\frac{\sigma_b^2}{\sigma_d^2}\right)}{\left(1 - \frac{A}{2\pi\sigma_d^2}\right)\left\{1 + (n-k)\left(\frac{\sigma_b^2}{\sigma_d^2}\right)\right\}} T_{k-1} \tag{D4-5.14}$$

Derivation 4-5. (Continued

An alternate form of Eq. D4-5.12, obtained by reversing the order of the series, is

$$(Pr)_{eh} = 1 - \sum_{k=0}^{n} T'_k \tag{D4-5.15'}$$

where

$$T'_o = \frac{n!\left(\frac{\sigma_b^2}{\sigma_d^2}\right)^n}{\prod_{j=0}^{n}\left\{1 + (n-j)\left(\frac{\sigma_b^2}{\sigma_d^2}\right)\right\}} \tag{D4-5.16}$$

and

$$T_k = \frac{\left(-\frac{A}{2\pi\sigma_d^2}\right)\left\{1 + (k-1)\frac{\sigma_b^2}{\sigma_d^2}\right\}}{k\left(\frac{\sigma_b^2}{\sigma_d^2}\right)} T'_{k-1} \tag{D4-5.17}$$

Vote: The symbol $\prod_{j=0}^{n}\{\;\}$ represents a multiplication of the n terms of the quantity inside he braces, with j taking on all integral values from 0 to n.

The two alternative forms are given because it is desirable to have the coefficients of the recurrence relationships given by Eqs. D4-5.12 and D4-5.15 less than unity. An approximate discriminant for use in selecting the form to be employed, provided by Tappert (Ref. 28) is as follows:

(a) If $\dfrac{\left(\frac{\sigma_b^2}{\sigma_d^2}\right)}{1 - \frac{A}{2\pi\sigma_d^2}} < 1$, use Eq. D4-5.1

(b) If $\dfrac{(\quad)^2}{\cdot\;\overline{2\pi\sigma_d^2}} > 1$, use Eq. D4-5.15

Derivation 4-5. (Continued)

NOTE ON AN ALTERNATIVE SOLUTION

The preceding solution of the integralof Eq. D4-5.8 is the one that was employed in the error analysis of the Vigilante Weapon System (see par 4-6). More recently, the simpler solution below has been developed at Frankford Arsenal. First, let

$$\alpha = \frac{A}{2\pi\sigma_d^2} \quad \text{and} \quad \beta = \frac{\sigma_b^2}{\sigma_d^2}$$

A binomial expansion of the integral in Eq. D4-5.8 then gives

$$\int_0^1 \left[1 - \alpha x^{\beta}\right]^n dx = \int_0^1 \sum_{k=0}^{n} (-1)^k \alpha^k \binom{n}{k} x^{k\beta} dx \tag{D4-5.18}$$

where

$$\binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{k!}$$

$$= \text{the binomial coefficient}^{*}.$$

The integration indicated on the right-hand side of Eq. D4-5.18 is straightforward, yielding

$$\int_0^1 \left[1 - \alpha x^{\beta}\right]^n dx = \sum_{k=0}^{n} (-1)^k \alpha^k \binom{n}{k} \int_0^1 x^{k\beta} dx = \sum_{k=0}^{n} \frac{(-1)^k \alpha^k}{k\beta + 1} \binom{n}{k}. \tag{D4-5.19}$$

Thus, an alternative solution for Eq. D4-5.8 has the form

$$(Pr)_{eh} = 1 - \sum_{k=0}^{n} \frac{(-1)^k \alpha^k}{k\beta + 1} \binom{n}{k} = -\sum_{k=1}^{n} \frac{(-1)^k \alpha^k}{k\beta + 1} \binom{n}{k} = \sum_{k=1}^{n} \frac{(-1)^{k+1} \alpha^k}{k\beta + 1} \binom{n}{k}$$

$$= \sum_{k=1}^{n} (-1)^{k+1} \frac{A^k \sigma_d^{2(1-k)}}{(2\pi)^k (k\sigma_b^2 + \sigma_d^2)} \binom{n}{k} \tag{D4-5.20}$$

* See Item 2 on page 1 of Reference 23.

Derivation 4-6. Derivation of Eq. 4-120

The proof of Eq.4-120 requires first a demonstration that the sum of the means equals the mean of the sum. An extension of Eq. 4-29 to two random variables, x_1 and x_2, gives for the average of a bivariate function, $f(x_1, x_2)$,

$$\overline{f(x_1, x_2)} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x_1, x_2)\, p(x_1, x_2)\, dx_1 dx_2 \tag{D4-6.1}$$

(The superscript bar denotes an ensemble average of the quantity superscribed.) If $f(x_1, x_2) = f_1(x_1) + f_2(x_2)$, and if x_1 and x_2 are independent, so that $p(x_1, x_2) = p(x_1)\, p(x_2)$, then Eq. D4-6.1 takes the form

$$\overline{[f_1(x_1) + f_2(x_2)]} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [f_1(x_1) + f_2(x_2)]\, p(x_1)\, p(x_2)\, dx_1 dx_2 \tag{D4-6.2}$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_1(x_1)\, p(x_1)\, p(x_2)\, dx_1 dx_2 + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_2(x_2)\, p(x_1)\, p(x_2)\, dx_1 dx_2 \tag{D4-6.3}$$

$$= \int_{-\infty}^{\infty} f_1(x_1)\, p(x_1)\, dx_1 \int_{-\infty}^{\infty} p(x_2)\, dx_2 + \int_{-\infty}^{\infty} p(x_1)\, dx_1 \int_{-\infty}^{\infty} f_2(x_2)\, p(x_2)\, dx_2 \tag{D4-6.4}$$

Since the inner integral of the first term and the outer integral of the second term on the right-hand side of Eq. D4-6.4 are each unity, Eq. D4-6.4 can be expressed in the form

$$\overline{[f_1(x_1) + f_2(x_2)]} = \int_{-\infty}^{\infty} f_1(x_1)\, p(x_1)\, dx_1 + \int_{-\infty}^{\infty} f_2(x_2)\, p(x_2)\, dx_2 \tag{D4-6.5}$$

$$= \overline{f_1(x_1)} + \overline{f_2(x_2)}\,. \tag{D4-6.6}$$

This result can be extended to n variables, by letting

$$y = \sum_{i=1}^{n} f_i(x_i). \tag{D4-6.7}$$

This procedure yields the relationship

$$\overline{y} = \overline{\sum_{i=1}^{n} f_i(x_i)} = \sum_{i=1}^{n} \overline{f_i(x_i)}. \tag{D4-6.8}$$

Derivation 4-6. (Continued)

Introducing the definition of the variance from Eq. 4-34 and comparing it with Eq. 4-31 shows that

$$\sigma_y^2 = \int_{-\infty}^{\infty} (y - \bar{y})^2 \, p(y) \, dy = \overline{(y - \bar{y})^2} \, . \tag{D4-6.9}$$

Substitution from Eqs. D4-6.7 and D4-6.8 into Eq. D4-6.9, and regrouping terms, gives

$$\sigma_y^2 = \overline{\left[\left\{f_1(x_1) - \overline{f_1(x_1)}\right\} + \left\{f_2(x_2) - \overline{f_2(x_2)}\right\} + \cdots + \left\{f_n(x_n) - \overline{f_n(x_n)}\right\}\right]^2} \tag{D4-6.10}$$

Expansion of the square gives

$$\begin{aligned}\sigma_y^2 = {} & \overline{\left\{f_1(x_1) - \overline{f_1(x_1)}\right\}^2 + \left\{f_2(x_2) - \overline{f_2(x_2)}\right\}^2 + \cdots + \left\{f_n(x_n) - \overline{f_n(x_n)}\right\}^2} \\ & \overline{+ 2\left\{f_1(x_1) - \overline{f_1(x_1)}\right\}\left\{f_2(x_2) - \overline{f_2(x_2)}\right\} + \cdots} \, .\end{aligned} \tag{D4-6.11}$$

Application of Eq. D4-6.8 then yields

$$\begin{aligned}\sigma_y^2 = {} & \overline{\left\{f_1(x_1) - \overline{f_1(x_1)}\right\}^2} + \overline{\left\{f_2(x_2) - \overline{f_2(x_2)}\right\}^2} + \cdots + \overline{\left\{f_n(x_n) - \overline{f_n(x_n)}\right\}^2} \\ & \overline{+ 2\left\{f_1(x_1) - \overline{f_1(x_1)}\right\}\left\{f_2(x_2) - \overline{f_2(x_2)}\right\} + \cdots} \, .\end{aligned} \tag{D4-6.12}$$

In order to show that the cross-product terms are zero, a typical term is expanded and averaged as follows:

$$\begin{aligned}\overline{2\left[f_1(x_1) - \overline{f_1(x_1)}\right]\left[f_2(x_2) - \overline{f_2(x_2)}\right]} &= 2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[f_1(x_1) - \overline{f_1(x_1)}\right]\left[f_2(x_2) - \overline{f_2(x_2)}\right] p(x_1, x_2)\, dx_1 dx_2 \\ &= 2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[f_1(x_1) f_2(x_2) - f_1(x_1)\overline{f_1(x_2)} - f_2(x_2)\overline{f_1(x_1)}\right. \\ &\qquad \left. + \overline{f_1(x_1)}\;\overline{f_2(x_2)}\right] p(x_1, x_2)\, dx_1 dx_2\end{aligned} \tag{D4-6.13}$$

Since x_1 and x_2 are independent,

$$p(x_1, x_2) = p(x_1)\, p(x_2) \tag{D4-6.14}$$

Derivation 4-6. (Continued)

and

$$\overline{2[f_1(x_1)-\overline{f_1(x_1)}][f_2(x_2)-\overline{f_2(x_2)}]} = 2\left\{\int_{-\infty}^{\infty} f_1(x_1)\,p(x_1)\,dx_1 \int_{-\infty}^{\infty} f_2(x_2)\,p(x_2)\,dx_2\right.$$

$$-\overline{f_2(x_2)}\int_{-\infty}^{\infty} f_1(x_1)\,p(x_1)\,dx_1 \int_{-\infty}^{\infty} p(x_2)\,dx_2$$

$$-\overline{f_1(x_1)}\int_{-\infty}^{\infty} p(x_1)\,dx_1 \int_{-\infty}^{\infty} f_2(x_2)\,p(x_2)\,dx_2$$

$$\left.+\overline{f_1(x_1)}\;\overline{f_2(x_2)}\int_{-\infty}^{\infty} p(x_1)\,dx_1 \int_{-\infty}^{\infty} p(x_2)\,dx_2\right\} \tag{D4-6.15}$$

Since

$$\int_{-\infty}^{\infty} f(x)\,p(x)\,dx = \overline{x} \tag{D4-6.16}$$

and

$$\int_{-\infty}^{\infty} p(x)\,dx = 1 \tag{D4-6.17}$$

then

$$\overline{2[f_1(x_1)-\overline{f_1(x_1)}][f_2(x_2)-\overline{f_2(x_2)}]} = 2\left\{\overline{f_1(x_1)}\;\overline{f_2(x_2)} - \overline{f_2(x_2)}\;\overline{f_1(x_1)} - \overline{f_1(x_1)}\;\overline{f_2(x_2)} + \overline{f_1(x_1)}\;\overline{f_2(x_2)}\right\} = 0. \tag{D4-6.18}$$

Since $\overline{[f_i(x_i) - \overline{f_i(x_i)}]^2} = \sigma_i^2$ by definition, Eq. D4-6.12 becomes

$$\sigma_y^2 = \sigma_1^2 + \sigma_2^2 + \cdots + \sigma_n^2 = \sum_{i=1}^{n} \sigma_i^2 \tag{D4-6.19}$$

which is given in the text as Eq. 4-120.

Derivation 4-7. The input-output relationship in the time domain for a system element.

Equation 4-173 can be re-expressed in the form

$$\phi_{yy}(\tau) = \int_{-\infty}^{\infty} r(\mu)\, d\mu \int_{-\infty}^{\infty} \phi_{xx}(\tau + \mu - \nu)\, r(\nu)\, d\nu \tag{D4-7.1}$$

since

$$\rho = \tau + \mu - \nu. \tag{D4-7.2}$$

Change the variable of integration by letting

$$t_2 = \nu - \mu. \tag{D4-7.3}$$

Then, Eq. D4-7.1 becomes

$$\phi_{yy}(\tau) = \int_{-\infty}^{\infty} r(\mu)\, d\mu \int_{-\infty}^{\infty} \phi_{xx}(\tau - t_2)\, r(\mu + t_2)\, d(\mu + t_2) \tag{D4-7.4}$$

$$= \phi_{xx}(\tau - t_2) \int_{-\infty}^{\infty} d\mu \int_{-\infty}^{\infty} r(\mu) r(\mu + t_2) d\mu + \int_{-\infty}^{\infty} r(\mu) d\mu \int_{-\infty}^{\infty} r(\mu + t_2)\, \phi_{xx}(\tau - t_2) dt_2. \tag{D4-7.5}$$

Changing the order of integration shows that

$$\phi_{yy}(\tau) = \phi_{xx}(\tau - t_2) \int_{-\infty}^{\infty} \phi_{rr}(t_2) d\mu + \int_{-\infty}^{\infty} \phi_{xx}(\tau - t_2) dt_2 \int_{-\infty}^{\infty} r(\mu) r(\mu + t_2) d\mu \tag{D4-7.6}$$

where

$$\phi_{rr}(t_2) = \int_{-\infty}^{\infty} r(\mu) r(\mu + t_2) d\mu \tag{D4-7.7}$$

= autocorrelation of the impulse response $r(\mu)$.

The integral in the first term on the right-hand side of Eq. D4-7.6 is zero. The second integral of the following term is $\phi_{rr}(t_2)$. Therefore,

$$\phi_{yy}(\tau) = \int_{-\infty}^{\infty} \phi_{xx}(\tau - t_2)\, \phi_{rr}(t_2) dt_2. \tag{D4-7.8}$$

Theexpression on the right-hand side of this equation is the convolution of the autocorrelation of the input with the autocorrelation of the impulse response.

Derivation 4-8. Derivation of the tracking error equation.

Figure D4-8.1 depicts the fundamental vector relationships associated with the tracking process that have been employed in the systems analysis of the Vigilante Antiaircraft Weapon System [58]. The convention employed in establishing the direction of the instantaneous tracking error vector $\vec{E}$ with respect to $\vec{D}_O$ and $\vec{D}_S$ is consistent with that associated with the error output of the comparator in the input portion of a feedback control system. There (see Chapter 1 of Reference 40, for example) the error is defined as the desired value of the system output minus the value of the actual output that is generated by the feedback control system; i.e.,

(Error) = (Ces red value of system output) – (Actual value of system output) (D4-8.1)

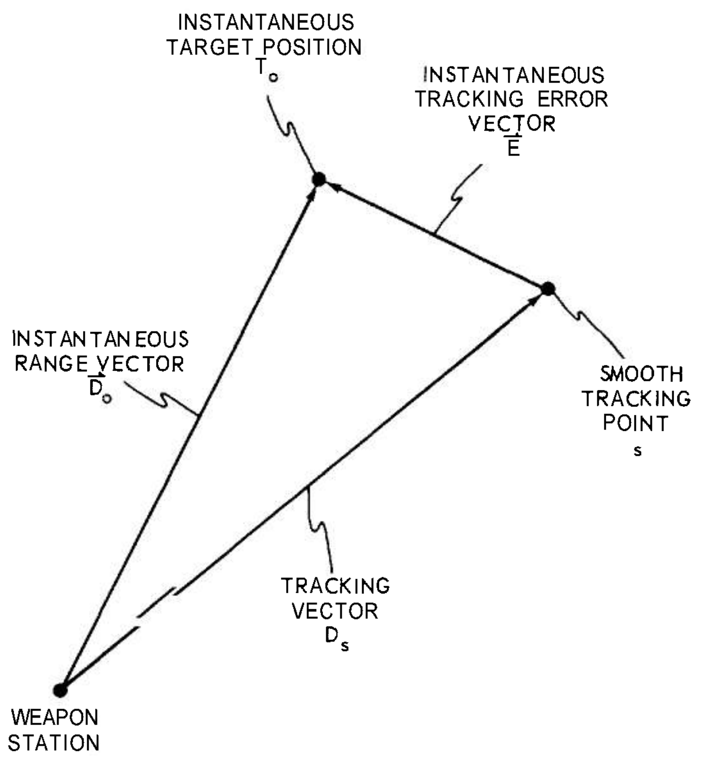


Figure D4-8.1. Vector relationships associated with tracking.

In the operation of a fire control system, the tracking system serves as a feedback control system. Its function is to keep the tracking vector $\vec{D}_S$ aligned as closely as possible with the instantaneous range vector $\vec{D}_O$. The vector $\vec{D}_O$ is thus the desired quantity associated with the tracking system, while the vector $\vec{D}_S$ is the actual output quantity generated by the tracking system and, ideally, is made to coincide with $\vec{D}_O$. Thus, by analogy with Eq. D4-8.1, the tracking error equation is

$$\vec{E} = \vec{D}_o - \vec{D}_s \qquad \text{(D4-8.2)}$$

Derivation 4-8. (Continued)

Since these vectors are all instantaneous quantities that vary with time during the course of a fire control engagement, then

$$\frac{d}{dt}\vec{E} = \frac{d}{dt}\vec{D}_o - \frac{d}{dt}\vec{D}_s \qquad \text{(D4-8.3)}$$

While the magnitude of a system error is always equal to the magnitude of the difference between the desired, or ideal, value of the system output and the actual value of the system output, the algebraic sign associated with this system error depends upon the particular approach used to define the error. For example, in contrast to the approach just described, an alternative approach is to view the instantaneous tracking error vector $\vec{E}$ in a context that is entirely separate from association with a feedback control system. In this other approach, the error relationship is based on the concept that the addition of an error to an ideal quantity yields an actual quantity; i.e.,

$$\text{(Ideal quantity)} + \text{(Error)} = \text{(Actual quantity)} \qquad \text{(D4-8.4)}$$

Thus, in that context,

$$\vec{D}_o + \vec{E} = \vec{D}_s \qquad \text{(D4-8.5)}$$

or

$$\vec{E} = \vec{D}_s - \vec{D}_o = -\vec{C} \qquad \text{(D4-8.6)}$$

where $\vec{C}$ is the instantaneous tracking correction required to null the instantaneous tracking error $\vec{E}$, Chapter 3 of Reference 4, for example, uses this alternative convention in discussing general features common to all fire control systems.

It is important to observe that whatever convention is employed initially in a given error analysis should be rigidly adhered to in the subsequent development in order to avoid errors that might otherwise result from confusion with regard to algebraic sign.

Derivation 4-9. Derivation of Eq. 4-346.

For the purposes of applying the generalized differential error Eqs. 4-123 to the computing servo represented in Fig. 4-71, the functional block diagram given in Fig. 4-71 can be converted to the equivalent form shown in Fig. D4-9.1.

The input quantity x_4, and hence the output quantities y_1, y_2, y_3, and y_4 also, are each assumed to contain an error component. The transfer functions associated with the servo elements (assumed to be ideal) are expressed inside the brackets in Fig. D4-9.1. Note that a separate block must be employed for each output quantity even though two blocks have a common input quantity.

Examination of Fig. D4-9.1 shows that

$$\left.\begin{aligned} \dot{y}_1 &= K y_4 \\ y_2 &= -k_g \dot{y}_1 \\ y_3 &= -k_p y_1 \\ y_4 &= y_2 + y_3 + x_4 \end{aligned}\right| \qquad \text{(D4-9.1)}$$

The equivalence of Fig. D4-9.1 to Fig. 4-71 is confirmed by the fact that these four simultaneous equations yield the two equations given on Fig. 4-71 when the identities noted on Fig. D4-9.1 are taken into account.

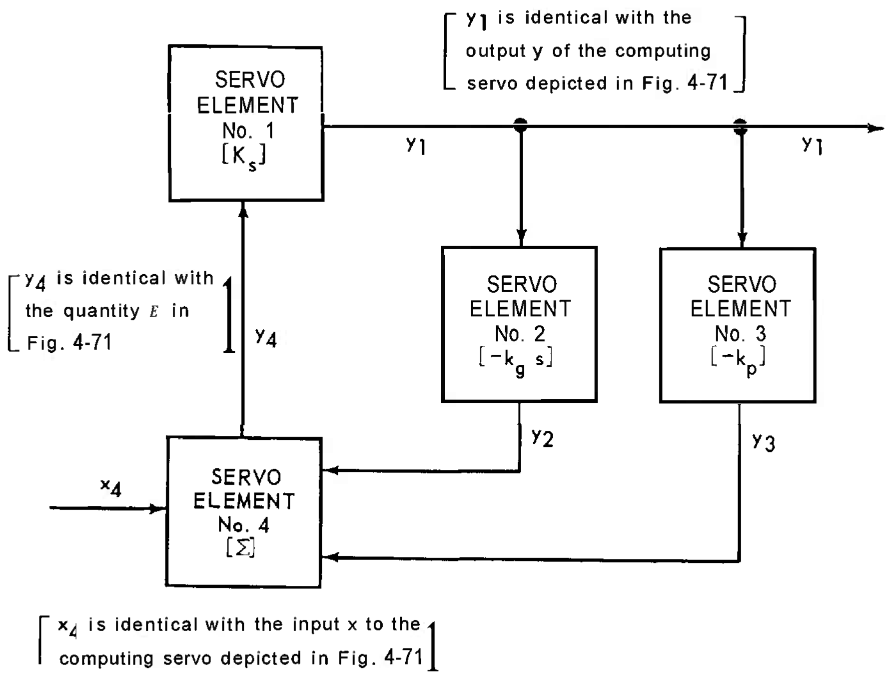


Figure D4-9.1. Equivalent form of Figure 4-71.

Derivation 4-9. (Continued)

Equations D4-9.1 can be converted as follows to the completely generalized functiona orm discussed in par 4-4.4.2:

$$
\left.
\begin{aligned}
f_1(y_1, y_4) &= f_1 = \dot{y}_1 - Ky_4 = 0 \\
f_2(y_1, y_2) &= f_2 = y_2 + k_g \dot{y}_1 = 0 \\
f_3(y_1, y_3) &= f_3 = y_3 + k_p y_1 = 0 \\
f_4(x_4, y_2, y_3, y_4) &= f_4 = y_4 - y_2 - y_3 - x_4 = 0
\end{aligned}
\right\} \qquad \text{(D4-9.2}
$$

Partial differentiation of Eqs. D4-9.2 with respect to the output quantities of the fou: servo elements (y_1, y_2, y_3, and y_4) shows that:

$$
\left.
\begin{array}{llll}
\dfrac{\partial f_1}{\partial y_1} = 0 & \dfrac{\partial f_1}{\partial y_2} = 0 & \dfrac{\partial f_1}{\partial y_3} = 0 & \dfrac{\partial f_1}{\partial y_4} = -K \\[2ex]
\dfrac{\partial f_2}{\partial y_1} = 0 & \dfrac{\partial f_2}{\partial y_2} = 1 & \dfrac{\partial f_2}{\partial y_3} = 0 & \dfrac{\partial f_2}{\partial y_4} = 0 \\[2ex]
\dfrac{\partial f_3}{\partial y_1} = k_p & \dfrac{\partial f_3}{\partial y_2} = 0 & \dfrac{\partial f_3}{\partial y_3} = 1 & \dfrac{\partial f_3}{\partial y_4} = 0 \\[2ex]
\dfrac{\partial f_4}{\partial y_1} = 0 & \dfrac{\partial f_4}{\partial y_2} = -1 & \dfrac{\partial f_4}{\partial y_3} = -1 & \dfrac{\partial f_4}{\partial y_4} = 1
\end{array}
\right\} \qquad \text{(D4-9.3}
$$

and

$$
\left.
\begin{array}{llll}
\dfrac{\partial f_1}{\partial \dot{y}_1} = 1 & \dfrac{\partial f_1}{\partial \dot{y}_2} = 0 & \dfrac{\partial f_1}{\partial \dot{y}_3} = 0 & \dfrac{\partial f_1}{a i 4} = 0 \\[2ex]
\dfrac{\partial f_2}{\partial \dot{y}_1} = k_g & \dfrac{\partial f_2}{\partial \dot{y}_2} = 0 & \dfrac{\partial f_2}{\partial \dot{y}_3} = 0 & \dfrac{\partial f_2}{\partial \dot{y}_4} = 0 \\[2ex]
\dfrac{\partial f_3}{\partial \dot{y}_1} = 0 & \dfrac{\partial f_3}{\partial \dot{y}_2} = 0 & \dfrac{\partial f_3}{\partial \dot{y}_3} = 0 & \dfrac{\partial f_3}{\partial \dot{y}_4} = 0 \\[2ex]
\dfrac{\partial f_4}{\partial \dot{y}_1} = 0 & \dfrac{\partial f_4}{\partial \dot{y}_2} = 0 & \dfrac{\partial f_4}{\partial \dot{y}_3} = 0 & \dfrac{\partial f_4}{\partial \dot{y}_4} = 0
\end{array}
\right\} \qquad \text{(D4-9.4}
$$

Derivation 4-9. (Continued)

Examination of Eqs. D4-9.2 shows that the partial derivatives for higher time derivatives of y_1, y_2, y_3, and y_4 are all zero.

Equations D4-9.2 can be similarly differentiated with respect to the single input quantity from outside the servo, namely x_4. The results are as follows:

$$\frac{\partial f_1}{\partial x_4} = \qquad \frac{\partial f_2}{\partial x_4} = 0 \qquad \frac{\partial f_3}{\partial x_4} = 0 \qquad \frac{\partial f_4}{\partial x_4}$$

$$\frac{\partial f_1}{\partial \dot{x}_4} = 0 \qquad \frac{\partial f_2}{\partial \dot{x}_4} = 0 \qquad \frac{\partial f_3}{\partial \dot{x}_4} = 0 \qquad \frac{\partial f_4}{\partial \dot{x}_4} = 0$$

Examination of Eqs. D4-9.2 shows that the partial derivatives for higher time derivatives of x_4 are all zero.

Equations D4-9.3 through D4-9.6 can be used to evaluate the applicable terms of the generalized differential error Eqs. 4-123, with the following results:

$$\left.\begin{aligned}
&\sum_{k=1}^{q} \frac{\partial f_1}{\partial y_k}\epsilon_{y_k} = \sum_{k=1}^{4} \frac{\partial f_1}{\partial y_k}\epsilon_{y_k} = -K\epsilon_{y_4}\\
&\sum_{k=1}^{q} \frac{\partial f_2}{\partial y_k}\epsilon_{y_k} = \sum_{k=1}^{4} \frac{\partial f_2}{\partial y_k}\epsilon_{y_k} = \epsilon_{y_2}\\
&\sum_{k=1}^{q} \frac{\partial f_3}{\partial y_k}\epsilon_{y_k} = \sum_{k=1}^{4} \frac{\partial f_3}{\partial y_k}\epsilon_{y_k} = k_p\epsilon_{y_1} + \epsilon_{y_3}\\
&\sum_{k=1}^{q} \frac{\partial f_4}{\partial y_k}\epsilon_{y_k} = \sum_{k=1}^{4} \frac{\partial f_4}{\partial y_k}\epsilon_{y_k} = -\epsilon_{y_2} - \epsilon_{y_3} + \epsilon_{y_4}\\
&\sum_{k=1}^{q} \frac{\partial f_1}{\partial \dot{y}_k}\dot{\epsilon}_{y_k} = \sum_{k=1}^{4} \frac{\partial f_1}{\partial \dot{y}_k}\dot{\epsilon}_{y_k} = \dot{\epsilon}_{y_1}\\
&\sum_{k=1}^{q} \frac{\partial f_2}{\partial \dot{y}_k}\dot{\epsilon}_{y_k} = \sum_{k=1}^{4} \frac{\partial f_2}{\partial \dot{y}_k}\dot{\epsilon}_{y_k} = -k_g\dot{\epsilon}_{y_1}\\
&\sum_{k=1}^{q} \frac{\partial f_3}{\partial \dot{y}_k}\dot{\epsilon}_{y_k} = \sum_{k=1}^{4} \frac{\partial f_3}{\partial \dot{y}_k}\dot{\epsilon}_{y_k} = 0\\
&\sum_{k=1}^{q} \frac{\partial f_4}{\partial \dot{y}_k}\dot{\epsilon}_{y_k} = \sum_{k=1}^{4} \frac{\partial f_4}{\partial \dot{y}_k}\dot{\epsilon}_{y_k} = 0
\end{aligned}\right\} \qquad (D4\text{-}9.7)$$

Derivation 4-9. (Continued)

and

$$
\left.
\begin{aligned}
&\sum_{n=1}^{r} \frac{\partial f_1}{\partial x_n}\,\epsilon_{x_n} = \frac{\partial f_1}{\partial x_4}\,\epsilon_{x_4} = 0\\
&\sum_{n=1}^{r} \frac{\partial f_2}{\partial x_n}\,\epsilon_{x_n} = \frac{\partial f_2}{\partial x_4}\,\epsilon_{x_4} = 0\\
&\sum_{n=1}^{r} \frac{\partial f_3}{\partial x_n}\,\epsilon_{x_n} = \frac{\partial f_3}{\partial x_4}\,\epsilon_{x_4} = 0\\
&\sum_{n=1}^{r} \frac{\partial f_4}{\partial x_n}\,\epsilon_{x_n} = \frac{\partial f_4}{\partial x_4}\,\epsilon_{x_4} = -\epsilon_{x_4}\\
&\sum_{n=1}^{r} \frac{\partial f_1}{\partial \dot{x}_n}\,\dot{\epsilon}_{x_n} = \frac{\partial f_1}{\partial \dot{x}_4}\,\dot{\epsilon}_{x_4} = 0\\
&\sum_{n=1}^{r} \frac{\partial f_2}{\partial \dot{x}_n}\,\dot{\epsilon}_{x_n} = \frac{\partial f_2}{\partial \dot{x}_4}\,\dot{\epsilon}_{x_4} = 0\\
&\sum_{n=1}^{r} \frac{\partial f_3}{\partial \dot{x}_n}\,\dot{\epsilon}_{x_n} = \frac{\partial f_3}{\partial \dot{x}_4}\,\dot{\epsilon}_{x_4} = 0\\
&\sum_{n=1}^{r} \frac{\partial f_4}{\partial \dot{x}_n}\,\dot{\epsilon}_{x_n} = \frac{\partial f_4}{\partial \dot{x}_4}\,\dot{\epsilon}_{x_4} = 0
\end{aligned}
\right|
\qquad \text{(D4-9.8)}
$$

The terms of Eqs. 4-123 involving higher derivatives of the input and output quantities are all zero. Also, the m terms of Eqs. 4-123 are all zero since the servo elements have been assumed to be ideal.

Substitution of the values given by Eqs. D4-9.7 and D4-9.8 into Eqs. 4-123 yields the following relationships:

$$
\left.
\begin{aligned}
&-K\,\epsilon_{y_4} + \dot{\epsilon}_{y_1} = 0\\
&\epsilon_{y_2} + k_g\,\dot{\epsilon}_{y_1} = 0\\
&k_p\,\epsilon_{y_1} + \epsilon_{y_3} = 0\\
&-\epsilon_{y_2} - \epsilon_{y_3} + \epsilon_{y_4} = \epsilon_{x_4}
\end{aligned}
\right|
\qquad \text{(D4-9.9)}
$$

Derivation 4-9. (Continued)

The first three relationships of Eqs. D4-9.9 can be substituted into the fourth relationship to

$$k_g \dot{\epsilon}_{y_1} + k_p \epsilon_{y_1} + \frac{\dot{\epsilon}_{y_1}}{K} = \epsilon_{x_4} \qquad \text{(D4-9.10)}$$

With a rearrangement of terms and taking into account the fact that y_1 and x_4 are respectively identical to y and x of Fig. 4-71, Eq. D4-9.10 becomes

$$k_p \epsilon_y + \frac{1 + k_g K}{K} \dot{\epsilon}_y = \epsilon_x \qquad \text{(D4-9.11)}$$

Derivation 4-10. Evaluation of the time averages of $V_i(t)$ and $L_i(t)$, the vertical and horizontal components of the firing error.

In accordance with the discussion of par 4-4.2.3, the time average of $V_i(t)$ can be defined by the relationship

$$\widetilde{V_i(t)} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} V_i(t)\, dt \qquad \text{(D4-10.1)}$$

where

t_1 = the time at which firing starts

t_2 = the time at which firing ceases

Since the Vigilante Antiaircraft Weapon System employs a one-second firing interval, $t_2 - t_1$ is equal to one second and Eq. D4-10.1 reduces to

$$\widetilde{V_i(t)} = \int_{t_1}^{t_2} V_i(t)\, dt \qquad \text{(D4-10.2)}$$

Inasmuch as $V_i(t)$ varies linearly with time, it can be represented as shown in Fig. D4-10.1. From this figure, it is evident that the slope α of $V_i(t)$ is given by

$$\alpha = \frac{V_i(t_2) - V_i(t_o)}{4/3} \qquad \text{(D4-10.3)}$$

Therefore, the value of $V_i(t)$ at any time t is

$$V_i(t) = V_i(t_o) + \alpha(t - t_o) \qquad \text{(D4-10.4)}$$

Substitution from Eq. D4-10.4 into Eq. D4-10.2 shows that

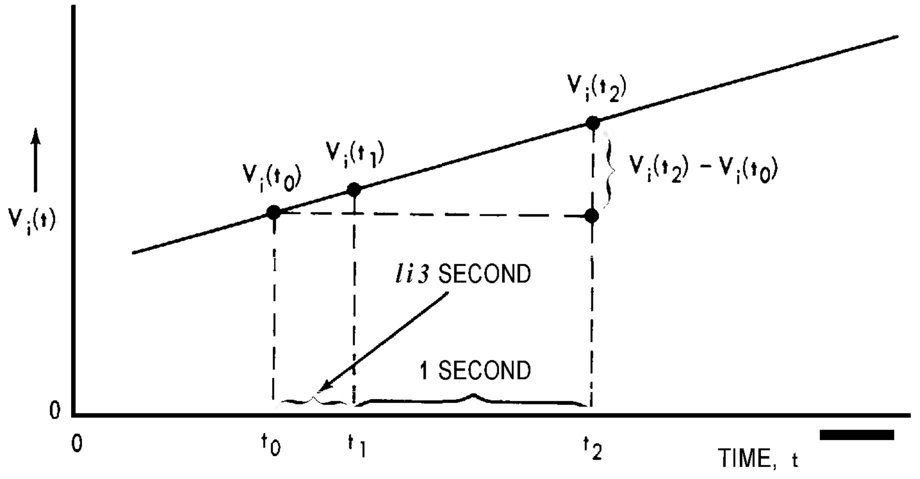
$$\widetilde{V_i(t)} = \int_{t_1}^{t_2} \left[V_i(t_o) + \alpha(t - t_o) \right] dt$$

$$= \int_{t_1}^{t_2} \left\{ \left[V_i(t_o) - \alpha t_o \right] + \alpha t \right\} dt$$

$$= \left[V_i(t_o) - \alpha t_o \right] \left[t \right]_{t_1}^{t_2} + \alpha \left[\frac{1}{2} t^2 \right]_{t_1}^{t_2}$$

$$= \left[V_i(t_o) - \alpha t_o \right] \left[1 \right] + \frac{\alpha}{2} \left[t_2^2 - t_1^2 \right] \qquad \text{(D4-10.5)}$$

Derivation 4-10. (Continued)



t_0 = START OF REGENERATIVE TRACKING

t_1 = START OF FIRING

t_2 = END OF FIRING AND REGENERATIVE TRACKING

Figure D4-10.1. Representation of $V_i(t)$, the vertical component of the firing error.

From Fig. D4-10.1, it is evident that

$$t_1 = t_o + \frac{1}{3} \text{ second} \tag{D4-10.6}$$

$$t_2 = t_o + \frac{4}{3} \text{ second} \tag{D4-10.7}$$

Application of these data to Eq. D4-10.5 shows that

$$\widetilde{V_i(t)} = V_i(t_o) - \alpha t_o + \frac{\alpha}{2}\left[t_o^2 + \frac{8}{3}t_o + \frac{16}{9} - t_o^2 - \frac{2}{3}t_o - \frac{1}{9}\right]$$

$$= V_i(t_o) - \alpha t_o + \frac{\alpha}{2}\left[2t_o + \frac{5}{3}\right]$$

$$= V_i(t_o) + \frac{5}{6}\alpha \tag{D4-10.8}$$

Substitution herein of the expression for x given by Eq. D4-10.3 yields

Derivation 4-10. (Continued)

$$\widetilde{V_i(t)} = V_i(t_o) + \frac{5}{6}\left\{\frac{3}{4}\left[V_i(t_2) - V_i(t_o)\right]\right\}$$

$$= V_i(t_o) + \frac{5}{8} V_i(t_2) - \frac{5}{8} V_i(t_o)$$

$$= \frac{5}{8} V_i(t_2) + \frac{3}{8} V_i(t_o) \qquad \text{(D4-10.9)}$$

The evaluation of $\widetilde{L_i(t)}$, the time average of the horizontal component of the firing error, is accomplished in an identical manner. The result is

$$\widetilde{L_i(t)} = \frac{5}{8} L_i(t_2) + \frac{3}{8} L_i(t_o) \qquad \text{(D4-10.10)}$$

Derivation 4-11. Derivation of the variance of a sinusoidal error.

Assume a sinusoidal error of the form

$$\epsilon = |\epsilon| \sin wt \tag{D4-11.1}$$

Then,

$$\lim_{T\to\infty} \int_{-T}^{T} \sin \omega t \, dt = 0 \tag{D4-11.2}$$

and

$$\overline{\epsilon^2} = \lim_{T\to\infty} \frac{|\epsilon|^2}{2T} \int_{-T}^{T} \sin^2 \omega t \, dt$$

$$= \lim_{T\to\infty} \frac{|\epsilon|^2}{2\omega T} \left[\omega T - \frac{1}{2} \sin 2\omega T \right]$$

$$= \lim_{T\to\infty} \left\{ \frac{|\epsilon|^2}{2} - \quad 2 \frac{\sin 2\omega T}{4\omega T} \right\}$$

$$= \frac{|\epsilon|^2}{2} \tag{D4-11.3}$$

Therefore,

$$\sigma^2 = \overline{\epsilon^2} - \bar{\epsilon}^2 = \frac{|\epsilon|^2}{2} \tag{D4-11.4}$$

Example 4-1. Fire-control example illustrating the concept of conditional probability.*

Problem A

A gun fires projectiles at a target until a first hit is scored. It is desired to determine the probability of the first hit occurring on any particular round.

Solution to Problem A

It is assumed that the event of a hit or a miss on any one round is independent of the event of a hit or a miss on any other round. Let the event of a hit on any one round be designated by the symbol H and the event of a miss on any one round be designated by the symbol M. Since any one round must result in either a hit or a miss, the probability of either a hit or a miss is unity, i.e.,

$$\Pr[H + M] = 1 \tag{E4-1.1}$$

where

$\Pr[H + M]$ = probability of either a hit or a miss on any one round.

Inasmuch as the two events concerned are mutually exclusive, Eq. 4-3 applies. From this equation, it is apparent that

$$\Pr[H + M] = \Pr[H] + \Pr[M] \tag{E4-1.2}$$

where

$\Pr[H]$ = probability of a hit on any one round (this probability is identical with the single-shot hit probability, $(\Pr)_{ssh}$, that is discussed in par 4-4.3.3)

$\Pr[M]$ = probability of a miss on any one round.

Therefore, from Eqs. E4-1.1 and E4-1.2, it is evident that

$$\Pr[H] + \Pr[M] = 1 \tag{E4-1.3}$$

Let the event of a hit on any particular round k be denoted by H_k and the event of a miss on that round be denoted by M_k. Because of the statistical independence of the hit or miss on any one round from the hit or miss on any other round, it is evident that

$$\Pr[H_k] = \Pr[H] \tag{E4-1.4}$$

and

$$\Pr[M_k] = \Pr[M]. \tag{E4-1.5}$$

Now, let the event "the first hit occurs in the kth round" — i.e., the first, second, . . . and (k-1)st rounds all miss, and the kth round hits — be denoted by ϵ_k. The probability of the event ϵ_k is therefore identical with the multiple joint probability of the events contained between the dashes, i.e.,

$$\Pr[\epsilon_k] = \Pr[M_1, M_2, \cdots, M_{k-1}, H_k] \tag{E4-1.6}$$

*Adapted from pages 27 and 28 of Reference 16.

Example 4-1. (Continued)

where

M_1, M_2 and M_{k-1} = the probability of a miss on the first round, on the second round, and on the (k-1)st round, respectively.

Since the events concerned are all statistically independent, the relationship for multiple joint probabilities that corresponds to Eq. 4-4 applies and shows that

$$\Pr[\epsilon_k] = \Pr[M_1] \cdot \Pr[M_2] \cdot \cdots \cdot \Pr[M_{k-1}] \cdot \Pr[H_k]. \tag{E4-1.7}$$

By virtue of Eqs. E4-1. 4 and E4-1. 5, Eq. E4-1. 7 can be rewritten as

$$\Pr[\epsilon_k] = \left\{ \Pr[M] \right\}^{k-1} \Pr[H]. \tag{E4-1.8}$$

Substitution from Eq. E4-1.3 into Eq. E4-1.8 shows that

$$\Pr[\epsilon_k] = \left\{ 1 - \Pr[H] \right\}^{k-1} \Pr[H]. \tag{E4-1.9}$$

Obviously, the probability that the first hit will occur in the kth round is less than the probability that the first hit will occur in the first round, which is Pr[H]. Equation E4-1.9 shows that the reduction factor is $\left\{ 1 - \Pr[H] \right\}^{k-1}$.

Problem B

Another problem of interest in connection with this fire-control example is the determination of the probability that more than two rounds will be required to score a hit, given the fact that the first round is a miss.

Solution to Problem B

First, let the event that more than two rounds are required to score a hit be denoted by the symbol A. Then

$$\Pr[A] = \Pr[\epsilon_3 + \epsilon_4 + \cdots] \tag{E4-1.10}$$

where

ϵ_3 = the event that the first hit occurs on the third round

ϵ_4 = the event that the first hit occurs on the fourth round.

In words, this equation states that the probability that more than two rounds are required is the probability that a hit occurs in either round No. 3 or some succeeding round. Since the events $\epsilon_3, \epsilon_4, \ldots$ are mutually exclusive, the relationship for multiple probabilities that corresponds to Eq. 4-3 applies and shows that

$$\Pr[A] = \Pr[\epsilon_3] + \Pr[\epsilon_4] + \cdots \tag{E4-1.11}$$

$$= \sum_{k=3}^{k=\infty} \Pr[\epsilon_k]. \tag{E4-1.12}$$

Example 4-1. (Continued)

Substitution from Eq. E4-1.8 into Eq. E4-1.12 shows that

$$\Pr[A] = \sum_{k=3}^{k=\infty} \left\{\Pr[M]\right\}^{k-1} \Pr[H] \tag{E4-1.13}$$

$$= \Pr[H]\left\{\Pr[M]\right\}^2 + \Pr[H]\left\{\Pr[M]\right\}^3 + \Pr[H]\left\{\Pr[M]\right\}^4 + \cdots \tag{E4-1.14}$$

$$= \Pr[H]\left\{\Pr[M]\right\}^2 \left(1 + \Pr[M] + \left\{\Pr[M]\right\}^2 + \cdots\right). \tag{E4-1.15}$$

The series inside the parentheses can be recognized as a geometrical progression. The previous discussion shows that Pr[M] must lie between 0 and 1, where 0 represents a very improbable event and 1 represents an almost certain event, i.e., $0 < \Pr[M] < 1$. Then, $0 < \left\{\Pr[M]\right\}^2 < 1$ also. In this case, the limit of the sum of an infinite number of terms of the series is $\frac{1}{1 - \Pr[M]}$.* Substitution of $1/(1 - \Pr[M])$ for the geometrical progression in Eq. E4-1.15 yields

$$\Pr[A] = \frac{\Pr[H]\left\{\Pr[M]\right\}^2}{1 - \Pr[M]}. \tag{E4-1.16}$$

With Pr[H] substituted for 1 - Pr[M] (see Eq. E4-1.3), Eq. E4-1.16 becomes

$$\Pr[A] = \left\{\Pr[M]\right\}^2. \tag{E4-1.17}$$

The result desired is the probability that more than two rounds are required, given the fact that the first round is a miss. This probability is a conditional probability that can be represented by the symbol $\Pr[A|M_1]$. From Eq. 4-6,

$$\Pr[A|M_1] = \frac{\Pr[A, M_1]}{\Pr[M_1]}. \tag{E4-1.18}$$

The numerator in the right-hand side of Eq. E4-1.18 is the joint probability of events A and M_1, i.e., the joint probability that (1) more than two rounds are required to score a hit, and (2) the first round is a miss. It is obvious that the joint probability $\Pr[A, M_1]$ is the same as the probability Pr[A] alone, i.e.,

$$\Pr[A, M_1] = \Pr[A]. \tag{E4-1.19}$$

* See item 26 of Reference 23.

Example 4-1. (Continued)

Accordingly, substituting into **Eq.** E4-1.18 from Eqs. E4-1.5 and E4-1.19 gives

$$Pr[A|M_1] = \frac{Pr[A]}{Pr[M]}. \tag{E4-1.20}$$

Substituting further, from Eq. E4-1.17 into Eq. E4-1.20 shows that

$$Pr[A|M_1] = \frac{\{Pr[M]\}^2}{Pr[M]} = Pr[M]. \tag{E4-1.21}$$

This result is, of course, exactly what one should logically expect. That is, if it is known that the first round is a miss, then more than two rounds will be required if and only if the second round is a miss also. The probability of the second round being a miss is, by virtue of Eq. E4-1.5, Pr[M].

So detailed a treatment of a simple problem would not be used in practice since the results are intuitively obvious. The intent of these examples is to demonstrate the application of the basic rules of probability in simple situations, where the solution is known in advance.

Example 4-2. Example of the experimental determination of the probability density function and the probability distribution function of a continuous random variable.*:

If experimental data of a random phenomenon are recorded on magnetic tape or by some similar means, it is possible to electronically compute approximations to the probability density function and the probability distribution function associated with these data. Examples of practical uses of such a technique are the recording and analysis of wind-gust loading on an aircraft wing or a radar antenna, or the recording and analysis of the shock loading on various parts of a weapon system as a result of gunfire.

In this technique (see Fig. E4-2.1), the recorded signals are amplified and passed

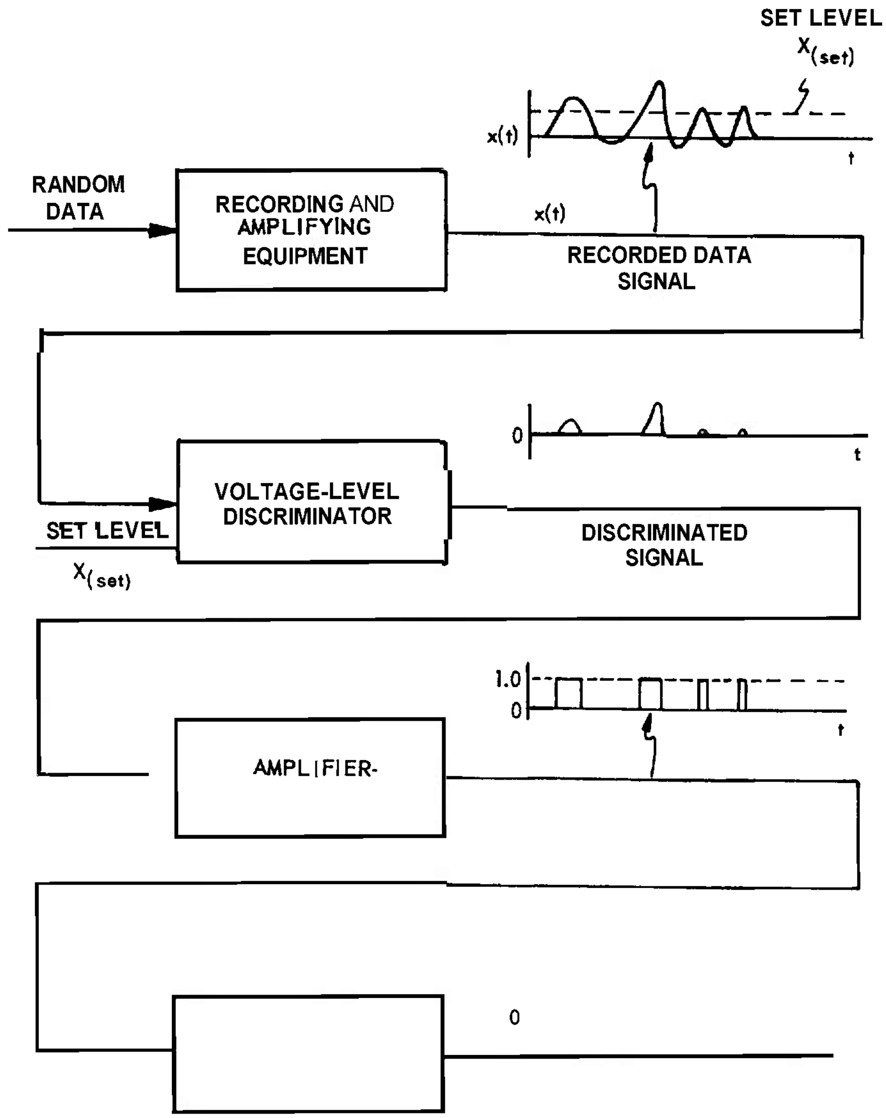


Figure E4-2.1. Functional representation of the means employed for computing probability distribution functions associated with experimental data.

*Adapted from pages 418 and 419 of Reference 25.

Example 4-2. (Continued)

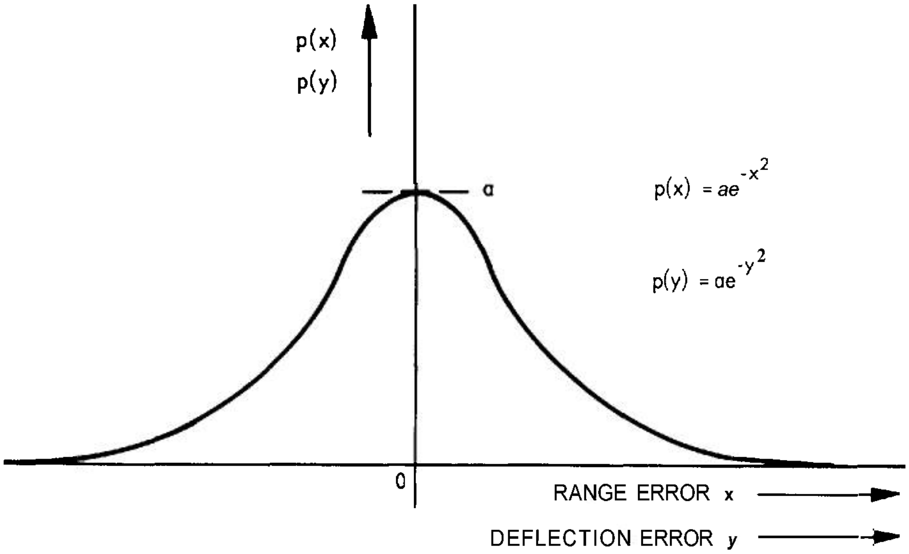
through a voltage-level discriminator which passes only those voltage amplitudes that exceed the set level, $X_{(set)}$, to which the discriminator is adjusted. The signal that passes the discriminator is then amplified and limited; in effect, this gives a signal that has an instantaneous value of either 0 or 1. The time average of this signal is then computed in an electronic integrator, yielding the percentage of the time that the voltage exceeds the set level.

The arrangement shown in Fig. E4-2.1 is thus a computer whose output is $1-P(X)$, where X is the set level and $P(X)$ is the associated probability distribution function. It is evident from Eq. 4-13 that the probability density function $p(x)$ can be obtained from the computer output by differentiating $P(X)$ with respect to the random variable $x(t)$.

Example 4-3. Example of the application of joint probability density functions to a fire-control problem.*

Given:

The errors in range of a certain artillery weapon are normally distributed. The deflection errors are similarly distributed and are independent of the range errors. The governing probability density functions are specified and plotted in Fig. E4-3.1.



$$p(x) = ae^{-x^2} \tag{E4-3.1}$$

$$p(y) = ae^{-y^2} \tag{E4-3.2}$$

Figure E4-3.1. Distribution of range and deflection errors.

To be found:

What is the probable distribution of the miss distance, i.e., the distance between the point of impact and the center of the target?

Solution:

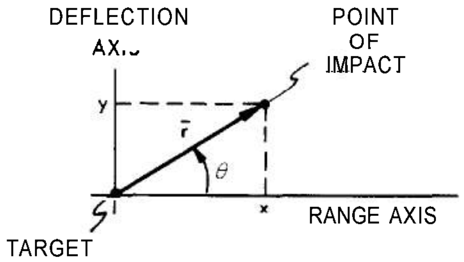
Because of the independence of the range and deflection errors, the joint distribution of errors is given (from Eq. 4-25) by the joint probability density function

$$p(x,y) = p(x)\,p(y) = a^2 e^{-(x^2+y^2)} \tag{E4-3.3}$$

The miss distance r is obtained from the transformation of x and y rectangular coordinates to r and θ polar coordinates. The following geometrical relationships apply (see Fig. E4-3.2).

* Adapted from Ex. 8-7 on page 117 of Reference 7.

Example 4-3. (Continued)



$$r^2 = x^2 + y^2 \quad \text{(E4-3.4)}$$

$$dxdy = rdrd\theta \quad \text{(E4-3.5)}$$

$$x = r\cos\theta \quad \text{(E4-3.6)}$$

$$y = r\sin\theta \quad \text{(E4-3.7)}$$

$\bar{r}$ = MISS-DISTANCE VECTOR
θ = ANGLE BETWEEN THE RANGE AXIS AND THE MISS-DISTANCE VECTOR
x, y = COMPONENTS OF $\bar{r}$ ALONG THE RANGE AND DEFLECTION AXES, RESPECTIVELY

Figure E4-3.2. Geometry associated with the miss distance.

In a manner analogous to Eq. 4-18, it is evident that the joint probability of a given range error x and a given deflection error y is established by the relationship

$$\Pr[x,y] = p(x,y)\,dxdy \quad \text{(E4-3.8)}$$

where Pr[x,y] is the probability that the point of impact falls in the elemental area dxdy. It is similarly possible to define an equivalent probability and probability density function for the elemental area $rdrd\theta$, i.e.,

$$\Pr[r,\theta] = p(r,\theta)\,rdrd\theta \quad \text{(E4-3.9)}$$

where $\Pr[r,\theta]$ is the probability that impact occurs in the elemental area $rdrd\theta$ and $p(r,\theta)$ is the corresponding probability density function. Since the elemental areas are equivalent (see Eq. E4-3.5), the probabilities — and thus the probability density functions — can be equated, i.e.,

$$\Pr[x,y] = \Pr[r,\theta] = p(x,y)\,dxdy = p(r,\theta)\,rdrd\theta \quad \text{(E4-3.10)}$$

and

$$p(x,y) = p(r,\theta). \quad \text{(E4-3.11)}$$

Substitution from Eqs. E4-3.3 and E4-3.5 into Eq. E4-3.10 yields

$$p(r,\theta)\,rdrd\theta = a^2 e^{-(x^2+y^2)}\,rdrd\theta \quad \text{(E4-3.12)}$$

$$= a^2 re^{-r^2}\,drd\theta \quad \text{(E4-3.13)}$$

Example 4-3. (Continucd)

so that, accordingly,

$$p(r,\theta) = a^2 e^{-r^2} . \tag{E4-3.14}$$

The miss distance r defines a circle of radius r at some point of which the impact occurs. Therefore, the probability of a particular miss distance is the probability that the point of impact falls in thc annular element of area $2\pi r dr$. This probability is designated Pr[r],and a corresponding radial probability density p(r) is defined by the relationship

$$Pr[r] = p(r)\,dr. \tag{E4-3.15}$$

Since the probability pcr unit area is $p(r, \theta)$, then

$$Pr[r] = 2\pi r p(r,\theta)\,dr \tag{E4-3.16}$$

and

$$p(r) = 2\pi r p(r,\theta) . \tag{E4-3.17}$$

Substitution from Eq. E4-3.14 into Eq. E4-3.17 gives

$$p(r) = 2\pi a^2 r e^{-r^2} . \qquad \text{(See Fig. E4-3. 3)} \tag{E4-3.18}$$

It is also possible to find the probability that the miss distance is less than a given value r. 'This is the probability distribution function of the miss distance and also the probability

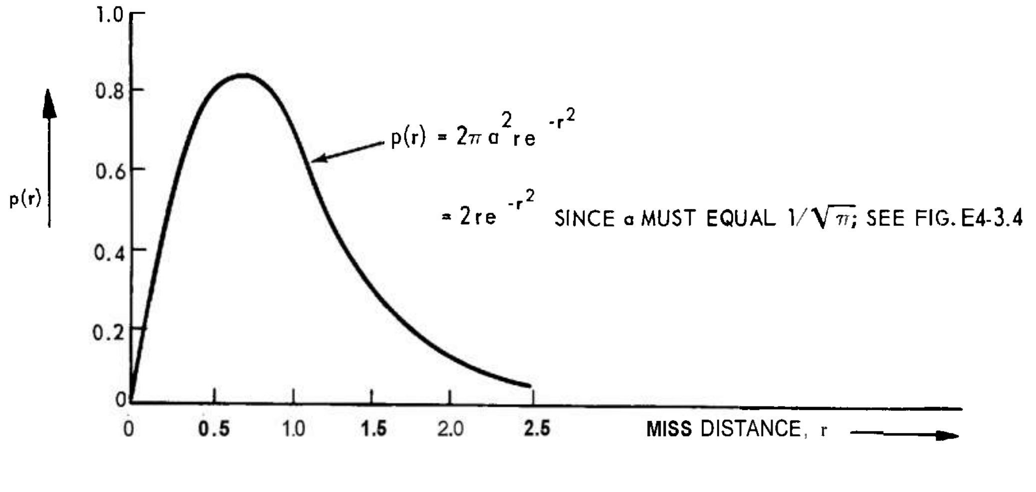


Figure E4-3.3. Probability density function of the miss distance.

Example 4-3. (Continued)

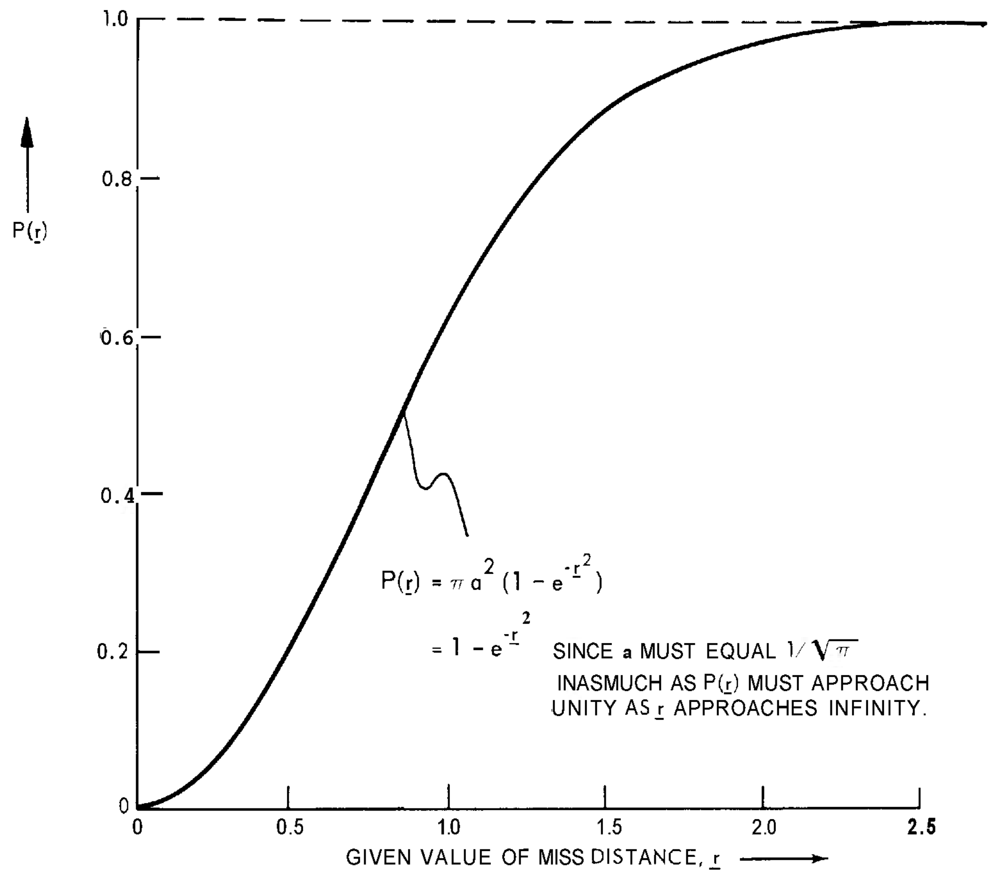


Figure E4-3.4. Probability distribution function of the miss distance.

that the impact point falls within the circular area of radius r. Use of Eq. 4-13 shows that

$$P(r) = \int_0^r p(r)\, dr = 2\pi a^2 \int_0^r re^{-r^2}\, dr. \tag{E4-3.19}$$

Equation E4-3.19 is readily integrated by making the substitution $\rho = r^2$; then

$$P(r) = \pi a^2 \int_0^{r^2} e^{-\rho}\, d\rho = \pi a^2 (1 - e^{-r^2}). \quad \text{(See Fig. E4-3.4)} \tag{E4-3.20}$$

The probability density function of the miss distance that is given by Eq. E4-3.18 and the corresponding probability distribution function given by Eq. E4-3.20 are plotted in Figs. E4-3.3 and E4-3.4, respectively. Note that, since P(r) must approach unity for large values

Example 4-3. (Continued)

of $\underline{r}$, the constant a must be equal to $1/\sqrt{\pi}$. As is shown later (see par 4-4.2.4), the probability density functions of Eqs. E4-3.1 and E4-3.2 are Gaussian, with a variance of 0.5. Figure E4-3.4 shows that, in spite of the fact that the range and deflection errors have a zero mean, the miss-distance probability — i.e., the probability that the impact distance falls within the circular area of radius $\underline{r}$ — increases very slowly for small values of the radius.

Example 4-4. Example showing the employment of probability density and distribution functions in a conditional probability problem.*

Given

A precision potentiometer used in a fire control computer has survived for t days. Its probability of failure within the next time period of operation Δt is 0.01 Δt and is independent of t.

Problem

If a certain potentiometer in the application noted has lasted 30 days, what is the probability of its failure within the next 10 days?

Solution

Let X be the random variable representing the life of the potentiometer in days. Let A be the event that the potentiometer has lasted t days, and let B be the event that it fails between t and t + dt, where dt is a time increment. The given conditions state that, for any time interval, the probability of failure in that interval (given that it has survived to the start of the interval) is 0.01 times the length of the interval. In symbols

$$Pr[B|A] = Pr\left[(t < X < t + dt) \mid (t < X)\right] = 0.01\, dt. \qquad (E4\text{-}4.1)$$

If the potentiometer fails between t and t + dt (event B), it must necessarily have lasted for t days (event A). Accordingly

$$Pr[A,B] = Pr[B]. \qquad (E4\text{-}4.2)$$

Therefore, from Eq. 4-6

$$Pr[B|A] = \frac{Pr[A,B]}{Pr[A]} = \frac{Pr[B]}{Pr[A]}. \qquad (E4\text{-}4.3)$$

$Pr[A]$ is the probability that $X > t$. Alternatively, $Pr[A]$ is unity minus the probability that $X \leqq t$. But the probability that $X \leqq t$ is the probability distribution function of t. Accordingly

$$Pr[A] = 1 - P(t). \qquad (E4\text{-}4.4)$$

Similarly, $Pr[B]$ is the probability that $t < X \leqq t + dt$ but is also related to the probability density function p(t) by Eq. 4-12 as follows

$$Pr[B] = p(t)\, dt. \qquad (E4\text{-}4.5)$$

Combination of the relationships of Eqs. E4-4.1, E4-4.3, E4-4.4 and E4-4.5 shows that

$$Pr[B|A] = 0.01\, dt = \frac{Pr[B]}{Pr[A]} = \frac{p(t)\, dt}{1 - P(t)}. \qquad (E4\text{-}4.6)$$

* Adapted from page 39 of Reference 16.

Example 4-4. (Continued)

From Eq. E4-4.6, it is evident that

$$p(t) = 0.01\,[1 - P(t)]\,. \tag{E4-4.7}$$

Differentiating Eq. E4-4.7 with respect to time yields the relationship

$$\frac{d}{dt}\,p(t) + 0.01\,\frac{d}{dt}\,P(t) = 0. \tag{E4-4.8}$$

As can be seen from Eq. 4-13, however, an alternative form of the probability density function is

$$p(t) = \frac{d}{dt}\,P(t). \tag{E4-4.9}$$

Therefore, Eq. E4-4.8 can be rewritten as

$$\frac{d}{dt}\,p(t) + 0.01\,p(t) = 0. \tag{E4-4.10}$$

This equation has the form of a linear differential equation with constant coefficients, and has the solution:*

$$p(t) = ce^{-0.01t} \tag{E4-4.11}$$

The constant c can be determined by the application of Eq. 4-15. Inasmuch as negative time is meaningless in the particular problem under consideration, Eq. 4-15 can be rewritten as

$$\int_0^{\infty} p(t)\,dt = 1. \tag{E4-4.12}$$

From Eq. E4-4.11, however,

$$\int_0^{\infty} p(t)\,dt = c\int_0^{\infty} e^{-0.01t}\,dt \tag{E4-4.13}$$

$$= c\left[-100\,e^{-0.01t}\right]_0^{\infty} = 100\,c. \tag{E4-4.14}$$

*See Art. 891.1 of Ref. 23.

Example 4-4. (Continued)

Therefore, from Eqs. E4-4.12 and E4-4.14

$$c = 0.01. \qquad \text{(E4-4.15)}$$

In terms of the problem statement, let G now be defined as the event that the potentiometer has lasted 30 days, and let H be the event that it fails between 30 and 40 days. Then, by reasoning similar to that by which Eq. E4-4.3 was obtained,

$$\Pr[H|G] = \frac{\Pr[G,H]}{\Pr[G]} = \frac{\Pr[H]}{\Pr[G]} \qquad \text{(E4-4.16)}$$

where $\Pr[H|G]$ is the probability that the potentiometer will fail between 30 and 40 days, given that it has lasted 30 days. From Eq. E4-4.3

$$\Pr[G] = 1 - P(30). \qquad \text{(E4-4.17)}$$

This equation can be evaluated by use of Eq. 4-13; i.e.,

$$\Pr[G] = 1 - P(30) = 1 - \int_0^{30} p(t)\,dt. \qquad \text{(E4-4.18)}$$

Substitution from Eq. E4-4.11 into Eq. E4-4.17 and using Eq. E4-4.15 in the result shows that

$$\Pr[G] = 1 - c\int_0^{30} e^{-0.01t}\,dt = 1 - 0.01\int_0^{30} e^{-0.01t}\,dt. \qquad \text{(E4-4.19)}$$

Performing the indicated integration shows that

$$\Pr[G] = 1 - 0.01\left[-100\,e^{-0.01t}\right]_0^{30} = 1 - 1 + e^{-0.3} = e^{-0.3}. \qquad \text{(E4-4.20)}$$

Since $\Pr[H]$ is the probability of failure in the period between 30 and 40 days, it is given by the difference of the two probability distribution functions as follows:

$$\Pr[H] = P(40) - P(30). \qquad \text{(E4-4.21)}$$

Substitution of Eq. E4-4.20 into Eq. E4-4.17 yields

$$P(30) = 1 - e^{-0.3}. \qquad \text{(E4-4.22)}$$

A similar derivation yields

$$P(40) = 1 - e^{-0.4}. \qquad \text{(E4-4.23)}$$

Example 4-4. (Continued)

Substitution of Eqs. E4-4.20, E4-4.21, E4-4.22, and E4-4.23 into Eq. E4-4.16 yields

$$\Pr[H|G] = \frac{(1 - e^{-0.4}) - (1 - e^{-0.3})}{e^{-0.3}} = 1 - e^{-0.1} = 0.095. \tag{E4-4.24}$$

Equation E4-4.24 states that the probability that the potentiometer under consideration will fail within 40 days, having lasted for 30 days, is 0.095.

Example 4-5. Illustrative application of the relationship between $(Pr)_{eh}$ and $(Pr)_{ssh}$.

Given:

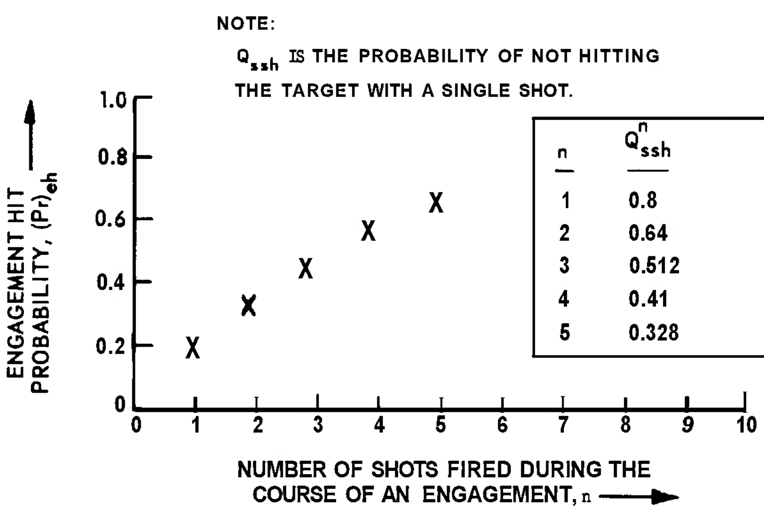
The single-shot hit probability $(Pr)_{ssh}$ of a given weapon system is 0.2.

Problem:

How many shots must be fired during the course of a given engagement in order to ensure an engagement hit probability $(Pr)_{eh}$ of 0.6?

Solution:

Since $(Pr)_{ssh} = 0.2$, Eq. 4-70 shows that $Q_{ssh} = 1 - (Pr)_{ssh} = 0.8$. From Eq. 4-73, $(Pr)_{eh} = 1 - Q^n_{ssh} = 1 - 0.8^n$. This function is plotted in Fig. E4-5.1 for the first five shots of an engagement.



n	Q^n_{ssh}	$(Pr)_{eh}$
1	0.8	0.2
2	0.64	0.36
3	0.512	0.488
4	0.41	0.59
5	0.328	0.672

Figure E4-5.1. Plot of $(Pr)_{eh}$ versus n.

As shown by Fig. E4-5.1 and the associated data, a minimum of 4 shots must be fired to ensure an engagement hit probability of about 0.6.

Example 4-6. The error associated with a simple amplifier.

Given

A simple amplifier has a nominal gain factor α. The actual gain is $\alpha + \epsilon_\alpha$, where ϵ_α is the gain error. There is also a fixed offset error Δ in the amplifier output. The input quantity $x + \delta$ includes a fixed error δ. The amplifier can, therefore, be represented as shown in Fig. E4-6.1. In this figure, $y + \epsilon_Y$ is the output of the amplifier and ϵ_Y is the total error in this output.

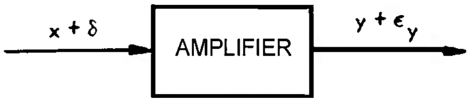


Figure E4-6.1. Functional diagram of a simple amplifier.

Problem

Derive an expression for ϵ_Y the total error in the output of the amplifier.

Solution A

The performance equation for the ideal amplifier is

$$y = \alpha x \qquad \text{(E4-6.1)}$$

The corresponding performance equation for the nonideal case is

$$y + \epsilon_y = (\alpha + \epsilon_\alpha)(x + \delta) + \Delta . \qquad \text{(E4-6.2)}$$

The expansion of Eq. E4-6.2 and substitution from Eq. E4-6.1 gives

$$\epsilon_Y = \epsilon_\alpha x + \alpha\delta + \epsilon_\alpha \delta + \Delta . \qquad \text{(E4-6.3)}$$

Since $\epsilon_\alpha \delta$ is the product of two small quantities, it can be neglected. Therefore, the total output error is

$$\epsilon_Y = \epsilon_\alpha x + \alpha\delta + \Delta . \qquad \text{(E4-6.4)}$$

Solution B

Equation E4-6.1 in functional form converts to

$$f_a = y - \alpha x = 0 \qquad \text{(E4-6.5)}$$

which corresponds to Eq. 4-100. Partial differentiation of Eq. E4-6.5 yields

$$\frac{\partial f_a}{\partial x} = -\alpha \qquad \text{(E4-6.6)}$$

$$\frac{\partial f_a}{\partial y} = 1 . \qquad \text{(E4-6.7)}$$

Example 4-6. (Continued)

Thecomplete set of error equations, Eq. 4-116, can now be applied to the simple amplifier. Since there is only one element involved, only one equation is required, i.e.,

$$\frac{}{\partial y}\,\epsilon_y \qquad \frac{}{\partial x}\,\epsilon_x + m\,\frac{}{\partial y} \tag{E4-6.8}$$

It is given that $\epsilon_\alpha = \delta$. Also, the quantity m is the output error of the element in the absence of an input error. Therefore

$$y + m = (\alpha + \epsilon_\alpha)\,x + \Delta. \tag{E4-6.9}$$

Substitution from Eq. E4-6.1 into Eq. E4-6.9 gives

$$m = \epsilon_\alpha x + \Delta. \tag{E4-6.10}$$

Then, substitution from Eqs. E4-6.6, E4-6.7 and E4-6.10 into Eq. E4-6.8 gives

$$\begin{aligned} \epsilon \quad &= -(-\alpha)\,\delta + \epsilon_{.}\,x + \Delta \\ &= \epsilon_\alpha x + \alpha\delta + A \end{aligned} \tag{E4-6.11}$$

forthe total error in the output of the amplifier. This result is the same as that of Solution A; see Eq. E4-6.4.

Example 4-7. Error propagation through a simple computer circuit.

The computer circuit represented in Fig. 4-25 has three inputs, one output, and three dements. (The resolver is considered as being made up of two elements since it has two outputs.) The circuit performance equations are

$$f_1 = y_1 - x_1 \cos x_3 - y_3 \sin x_3 = 0 \tag{E4-7.1}$$

$$f_2 = y_2 + x_1 \sin x_3 - y_3 \cos x_3 = 0 \tag{E4-7.2}$$

$$f_3 = y3 - x_2 - y_2 = 0. \tag{E4-7.3}$$

These three equations can be solved in the manner below to give the output of the computer circuit y_1 in terms of the three inputs: x_1, x_2, and x_3. From Eq. E4-7.3

$$y3 = x_2 + y_2. \tag{E4-7.4}$$

From Eq. E4-7.2

$$y_2 = y_3 \cos x_3 - x_1 \sin x_3. \tag{E4-7.5}$$

Substitution from Eq. E4-7.5 into Eq. E4-7.4 shows that

$$y_3 = x_2 + y_3 \cos x_3 - x_1 \sin x_3. \tag{E4-7.6}$$

Rearrangement of terms gives

$$y_3 (1 - \cos x_3) = x_2 - x_1 \sin x_3. \tag{E4-7.7}$$

Therefore,

$$y_3 = \frac{x_2 - x_1 \sin x_3}{1 - \cos x_3} \tag{E4-7.8}$$

From Eq. E4-7.1

$$y_1 = x_1 \cos x_3 + y_3 \sin x_3. \tag{E4-7.9}$$

Substitution from Eq. E4-7.8 into Eq. E4-7.9 shows that

$$y_1 = x_1 \cos x3 + \frac{x_2 - x_1 \sin x_3}{1 - \cos x3} \sin x_3 \tag{E4-7.10}$$

$$= \frac{x_1 \cos x_3 - x_1 \cos^2 x_3 + x_2 \sin x_3 - x_1 \sin^2 x_3}{1 - \cos X3} \tag{E4-7.11}$$

$$\frac{\sin x_3 + \cos x_3 - \cos x_3}{1 - \cos x_3} + \frac{x_2 \sin x_3}{1 - \cos x_3} \tag{E4-7.12}$$

Therefore, since $\sin^2 x + \cos^2 x = 1$

$$y_1 = -x_1 + \frac{x_2 \sin x_3}{1 - \cos x_3} \tag{E4-7.13}$$

Equation E4-7.13 expresses the output of the computer circuit in terms of the three inputs.

The required partial derivatives can be determined by successively differentiating Eq. E4-7.1 with respect to y_1, y_2, y_3, x_1, x_2 and x_3 to obtain $\frac{\partial f_1}{\partial y_1}$, $\frac{\partial f_1}{\partial y_2}$, $\frac{\partial f_1}{\partial y_3}$, $\frac{\partial f_1}{\partial x_1}$, $\frac{\partial f_1}{\partial x_2}$, and $\frac{\partial f_1}{\partial x_3}$ respectively, and then repeating the process with Eqs. E4-7.2 and E4-7.3 to yield the remaining partial derivatives. The results are tabulated below. ∎

$$\begin{array}{lll}
\frac{\partial f_1}{\partial y_1} = 1 & \frac{\partial f_2}{\partial y_1} = 0 & \frac{\partial f_3}{\partial y_1} = 0 \\[2ex]
\frac{\partial f_1}{\partial y_2} = 0 & \frac{\partial f_2}{\partial y_2} = 1 & \frac{\partial f_3}{\partial y_2} = -1 \\[2ex]
\frac{\partial f_1}{\partial y_3} = -\sin x_3 & \frac{\partial f_2}{\partial y_3} = -\cos x_3 & \frac{\partial f_3}{\partial y_3} = 1 \\[2ex]
\frac{\partial f_1}{\partial x_1} = -\cos x_3 & \frac{\partial f_2}{\partial x_1} = \sin x_3 & \frac{\partial f_3}{\partial x_1} = 0 \\[2ex]
\frac{}{\partial x_2} = 0 & \frac{\partial f_2}{\partial x_2} = 0 & \frac{\partial f_3}{\partial x_2} = -1 \\[2ex]
\frac{\partial f_1}{\partial x_3} = x_1 \sin x_3 & \frac{\partial f_2}{\partial x_3} = x_1 \cos x_3 & \frac{\partial f_3}{\partial x_3} = 0 \\
\quad - y_3 \cos x_3 & \quad + y_3 \sin x_3 &
\end{array} \tag{E4-7.14}$$

Equations 4-116 show that the error for computer element No. 1 in Fig. 4-25 can be expressed in the generalized form

$$\sum_{k=1}^{3} \frac{\partial f_1}{\partial y_k} \epsilon_{y_k} = - \sum_{n=1}^{3} \frac{\partial f_1}{\partial x_n} \epsilon_{x_n} + m_1 \frac{\partial f_1}{\partial y_1} \tag{E4-7.15}$$

Substitution of the values of the partial derivatives given by Eqs. E4-7.14 into Eq. E4-7.15 then yields the error equation

$$\epsilon_{y_1} - (\sin x_3)\, \epsilon_{y_3} = (\cos x_3)\, \epsilon_{x_1} + (y_3 \cos x_3 - x_1 \sin x_3)\, \epsilon_{x_3} + m_1. \tag{E4-7.16}$$

Example 4-7. (Continued)

A similar procedure for computer elements No. 2 and No. 3 yields the respective erro equations

$$\epsilon_{y_2} - (\cos x_3)\,\epsilon_{y_3} = -(\sin x_3)\,\epsilon_{x_1} - (x_1 \cos x_3 + y_3 \sin x_3)\,\epsilon_{x_3} + m_2 \qquad \text{(E4-7.17)}$$

and

$$-\epsilon_{y_2} + \epsilon_{y_3} = \epsilon_{x_2} + m_3. \qquad \text{(E4-7.18)}$$

Solving Eqs. E4-7.16, E4-7.17 and E4-7.18 explicitly for ϵ_{y_1}, ϵ_{y_2} and ϵ_{y_3} gives

$$\epsilon_{y_1} = (\sin x_3)\,\epsilon_{y_3} + (\cos x_3)\,\epsilon_{x_1} + (y_3 \cos x_3 - x_1 \sin x_3)\,\epsilon_{x_3} + m_1. \qquad \text{(E4-7.19)}$$

$$\epsilon_{y_2} = (\cos x_3)\,\epsilon_{y_3} - (\sin x_3)\,\epsilon_{x_1} - (x_1 \cos x_3 + y_3 \sin x_3)\,\epsilon_{x_3} + m_2 \qquad \text{(E4-7.20)}$$

$$\epsilon_{y_3} = \epsilon_{y_2} + \epsilon_{x_2} + m_3. \qquad \text{(E4-7.21)}$$

The block diagram shown in Fig. 4-26 can be readily formed from Eqs. E4-7.19, E4-7.21 and E4-7.21. This diagram aids in visualizing the effect of a given input error on the outpu error ϵ_{y_1}.

Equations E4-7.19, E4-7.20, and E4-7.21 can also be solved as shown below to give ϵ_{y_1}, the error in the output, explicitly in terms of the input errors (ϵ_{x_1}, ϵ_{x_2} and ϵ_{x_3}) and the element errors (m_1, m_2 and m_3). Substitution from Eq. E4-7.20 into Eq. E4-7.21 gives:

$$\epsilon_{y_3} = (\cos x_3)\,\epsilon_{y_3} - (\sin x_3)\,\epsilon_{x_1} - (x_1 \cos x_3 + y_3 \sin x_3)\,\epsilon_{x_3} + m_2 + \epsilon_{x_2} + m_3. \qquad \text{(E4-7.22)}$$

Then, solving for ϵ_{y_3} shows that

$$\epsilon_{y_3} = \frac{-(\sin x_3)\,\epsilon_{x_1} - (x_1 \cos x_3 + y_3 \sin x_3)\,\epsilon_{x_3} + m_2 + \epsilon_{x_2} + m_3}{1 - \cos x_3} \qquad \text{(E4-7.23)}$$

Example 4-7. (Continued)

Substitution from Eq. E4-7.23 into Eq. E4-7.19 shows that

$$\epsilon_{y_1} = \frac{-(\sin^2 x_3)\,\epsilon_{x_1} - (x_1 \sin x_3 \cos x_3)\,\epsilon_{x_3} - (y_3 \sin^2 x_3)\,\epsilon_{x_3} + (\sin x_3)\, m_2}{1 - \cos x_3}$$

$$+ \frac{(\sin x_3)\,\epsilon_{x_2} + (\sin x_3)\, m_3}{1 - \cos x_3} + (\cos x_3)\,\epsilon_{x_1} + (y_3 \cos x_3)\,\epsilon_{x_3} - (x_1 \sin x_3)\,\epsilon_{x_3} + m_1$$

$$= \frac{1}{1 - \cos x_3}\Big[-(\sin^2 x_3)\,\epsilon_{x_1} - (x_1 \sin x_3 \cos x_3)\,\epsilon_{x_3} - (y_3 \sin^2 x_3)\,\epsilon_{x_3}$$

$$+ (\sin x_3)\, m_2 + (\sin x_3)\,\epsilon_{x_2} + (\sin x_3)\, m_3 + (\cos x_3)\,\epsilon_{x_1}$$

$$+ (y_3 \cos x_3)\,\epsilon_{x_3} - (x_1 \sin x_3)\,\epsilon_{x_3} + m_1 - (\cos^2 x_3)\,\epsilon_{x_1}$$

$$- (y_3 \cos^2 x_3)\,\epsilon_{x_3} + (x_1 \sin x_3 \cos x_3)\,\epsilon_{x_3} - (\cos x_3)\, m_1\Big]. \tag{E4-7.24}$$

Removal of those terms of Eq. E4-7.24 that cancel one another, simplification by use of the trigonometric identity $\sin^2 x + \cos^2 x = 1$, and rearrangement of terms show that

$$\epsilon_{y_1} = \frac{1}{1 - \cos x_3}\Big[-(1 - \cos x_3)\,\epsilon_{x_1} + (\sin x_3)\,\epsilon_{x_2} - y_3 (1 - \cos x_3)\,\epsilon_{x_3}$$

$$- (x_1 \sin x_3)\,\epsilon_{x_3} + (1 - \cos x_3)\, m_1 + (\sin x_3)\, m_2 + (\sin x_3)\, m_3\Big]. \tag{E4-7.25}$$

Substitution from Eq. E4-7.8 into Eq. E4-7.25 and simplifying terms yield

$$\epsilon_{y_1} = -\epsilon_{x_1} + \frac{\sin x_3}{1 - \cos x_3}\,\epsilon_{x_2} - \frac{x_2}{1 - \cos x_3}\,\epsilon_{x_3} + m_1 + \frac{\sin x_3}{1 - \cos x_3}\, m_2 + \frac{\sin x_3}{1 - \cos x_3}\, m_3. \tag{E4-7.26}$$

Equation E4-7.26 expresses the error in the output of the computer circuit in terms of th errors in the elements and the errors in the inputs. Note that the element-error terms an the input-error terms are all entirely separate, i.e., they are independent of one another

Example 4-8. Example of the error-summation procedures described in the text.

Assume the following values of systematic and random component errors for the error equation derived in Example 4-7 (see Eq. E4-7.26):

Table E4-8.1

Error	Systematic Component (peak errors)	Random Component (standard deviations)
ϵ_{x_1}	$c_1 x_2$	σ_1
ϵ_{x_2}	c_2	0
ϵ_{x_3}	$c_3(1-\cos x_3)$	0
m_1	d_1	$\sigma_2 \sin x_3$
m_2	d_2	$a_3(1-\cos x_3)$
m_3	d_3	0

where c_1, c_2, c_3, d_1, d_2, and d_3 are constants.

When the values of the systematic components given in Table E4-8.1 are substituted in Eq. E4-7.26 of Example 4-7, the resulting relationship yields the systematic component of ϵ_{y_1}, which is designated as ϵ_{y_1s}, in the form

$$\epsilon_{y_1s} = -c_1 x_2 + \frac{c_2 \sin x_3}{1-\cos x_3} - c_3 x_2 + d_1 + \frac{d_2 \sin x_3}{1-\cos x_3} + \frac{d_3 \sin x_3}{1-\cos x_3} \tag{E4-8.1}$$

$$= -(c_1 + c_3)\, x_2 + d_1 + (c_2 + d_2 + d_3)\, \frac{\sin x_3}{1-\cos x_3} \, . \tag{E4-8.2}$$

This relationship can be rearranged and simplified to the form

$$\epsilon_{y_1s} = c_4 x_2 + d_1 + c_5 \cot \frac{x_3}{2} \tag{E4-8.3}$$

where

$$c_4 = -(c_1 + c_3) \tag{E4-8.4}$$

$$c_5 = c_2 + d_2 + d_3 \tag{E4-8.5}$$

and, by a fundamental trigonometric identity,

$$\cot \frac{x_3}{2} = \frac{\sin x_3}{1-\cos x_3} \tag{E4-8.6}$$

Example 4-8. (Continued)

The dependent terms of Eq. E4-8.3 are plotted in Fig. E4-8.1. Obviously, the resolver computer circuit that is the subject of this example (see Figs. 4-25 and 4-26) becomes useless for values of x_3 that approach zero degrees and 360 degrees since the systematic component of ϵ_{y_1} then approaches an infinite magnitude. The variable x_3 must accordingly be limited to the range $x_{3(min)} < x_3 < x_{3(max)}$, as shown in Fig. E4-8.1(B). This range is determined explicitly by Eq. E4-8.3 if the maximum value of $\epsilon_{y_1 s}$ and the maximum value of x_2 (designated $x_{2\,(max)}$) are specified. If, on the other hand, for example, $x_{2\,(max)} = 1$, $x_{3\,(min)} = 45°$ and $x_{3\,(max)} = 315°$, then the maximum value of $\epsilon_{y_1 s}$ is either

$$\epsilon_{y_1 s} = c_4 + d_1 + 2.4\, c_5 \tag{E4-8.7}$$

for

$$x_3 = x_{3(min)} = 45°,$$

or

$$\epsilon_{y_1 s} = c_4 + d_1 - 2.4 c_5 \tag{E4-8.8}$$

$$x_3 = x_{3\,(max)} = 315°$$

Whether Eq. E4-8.7 or Eq. E4-8.8 will yield the maximum systematic component of ϵ_{y_1} for a particular numerical example will, of course, depend on the algebraic sign of $c_4 + d_1$. The total random error is obtained from the relationship

$$\sigma^2_{\epsilon_{y_1}} = \sigma^2_{\epsilon_{[illegible]}} + \sigma^2_{\epsilon_{[illegible]}} + \sigma^2_{\epsilon_{[illegible]}} + \sigma^2_{[illegible]} + \sigma^2_{m_2} + \sigma^2_{[illegible]} \tag{E4-8.9}$$

where $\sigma^2_{\epsilon_{y_1}}$ is the variance of the output error, $\sigma^2_{\epsilon_{x_1}}$ is the variance of the first term of Eq. E4-7.26 in Example 4-7, and the remaining variances pertain to the remaining terms, in order. Substitution of the values of the random errors given in Table E4-8.1 into Eq. E4-8.9, which replaces Eq. E4-7.26 of Example 4-7 for the purposes of random-error summation, yields the equation

$$\sigma^2_{\epsilon_{[illegible]}} = \sigma_1^2 + \sigma_2^2 \sin^2 x_3 + \sigma_3^2 \sin^2 x_3 \tag{E4-8.10}$$

$$= \sigma_1^2 + (\sigma_2^2 + \sigma_3^2) \sin^2 x_3 \tag{E4-8.11}$$

$$= \sigma_1^2 + \sigma_t^2 \sin^2 x_3 \tag{E4-8.12}$$

where

$$\sigma_t^2 = \sigma_2^2 + \sigma_3^2 . \tag{E4-8.13}$$

Example 4-8. (Continued)

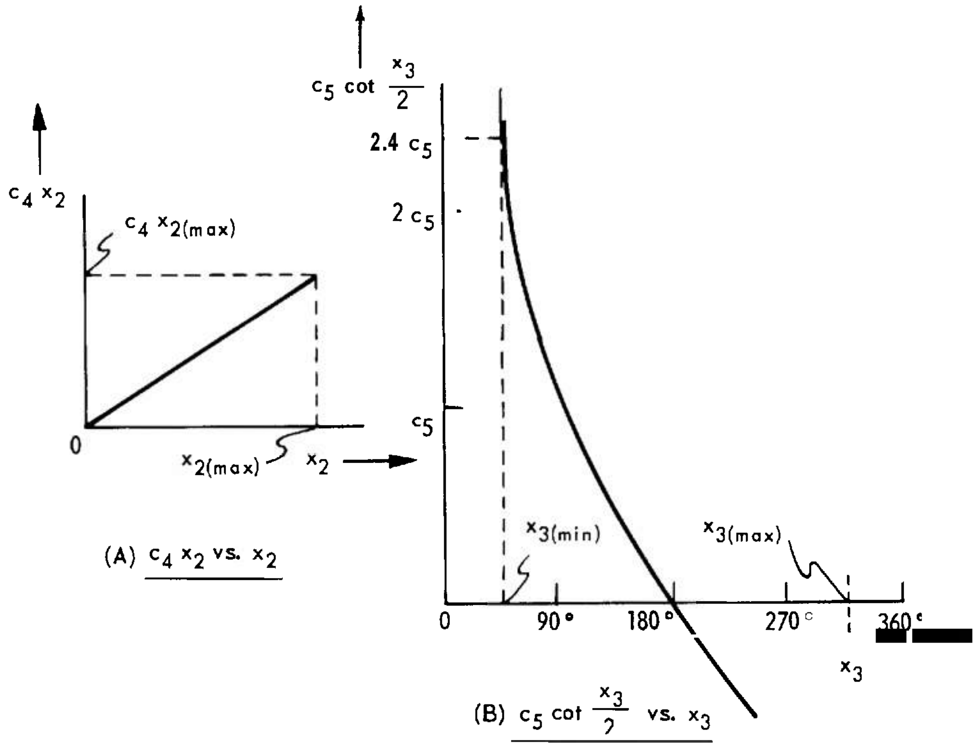


Figure E4-8.1. Plots of the dependent terms of Eq. E4-8.3.

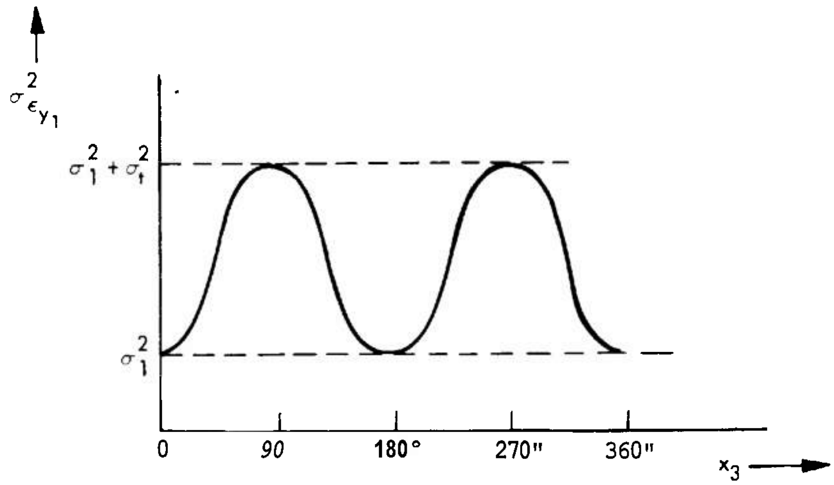


Figure E4-8.2. Plot of $\sigma^2_{\epsilon_{y_1}}$ as a function of x_3.

Example 4-8. (Continued)

The plot of $\sigma^2_{\epsilon_{y_1}}$ as a function of x_3 is given in Fig. E4-8.2 and shows that the maximum value of $\sigma^2_{\epsilon_{y_1}}$ is $\sigma^2_1 + \sigma^2_t$. If all values of x_3 are equally likely, the mean value of $\sigma^2_{\epsilon_{y_1}}$ is given by the relationship

$$\overline{\sigma^2_{\epsilon_{y_1}}} = \sigma_1^2 + \frac{1}{2}\int^{\pi} \sigma_t^2 \sin^2 x_3\, dx \tag{E4-8.14}$$

$$= \sigma_1^2 + \left[\frac{\sigma_t}{4} x_3 - \frac{\sigma_t^2}{a} \sin 2x_3 \right|_o^{\pi} \tag{E4-8.15}$$

$$= \sigma_1^2 + \frac{\pi\sigma_t^2}{4} \tag{E4-8.16}$$

$$= \sigma_1^2 + \frac{\pi}{4}\left(\sigma_2^2 + \sigma_3^2\right). \tag{E4-8.17}$$

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ACKNOWLEDGMENT AND TRIBUTE TO

DR. JOHN G. TAPPERT

1906 - 1959

One cannot read the Fire Control Series of the Engineering Design Handbook without being impressed by the numerous references to the work of the late Dr. John G. Tappert of Frankford Arsenal. His original contributions to fire control technology and the inspiration and guidance he provided to his colleagues rank him among the foremost of the engineer-scientists who have served the Army. It is therefore fitting that this acknowledgment and tribute should appear in the Fire Control Series.

In the years immediately preceding the United States' entry into World War II, a most urgent requirement existed for improved defenses against strategic and tactical air attack. The high-altitude weapon was the 90 mm antiaircraft gun, manually trained and elevated. The 37 mm gun had recently been developed for the low-altitude role and was later supplanted by the more reliable 40 mm gun.

Fire control for the 90 mm gun was provided by an off-carriage director and for the 37 mm gun by direct laying, the line of sight being offset from the gun axis by a lead computer. Radar was not yet out of the laboratory. Target detection and acquisition were by visual means during the day and by sound locator and searchlight at night. Manual optical tracking and ranging was the only means of position finding.

The stereoscopic heightfinder was potentially the most accurate altitude instrument available, but even the most skilled operators could not maintain continuous stereo coincidence between the reticle and target images. Unfortunately, the gun directors then in use required continuous altitude data input to be effective against diving or climbing targets. Without it they could provide no defense against the dive-bombing tactics used with such devastating effect by the Luftwaffe.

After several frustrating years of trying to fix the shortcomings of existing antiaircraft predictors, Dr. Tappert turned his attention to the conception and study of radically different approaches. During the years 1939 – 1940 he considered nearly a half dozen original predictor schemes from the point of view of potential effectiveness and feasibility of development.

Unlike other designers, he preferred to treat the entire prediction problem in polar coordinates. He developed techniques for maintaining the systematic errors of velocity measurement in polar coordinates within tolerable limits, and he believed that the difficulties of velocity-measurement parallax correction in polar coordinates were more than offset by the accuracy advantages. Coordinate transformations, at unfavorable scale factor, from polar to rectangular and back to polar again could be avoided, and gun-laying orders could be obtained by computing relatively small lead angles at favorable scale factor to be added to present elevation and azimuth.

In 1940, Dr. Tappert undertook development of the Director T12 for 90 mm AA guns, which was completed in 1943. Successful completion of this project was frustrated by an unfortunate choice of servo technology, but the Director T12 did have several unique features. In addition to employing polar coordinates throughout for velocity measurement, prediction and parallax correction, it incorporated a regenerative altitude circuit capable of generating continuous altitude and altitude rate data from sampled input data as obtained from the stereoscopic heightfinder. The accuracy demonstrated by dynamic tests tended to justify Dr. Tappert's faith in the polar coordinate approach, and he used it for all of his subsequent predictors.

Dr. Tappert made many original and novel contributions to mechanical analog computer technology. Perhaps the most significant of these was the development of practical techniques for the manufacture of noncircular spur gears, which were used extensively in his computers for the generation of arbitrary continuous and monotonic functions of a single variable. The sum of the pitch radii of driver and follower is held constant while their ratio is varied in proportion to the derivative of the function to be generated. The manufacturing difficulty lies in cutting properly mating teeth on the noncircular pitch contours of driver and follower. Dr. Tappert solved this problem originally by approximating the pitch curves with a series of circular arcs and cutting the teeth with a circular cutter on a standard gear shaper, using multiple set-up operations. Later, in collaboration with another Frankford Arsenal engineer, he developed and patented a modification for the gear shaper to cut teeth on the entire pitch curve in a single operation by continuously varying the center distance and speed ratio between cutter and blank in accordance with prepared templates.

In the design of his computers, Dr. Tappert made extensive use of mechanical linkages as a reliable and comparatively inexpensive means for generating arbitrary functions of one or more independent variables. The design of such linkages typically

involves optimization of the linkage parameters to best fit the linkage function to the desired function. Dr. Tappert developed practical iterative methods for refining an initial set of parameters such as to converge reliably on the optimum values. An interesting application of this technique was the development of a linkage multiplier with better scale factors and higher mechanical efficiency than the conventional sliding bar multiplier based on the proportionality of the sides of similar triangles. Shortly before his death, Dr. Tappert was applying this empirical design approach to the design of a lead computer for the Twin 40 mm Antiaircraft Gun Turret (Duster). The entire computer was treated as a complex mechanical function generator whose parameters were determined to best approximate the lead functions for a representive family of tactical aircraft attack courses.

Dr. Tappert's contributions to the art of numerical analysis were also voluminous. He developed practical methods for smoothing, differentiating, interpolating, extrapolating and integrating tabular data and published them in the form of a very popular Computer's Handbook. Also included were practical methods for the integration of differential equations with the aid of a desk calculator. His methods of system accuracy analysis were used very effectively in the evaluation of the Vigilante Antiaircraft Weapon System and are described in Chapter 4.

In addition to his achievements in fire control technology, Dr. Tappert will long be remembered among his colleagues for his exacting standards of performance and for his contagious interest in the diverse branches of science and philosophy.

CHAPTER 5

DESIGNING FOR RELIABILITY, MAINTAINABILITY, EASE OF OPERATION, AND SAFETY

5-1 GENERAL CONSIDERATIONS

5-1.1 FACTORS AFFECTING EFFECTIVENESS IN THE FIELD

Most Army fire control equipment must perform in any climate and over all terrain. Components and systems are either installed on vehicles--tanks, self-propelled or towed guns, and the like--or are intended for temporary ground installation, and must be designed for portability by soldiers, ground vehicles, and aircraft and for quick assembly and disassembly. Much of the equipment may be air-dropped for immediate use.

Equipment that is designed to be 100 percent effective will clearly be only 50 percent effective under battle conditions if it is inoperative half the time because of low reliability or difficulty in maintaining it in the field. Similarly, the equipment's accuracy and effectiveness are diminished in proportion to the operator's inability to handle it under the conditions at hand. The designer, then, must include (1) maximum reliability under all foreseeable conditions, (2) ease of maintenance, (3) human engineering to ensure effective operation, and (4) in most cases portability as integral to his design. To the extent that he fails in any of these respects, he nullifies any other advantage in the design.

It is worth emphasizing here that most of the resources available to industry for operating and maintaining complex and difficult equipment are simply not available to the Army in the field. For example, industry can hire and develop skilled mechanics. One major corporation employs only persons with high school diplomas or better as mechanics for electronic controls; trains them for at least two years in practical and theoretical post-graduate work; and trains them for at least two more years on the job before they are considered "qualified craftsmen."[1] Requirements of many other companies are equally rigorous. The Army, on the other hand, has among its recruits non-high-school graduates, and the total term of enlistment of recruits is often shorter than the minimum education and training period of industry. Accordingly, Army fire control equipment must be designed to be operated and maintained by men with only a small fraction of the education and training available to industry--men with perhaps a partial high school education and a limited amount of specialized in-service training, In times of national emergency, with civilians from all walks of life entering the service and with training periods of even shorter duration, the proportion of trained personnel will decline.

Then, too, industry can control the environment in which much of its equipment operates. Humidity and temperature are controlled in many plants, and it is common practice now to perform delicate assembly, test, and repair operations in special "clean rooms" with nearly all dust eliminated, with humidity and temperature maintained within narrow ranges, and with vibrations and sound drastically attenuated. At the other extreme, most Army fire control equipment must operate outdoors in all types of weather and, usually, in an environment of intensive shock and vibration.

Also, industry can schedule systematic maintenance programs based on production schedules. Where downtime is extremely costly, industry often finds it economical to develop unusually sophisticated devices for rapid trouble-shooting, overhaul, and testing. Neither course is open to the Army. Fire control equipment must be operable 100 percent of the time, if possible. The

use of elaborate support equipment is often precluded by field conditions and, at any rate, extra maintenance equipment and personnel must be supported at the expense of combat weapons and soldiers.

5-1.2 ECONOMY AND EFFICIENCY OF PRODUCTION

The cost of modern weapons is extremely high, even compared with those of World War II and the Korean War. Economy, consistent with reliability and the required accuracy, is a major consideration. Similarly, rapid and efficient production of fire control equipment, particularly in times of war or national emergency, is one of the dominant considerations in selecting a design and packaging the equipment, since equipment that cannot be produced in time to be used might as well not be produced at all.

Fortunately, most of the design considerations discussed so far are mutually reinforcing. Designing for maximum reliability implies maximum use of proven, standard components, which also contributes to economy and efficiency of manufacturing and provisioning. Ease of maintenance implies simplicity, modular construction, ready access to components, and interchangeability of parts, which in turn also lead to economy and efficiency.

Factors that should not be overlooked are the cost and criticality of the material chosen for use in the design under development. In a developmental contract, these factors may be insignificant. They may become a major factor if the equipment goes into large-scale production.

The designer should also be aware of the fact that the type of material cross section used often determines the most economical fabricating process. Commercial steel and aluminum, for example, are available in a large variety of shapes and a wide range of sizes. The designer can choose between sheets that are measured in thicknesses of a few thousandths of an inch or thick plates and odd shapes. The more complicated the shape, the more expensive it is, because of the amount and complexity of processing. Therefore, the design engineer should consider the cost of the commerical shapes to be selected. Sheets, plates, and common bar shapes are the least expensive; structural shapes are a little more expensive; closed pipe and tubing are the most expensive.

In general, the designer should use commercially available standard shapes because of the savings made possible in the initial processing at the mill. (The advantages of this practice, while self-evident, are surprisingly often overlooked.) Further economies can be achieved by specifying shapes that are standard to the plant producing the equipment, i.e., that are stocked regularly and need not be specially ordered.

The fire control equipment designer should keep in mind that in time of national emergency manufacturers of many types of commercial equipment will be pressed into service to produce fire control items. They will not be experienced in this type of work; will not be set up for it initially; and will probably not be used to working to close tolerances. Therefore, in summary, to make the transition easy and help provide as broad a base as possible for wartime production, the design engineer should:

1. Provide the simplest design compatible with the requirements, make maximum use of standard parts and shapes, and avoid the need for special jigs or tools in production.

2. Strive for flexibility. If several means of fabrication — e.g., casting, welding, stamping and forming — or variations in type or model of components are permissible, the alternatives should be stated.

3. Avoid too-tight tolerances and fits. Commercially oriented manufacturers will simply not be able to maintain very close dimensions without a great deal of training, retooling, and devising highly accurate jigs. If possible, specify tolerances in the order of 0.001 inch or looser (except for special items like shaft centers, where tighter tolerances may be a necessity). In fire control equipment, tolerance requirements can often be relaxed by optimum design of mechanical and electrical linkages without any loss of accuracy. For example, correct choice of gear ratios will minimize cumulative effects of backlash. Sometimes, a new design approach may be necessary. If system accuracy depends on very close tolerances, it is probably being designed too close to the margin for practical purposes.[2]

The designer, however, must eschew the false economies of specifying materials, components, or construction methods that fall short of the most rigorous conditions likely to be encountered by the equipment.

5-1.3 AVAILABILITY OF MATERIALS AND COMPONENTS

An extremely important consideration--one that may be decisive in achieving any of the goals discussed in this Handbook--is the availability of materials in time of war. This includes not only raw materials — such as copper which was a critical item in World War 11— but shapes of various types and methods of producing them (forgings in general were short in that war). Likewise, some types of components may be difficult to manufacture under wartime conditions. Most items may be plentiful in peace time; the problem is to predict the ones that will be in short supply in war time.

A National Stockpile of Strategic and Critical Materials was established by Congress in 1946. Its purpose is "to avoid dangerous and costly dependence on foreign sources of materials for meeting essential needs in limited war. . , and general war. . ." It is administered by the Office of Emergency Planning, which reports directly to the President. Similarly, the Department of Defense was given authority by Congress in 1950 to expand• productive industrial capacity, purchase materials, and encourage the exploration and development of new sources to meet the needs of national security. In this connection, a Defense Production Act (DPA) Inventory is maintained.

From the National Stockpile, DPA Inventory, and supplementary sources, the Office of Emergency Planning has compiled a book, Strategic and Critical Materials, Descriptive Data.[3] It summarizes the uses of 80 such materials; lists their sources; and (very usefully) describes the substitutes that can be used in various applications.

The design engmeer can assume that irregular or nonstandard components and shapes, items that require delicate or complex operations for their manufacture, and raw materials that are rare in North America will present difficulties in time of war. Accordingly, it is well to state permissible alternatives to specified materials and components wherever practical.

5-2 CLIMATIC CONDITIONS AND OTHER ENVIRONMENTAL FACTORS

5-2.1 BASIC PRINCIPLES

Most types of Army fire control equipment should normally be designed for any ground environment anywhere in the world. (Much Navy equipment, by contrast, is designed for specific environments that may be violent or extreme but are much more narrowly defined. With underwater ordnance equipment, for example, the designer may be able to plan for a relatively narrow temperature range extending over about 60° F, water of fairly constant salinity and predictable pressures, and a certain amount of shock and vibration.)

With today's mobile Army, it is also vital to plan for air transportation at high altitudes and speeds, and for air drop in the design and packaging of the equipment.

It is usually impossible to design optical, electronic, or mechanical equipment so that it can simply be picked up in one extreme—the tropics, for example — and used without alteration in an opposite extreme, such as the arctic regions. Accordingly, the designer must design adaptability into the equipment to permit accommodation to differing environments. For example, a device might contain either compensating elements to allow for temperature differentials, or elements that can be adjusted in the field. The optimum solution will be a function of the kind of equipment and the environmental extremes involved. A few examples are: adjustable heating and cooling elements; knob adjustments to compensate for optical changes due to thermal contraction and expansion, or variations in electrical characteristics with temperature; easily installed protective coverings against various environments; and (if the equipment itself must be altered) modular construction to permit quick replacement of the parts affected by environmental change.

Designing for environmental extremes must be considered from the points of view of (1) designing against the destructive or distorting effects of the environment (par

5-3.3 through 5-3.6), and (2) designing for ease of maintenance (par 5-4.7.3) and operation (par 5-5) in extreme environments.

The environments which will be considered in detail are climatic extremes, mechanical forces, and interferences from various sources, man-made and natural.

1. Climatic Extremes: These include thermal and humidity stress, precipitation, wind, and penetration and abrasion of blowing sand, dust and snow. An indirect product of climate is fungus which can be devastating to improperly chosen or protected materials in the tropics. Atmospheric pressure is an increasingly important consideration, particularly in air transport since fire control equipment must be airborne at times.

2. Mechanical Forces: These include shock, vibration, and acceleration. These forces may be transmitted from the vehicle in which the equipment is mounted, from explosions due to own or enemy fire, from an aircraft or other vehicle in which the equipment is being transported, and from landing on hard ground during an air drop.

3. Interferences: These include RF and other emissions from adjacent equipment, and deliberate jamming by the enemy; emissions from radioactive sources are also grouped in this category.

5-2.2 CLIMATIC EXTREMES

Data on climatic extremes are still incomplete and unreliable in many areas where data exist, particularly at high altitudes. However, for the purposes of Army weapons, the data in such documents as AR 705-15, Operation of Materiel Under Extreme Conditions of Environment,[7] will suffice for most purposes. This document divides climatic conditions into five classifications for design purposes:

1. Hot-dry (temperatures to 120°F)
2. Warm-wet (temperatures to 95°F, precipitation to 7.18 in. per month average)
3. Intermediate (temperatures 105°F to -25°F, moderate rainfall)
4. Cold (temperatures to -50°F)
5. Extreme cold (temperatures to -80°F)

Each of these conditions is defined in terms of extreme air temperatures and solar radiation as a function of altitude, water temperatures, maximum precipitation over short periods, snow loads, icing phenomena, winds, atmospheric pressures, and blowing snow, sand, and dust. Charts show where in the world each condition occurs in each season of the year.

AR 705-15 recommends that all combat and support equipment be designed to operate under intermediate conditions, and that modification kits be supplied wherever possible to adapt equipment to cold, hot-dry, and warm-wet conditions. Only in the extreme cold areas is it expected that operations may require a preponderance of equipment specially designed for an extreme cold climate.

It should be emphasized that nearly everywhere conditions change a great deal with the season. Parts of India, for example, experience hot-dry, warm-wet, and intermediate weather, depending on the time of year. In Greenland, the weather varies from intermediate to extreme cold.

MIL-STD-210 A4 gives a breakdown of extreme ground conditions—in terms of extremes of heat, cold, humidity, precipitation, wind, snow, dust, and atmospheric pressure — with such additional details as the range of infrared, ultraviolet, and visible radiation intensities and the duration of maximum temperatures during 24-hour cycles.

MIL-STD-210 A further specifies the extremes that will be encountered in the following categories of operations: (1) operation ground, world wide; (2) operation ground, arctic winter; (3) operation ground, moist tropics; (4) operation ground, hot desert; (5) operation shipboard, world wide; and (6) storage and transit, short-term, world wide. Fire control equipment should, where possible, be designed for the extremes in categories (1) and (6).

MIL-STD-210 A also includes a detailed tabulation of various atmospheric extremes up to 100,000 feet — chiefly of interest to aircraft and missile designers but also of concern to designers of ground equipment that may be transported by high-altitude aircraft. For example, embrittlement of cold steels can cause devastating damage under conditions of shock and vibration. It may be necessary to specify that the equipment be transported in a heated, pressurized cabin.

For weatherproofing, see par 5-3.3.2.

5-2.3 VIBRATION, SHOCK, AND HIGH-G FORCES

Fire control equipment is subject to mechanical vibrations, shock, and high-acceleration forces as a result of the following:

1. Transport by air, sea, or land.
2. Motion of the vehicle on which the equipment is mounted. (Vibrations and shock arise from rough terrain, wheel shimmy, engine and tire vibrations, structural vibrations, and— on tracked vehicles — the track striking the ground.)
3. Firing of guns or other weapons on or near which the equipment is mounted.
4. Detonations of bombs, projectiles, and related explosive ordnance.

Damage during transport can be guarded against by proper packaging and handling procedures, Damage due to vehicular motions, weapons, and explosions can be eliminated only by the proper design of the equipment itself and its mounting.

With ground equipment, high-G forces are chiefly of concern as a part of the vibration and shock problem; higher forces are not likely to be encountered that are due, for example, to the acceleration of the vehicle in which they are mounted or carried. Vibration is a continuing periodic motion induced by an oscillating force that results from an unbalanced mass, mechanically, or from a fluctuating magnetic force, electrically.[5] Shock, on the other hand, is the effect of a suddenly applied force on a structure or a sudden change in the motion of the structure.' On-carriage fire control equipment, for example, experiences shock effects when the weapon is fired or when nonpenetrating ballistic impacts are achieved nearby. Transient vibrations that may be of high frequency and high amplitude are produced; the amplitude may become so high that brittle materials will fracture or ductile materials will yield. As an example, high-frequency vibrations may occur at the resonant frequency of an optical element within an optical sight and cause it to fail or shatter. Another characteristic of shock resulting from abrupt changes in motion is the presence of large accelerations that can be transmitted to components, causing physical damage or loss of accuracy under extreme conditions. The destructive frequencies induced by shock or vehicular motion are generally high frequencies in the order of 10 to 25 cps. Therefore, an important goal of designing and mounting is to ensure that the equipment will not have natural frequencies in that range (or its harmonics); natural frequencies should be higher than the destructive range, Low frequencies, such as those induced by the natural frequency of a vehicle's suspension, are not normally damaging, providing the amplitude is not excessive.[6]

Designing to minimize vibration and shock is discussed in par 5-3.4.

5-2.4 RF AND OTHER INTERFERENCES

Radio-frequency interference is a major problem in designing fire control equipment for the Army. In many other types of installations of electronic equipment, the sources, frequencies, and amplitudes of interference can be predicted quite accurately, and proper shielding can be provided. Most Army fire control equipment, however, being portable and subject to use almost anywhere--near radio and radar stations, high-kilovolt generating and transmission systems, and military electronic equipment of many types-must be designed against interference over a broad spectrum.

Added to the problem of accidental or random interference is the hazard of deliberate enemy interference. It must be anticipated that an alert enemy will beam RF interference over as wide a band as possible.

Radioactivity may be expected to present less of a problem to the designer of fire control equipment. While nuclear radiations do damage materials, the dosage rate in most cases must be so high that the associated effects of blast or heat on the equipment or the biological effects of radiations on the operators become the limiting factors in atomic warfare. The effects of blast can be minimized by shock mounting and shock-resistant construction (see par 5-3.4); the effects of heat and biological effects of radiation can be minimized in some cases by shielding which is generally the concern of the weapon system designer rather than the designer of the fire control elements.

The fire control designer, however, must be directly concerned with the light generated in an atomic explosion. It can

blind or seriously damage the eyes of men looking through optical sights. Current development is concentrated on shutters that will be triggered by nuclear blasts and will shut very rapidly, before the observer's eyes are burned.

5-3 DESIGNING FOR RELIABILITY

5-3.1 BASIC PRINCIPLES

Reliability and maintainability are discussed under different headings in this chapter because different aspects of design are involved. However, by producing reliable equipment, the manufacturer has done much to solve the problems of maintainability; equipment that is 100 percent reliable for its intended life requires no corrective maintenance. Reliability of course depends to a large extent on proper manufacturing procedures and quality control tests, but the design engineer can make major contributions in this field — reliability must be designed into the equipment; it cannot be built in.

Reliability has become a major problem in recent years — so much so that reliability engineering has become a speciality in its own right. The reasons for this are:

1. The increased complexity of equipment.
2. The short transition time between the theoretical laboratory stage and engineering design and production in many fields, so that man is working increasingly close to "the limits of experience."
3. The very close tolerances and accurate alignments, and consequent careful process controls required in many fields of technology.
4. The increasing complexity of industrial and military organizations.
5. The increased chance of human error resulting from the foregoing.
6. Insurance that reliability will be considered and demonstrated at each step of the development process.

Modern reliability methodology is derived from the mathematics of probability and statistics and, when effectively applied, demands the close cooperation of all engaged in the design, production, and testing of the item. A detailed discussion cannot be presented here; the reader is, therefore, referred to such books as Lloyd and Lipow,[8] from which much of the material on these pages is derived. The various means of achieving reliability are summarized in the paragraphs which follow.

It must be remembered that reliability must be considered in the practical context of the equipment being designed, i.e., the concern is with the reliability of fire control equipments in the actual environments described in par 5-2 rather than in any abstract or theoretical context. In designing for reliability, the engineer must always keep in mind the main purpose of the equipment — to score hits — and secondary goals, such as light weight, low silhouette, portability, and ease of maintenance and operation,

5-3.1.1 Reliability Through Simplicity and Redundancy

The more complex a system, the greater the chance of failure (other things being equal). If reliability is expressed as the probability of successful operation, then the reliability of the system is equal to the product of the probabilities of successful operation of its parts, provided they are statistically independent. Thus, it is possible to increase the reliability of a fire control system directly by making it as simple as its performance requirements permit — by using as few subsystems and components as possible that must function to carry out the end function of the system as a whole. (This rule does not apply, however, to parallel or back-up subsystems; see the following discussion of redundancy.)

Redundancy--*overdesigning*, or providing back-up or alternate systems--often provides a direct counter to the product rule of reliability, i.e., the more alternate subsystems there are, the greater the probability that one of them--and therefore the system as a whole--will operate satisfactorily. While redundancy seems at first glance to be incompatible with the ideals of light weight and compact construction, it should not be dismissed without careful consideration because:

1. In some fields of modern technology, such light and compact components have been achieved that alternate subsystems can be added without any serious increase in total

weight and size. This is not only true in such obvious fields as solid-state electronics (particularly microelectronics) but also in gyroscope technology, hydraulics, and other fields.

2. By carefully designing for a maximum strength-to-weight ratio, the designer can often improve safety factors--"overdesign" if you will--without actually increasing size and weight. Designers sometimes overlook the reserve strength of materials and, instead, arbitrarily apply (or misapply) factors of safety. In selecting materials, of course, their cost and availability must be considered as well as their strength characteristics (see par 5-1.2 and 5-1.3).

The increase in reliability; increase in weight, size, and complexity caused by adding back-up systems; and the extent to which the equipment itself is critical should all be analyzed to determine whether redundancy should be designed into the system.

5-3.1.2 Reliability Through Use of Standard Components and Proven Design Techniques

The word "design" implies, of course, that something new is being developed. If a new system is made up largely of relatively untried components and subsystems, then the probability of failure is multiplied (see the product rule in par 5-3.1.1). The main point about recently developed components, from the reliability standpoint, is that relatively little is known about their behavior in many environments; when two or more such items are combined, their interaction with one another may be ascertained only after extensive testing.

A standard item may be defined generally as any commercially available item or any item in the Federal Stock System (par 5-4.4), i.e., an item that does not have to be specially designed for some particular use. In this context, "standard" also implies conformity to military or other federal specifications and such industrial standards as those of the National Electrical Manufacturers' Association (NEMA).

The use of components, subsystems, and materials that have been proven in many types of operation will greatly increase reliability. The engmeer should, therefore, operate "at the limit of experience" only when this is necessary to achieve a specific requirement. When new systems or components must be used, the designer may be able to take advantage of the resources available for accelerated life and environmental tests--keeping in mind that these inevitably extend lead time.

5-3.1.3 Other Factors Contributing to Reliability

Training production workers to reduce human error, use of proper organization to ensure exchange of information within the design and production facilities, careful quality control during production, and adequate testing are just as importarit to reliability as the factors discussed in par 5-3.1.1 and par 5-3.1.2. While they are not primarily the concern of the design engmeer, he may be the first man to recognize that special problems will occur in one of these areas. As soon as he recognizes that the problem will occur, he should (1) take steps to design for greater simplicity of manufacture or (2) notify the cognizant managers in time for them to setup proper facilities and procedures.

Reliability is further increased in the detail design phases by specifying the proper materials and protective finishes, by designing against the extreme environments encountered by fire control equipment (particularly climatic extremes, shock, and vibration), and by providing the best and most practical types of lubrication for field conditions. These subjects are discussed in par 5-3.2 through par 5-3.6.

5-3.2 MATERIALS

A large body of experience has been accumulated in the choice of materials for fire control equipment; much of this is reflected in the various general and detailed specifications pertaining to the design of fire control equipment. MIL-F-14252(ORD)[9] and MILF-13926A(MU)[10] cover fire control material design, manufacture, and inspection in general; MIL-0-13830A[11] covers optical components of fire control instruments; MIL-E-4158C(USAF)[12] covers electronic instruments; and MIL-F-45133B(ORD)[13] covers

requirements for the procurement of fire control components and assemblies. These specifications, and the detailed specifications and standards referenced in them, can be particularly helpful to the design engineer in choosing materials, even when they are not required under the contract.

The specifications also list the environmental test requirements for fire control equipment and components, and hence serve as a guide for simulating environmental extremes for similar items of a developmental nature.

Selection of materials €or fire control equipment must include consideration of:

1. Stress--and the designer must fully understand the stress concentrations, distributions, and fluctuations in each part.
2. Impact
3. Abrasion resistance
4. Corrosion resistance
5. Effects of high and low temperatures
6. Weight (portability).

The limitations imposed by materials in one or more of the foregoing ways are often the controlling factors in a design. More than in most designs of electrical and mechanical equipment, the design of fire control equiprnent requires careful attention to choice of materials and a careful weighing of opposing considerations to achieve optimum trade-offs. A design problem relating to strength vs corrosion resistance, for example, may lead to such considerations as the following:

1. Pure aluminum will meet the gven corrosion problems anticipated for a particular item but may not be strong enough to withstand anticipated stresses,
2. High-strength aluminum alloys will withstand the stresses but they are not as corrosion-resistant as the pure metal.
3. High-alloy steels, such as the plain chromium and chromium-nickel stainless steels, have good strength and corrosion-resistant properties but they introduce additional weight.
4. A compromise--use of steel in the portions subject to greatest stress and aluminum in the positions most exposed to moisture--may create problems of galvanic action between the two metals.

In the above example, the design engineer must be able to "plug in" quantitative data before he can make the proper choice. However, it may also be necessary to compromise between reliability of material and availability in time of war (see par 5-1.3).

5-3.3 DESIGNING AGAINST ENVIRONMENTAL DETERIORATION

5-3.3.1 Basic Approaches

From the standpoint of design, improvements against the destruction effects of environment are continually being made. Development programs are in progress in an effort to develop component parts that will successfully meet this often severe problem. There are three basic approaches:[14]

1. To develop good protective devices while using conventional components and materials in the equipment itself. Conventional equipment can be protected from severe conditions by shielding, insulating, and cooling. This approach does not preclude the use of advanced techniques; for example, it may be possible to achieve cooling by utilizing the latent heats of melting or evaporation. Similarly, shock forces on conventional components can be greatly reduced by mounting and supporting them by the best techniques (see par 5-3.4.1 and par 5-3.4.2). Using this approach, however, the designer must make judicious selection of the conventional materials and components to be used in the equipment.

2. To extrapolate known data on the properties of materials, to use known concepts and design techniques, and to develop, by experiment and testing, components that will withstand environmental extremes. In electronic equipment the designer might rely on high-temperature materials such as ceramics to replace materials affected by heat. Materials satisfying high-temperature requirements may also be capable o€ surviving under other severe environmental conditions. For example, ceramic materials not only resist high temperature but also tolerate nuclear radiation and shock.

3. To develop new concepts of constructing materials (especially electronic materials), basically new devices, and wholly new design concepts. This approach is in the developmental stage. For example, to enable the equipment designer to devise com-

ponents capable of survival in the desert environment, considerably more data must be obtained on the electrical and physical properties of the available materials, followed by the development of new and superior materials.

5-3.3.2 Weatherproofing

Weatherproofing is a complex and difficult subject since it cuts across many disciplines and is based in large part on experience. Fortunately, the design engineer has recourse to military specifications which reflect in detail the research and experience gained by the Army and other organizations in weatherproofing fire control equipment.

Weatherproofing can be accomplished in a number of ways, and the designer must adapt one or more of these to suit his needs. These include the use of corrosion-resistant paints and finishes, sealing, potting, wholly or partly enclosing, and the careful choice of materials and components. These methods are described in the paragraphs which follow. Also, it is important to remember that fire control equipment will be transported through and used in swampy terrain, sandstorms, deep snows, and the like; accordingly, it must be proofed from below and all sides — not just the top — against moisture (including salt water), blown sand, and snow.

1. Paints and finishes. Processes available include (a) mechanical finishing for smooth, polished surfaces; (b) hot dip, electroplating, and molten metal spray processes using copper, nickel, chromium, tin, cadmium, zinc and lead plating, and aluminum and anodic coatings; (c) case hardening processes; (d) phosphate and black oxide chemical coatings; and (e) numerous organic paints, including top coats and primers. These items are covered in MIL-STD-171B[15] and MIL-STD-194A(ORD).[16]

Corrosion protection is best furnished by an inorganic surface treatment (chemical or electrochemical) plus an organic finish (primer and top coat). Organic coatings should be baked on where possible, rather than air dried.[2]

Often, some compromise must be made between optimum corrosion resistance and other properties that can be imparted or improved by various surface treatments, including wear resistance, friction characteristics, hardness, conductivity, reflectance, and appearance (or more often disappearance, i.e., camouflaging). On working surfaces, chemical treatment or electroplating (without the addition of organic paint) offers a considerable degree of protection against corrosion.[2] Finally, the designer should remember that finish processes generally build up the surface; where close tolerances are involved, dimensions for the final, surface-treated items should be specified--not only in the working drawings but in the instructions for overhauling the equipment.

2. Seals and enclosures. It is a practical necessity to seal optical elements of a fire control system from moisture, dust, and fungus spores; gyroscopic elements and some electronic components should be similarly sealed. Within a sealed element the pressure varies with respect to ambient pressure as a function of temperature, altitude, and normal changes in barometric pressure, These variations can cause "breathing": the inward or outward movement of air, together with the transfer of moisture, fungus spores, and dust. If there are no mechanical inputs or outputs, use of good static sealing techniques can eliminate breathing and produce a true hermetic seal. These techniques include:

a. Sealing the pores of the case by impregnating. MIL-I-13857(ORD) specifies proper impregnations for metal castings, and Type I of the specification defines correct impregnation processes for fire control equipment, (Impregnating with varnishes or other volatile materials should be avoided within optical equipment since they will eventually vaporize and form a film on the glass.)

b. Sealing any access openings by keeping the cover contact area at a minimum and by use of various coatings and gasketing materials, with the cover held on by positive (spring or screw) pressure. For example, polytetrafluoroethylene-coated parts or covers with lips of this material make effective seals when held on by pressure. Grooves around the area to be sealed, filled with viscous gasket

material after the cover is in place, have been used with success on some types of equipment. This method should be used as a last resort only because removal and replacement of the covers then becomes a major problem.

c. Filling the interior with inert gas (nitrogen or argon), slightly pressurized so that if there is any breathing, it wilt be outward.

For rotating or sliding shafts, good dynamic seals can be gained by synthetic rubber O rings or other seal rings. Where limited longitudinal motion only is involved, a flexible metallic bellows provides a good seal. However, no truly hermetic dynamic seals have been developed as yet; therefore, it is good practice to sectionalize for maximum protection. For example, place electronic components in a hermetically sealed compartment, and place mechanical units in an adjacent, dynamically and statically sealed compartment, with hermetic seals for electrical interconnections.[2]

It should be assumed that all sealed equipment will be carried at some time or other in unpressurized air transport planes for periods of several hours at altitudes up to about 25,000 feet (which will create an internal pressure of about 10psi for an item sealed at sea-level pressure) and will be operated for indefinite periods at high altitudes in mountainous terrain.

Potting is a special form of hermetic sealing in which the elements concerned are surrounded with a special potting compound that serves to insulate the elements against variable temperatures and stray electric currents. It is an excellent means of sealing sensitive electric and electronic elements that at the same time do not require maintenance or replacement of component parts. In the application of this technique one must ensure that the components are not damaged by the potting procedure.

There are many other degrees of enclosure defined in terms of the amount of protection involved (such as drip-proof, dust-tight, or explosion-proof), the contents (such as oil-filled), and the means of cooling the equipment (such as water- or air-cooled). MIL-STD-108D[17] defines the basic requirements for enclosures of electric and electronic equipment, and further information is generally given in the detailed specifications for various types of equipment.

In any equipment that is not sealed, means should be included for rapid draining in the event that water does get in. Particular care should be taken to avoid pockets where water can gather in exposed equipment and to prevent such protective items as sleeves and conduit from reversing their functions and acting as reservoirs.

3. Choice of materials. Corrosion-resistant properties of materials are given in the various detailed specifications for the individual materials. The general specification for fire control materiel, MIL-F-14252-(ORD),[9] serves as a good index of detailed specifications for the various materials, and MIL-STD-454[18] contains a convenient list of fungus resistant materials--an extremely important property for equipment that will inevitably be used in the tropics. In general, the use of organic materials should be avoided because of their vulnerability to moisture.

It is further important to remember that the use of dissimilar materials will cause unequal expansions that may be considerable in the extremes of temperature to be encountered. Components made of materials with different coefficients of expansion should not be packed too close together or connected rigidly. Likewise, moisture promotes galvanic action between certain dissimilar metals and serious corrosion can result. (Magnesium alloys are particularly vulnerable but other metals may react disastrously also.) If such metals must be used together, plastic coatings may be used to provide electrical insulation.' Special caution must be exercised in specifying electroplating, hot dip, or molten metal spray, since they may introduce new sources of galvanic action.

5-3.4 MOUNTING AND DESIGNING FOR RESISTANCE TO SHOCK AND VIBRATION

Unlike designers of commercial equipment, the fire control designer is designing equipment that will be abused, subjected to severe shock and vibration, and may be poorly maintained. The challenge, therefore, is to produce equipment that will per-

form within the required accuracy under those conditions.

In no branch of ordnance materiel design is more knowledge, ingenuity, and care required than in designing fire control equipment for shock and vibration resistance. The problem more often than not is one of designing a sensitive and accurate piece of electrical, optical, and mechanical equipment that will be lightweight and compact, present a low silhouette, and yet can stand up under repeated shocks or vibrations with high-G forces and can be dropped from the air onto hard terrain.

With these end purposes in mind, the design engineer must consider alternative solutions to problems and be prepared to innovate and use ingenuity. A lightweight instrument in a well-designed case may be the answer; larger and more complex equipment may still lend itself to division into separate more or less plug-in modules, each with its own case. It may be possible to design the equipment so that it can be mounted separately from a gun or rocket launcher during the actual firing, or on the part of the gun, launcher, or vehicle that is least subject to shock. Such devices as the placement of hydraulic/pneumatic springs or cushions, or impregnated materials such as Fabreeka,* between metal plates have proved successful in practice.

Beyond this, however, the design engineer must make a thorough stress analysis, if possible; and if not, then he should at least have a qualitative appreciation of where the problems are. For example, in a recoilless weapon, the shock during firing tends to be radial from the tube axis; in a weapon with a recoil mechanism, it tends to be fore and aft along the tube axis.

Most design problems will divide conveniently into four parts:

1. Determining the over-all requirements — the maximum stresses the equipment will be subjected to and, equally important, the overall accuracy it must maintain during maximum vibrations.

2. Determining the concentrations and distributions of stress.

3. Selecting or designing shock mounts to alleviate shock and damp vibrations, (As noted in par 5-3.1.2, the designer should select standard, proven mounts, designing new ones only when special problems demand it.)

4. Designing or altering the equipment itself for maximum shock and vibration resistance.

In all designing against shock and vibration, it is extremely important to avoid resonance. In one case, an optical element shattered when the weapon on which it was mounted was fired because the high-frequency vibrations originating in the weapon tube were of the same frequency as the resonant frequency of the optical element. In another case, equipment secured at one end only on a self-propelled weapon was in resonance with vibrations set up by the treads of the vehicle at certain road speeds; it broke away from the mount at these speeds.[2] Such failures can normally be prevented by adhering to the principles discussed in par 5-3.4.1 and par 5-3.4.2.

5-3.4.1 Shock Mounting

Shock mounts basically store the energy of shock and vibration and release it at a slower rate. While the amount of shock absorption varies approximately with the stiffness of the mountings, care must be taken not to amplify vibrations by using shock mountings that may be in resonance with externally induced high-frequency vibrations. In practice, resilient shock mounts that are resonant at about 20 cps generally protect against shock effects and will not amplify vibrations since the chief external vibrations are at a lower frequency.[5] However, to allow a safety factor, it is well to design for resonance at up to 30 cps (see par 5-2.3).

With a low center of gravity, bottom mounting may be satisfactory. With a high center of gravity — e.g., in a unit of relatively uniform mass whose height is greater than its smallest base dimension — additional mounts should be placed near the top, or the mounts should be in a horizontal plane passing near the center of gravity, to offset rotation modes of vibration.[19]

* Fabreeka - Manufactured by the Fabreeka Products Company of Boston, Massachusetts - is material designed for the absorption of impact shock and vibration.

5-3.4.2 Shock- and Vibration-Resistant Design

Much can be done in the design of the equipment itself (aside from the mounting) to reduce the effects of shock and vibration.

First, the designer should be particularly careful not to specify detail parts that are unusually vulnerable to shock and vibration. This includes not only such obvious items as thin glass and certain plastics, but also the use of components that must be kept within specified distances of each other (to maintain a critical air gap, for example) and moving parts that may not function properly under shock or vibration. Standard worm and gear assemblies, for example, may drift 5° with repeated vibrations; accordingly, use a specially designed worm and gear, or use another arrangement, if accuracy is affected by this drift.

Second, the shock and vibration can be greatly reduced by proper choice and orientation of supporting members and arrangement of components. The paragraphs below, largely derived from Reference 19, may serve as a useful guide.

1. Decrease useless weight: Keep the weight of components as small as possible and make structural members as strong as possible. Shock forces applied to the structure are largely inertial forces, and their magnitudes are proportional to the masses from which the forces are derived. Note also that a heavy component distant from the mounting base will apply excessive shock forces to all structural members between it and the base; place heavy components as near the mounts as possible. Thus, fire control equipment should always be designed from the standpoint of minimal volume in order to live in the inherent environment of shock and vibration. Fortunately, this requirement is consistent with the severe space limitations that frequently exist, for example, in tanks. By designing initially for minimal volume, the need for redoing the design — often with short notice — because of reductions in the allowable volume can usually be avoided.

2. Orient longitudinal members properly, and use I-beams, channels and the like. When a member of rectangular cross section is positioned with the long dimension in the direction of the vertical load, its cross-sectional moment of inertia is eight times larger and its sectional modulus is four times larger than those of the same member when its short dimension is in the direction of the vertical load. Even better results may be obtained with I-beams, channels, and other members in which much of the mass is away from the neutral longitudinal axis. An effective method of increasing shock resistance is to design mounting feet, supports, levers, and other members with ribs or other stiffening arrangements in preference to the use of flat sections. It should be noted that many materials are stronger in one direction than the other, for example, cold-rolled steel is stronger in the rolled direction. In fact, the designer should always design with a knowledge of the structural grain. This is especially pertinent with highly loaded materiel such as leaf springs.

3. Introduce flexibility at strategic locations. This principle, which of course is the basis of shock mounting, can also be applied within the equipment in various ways. For example, consider a mass supported from a cantilever beam; if the flexibility of the supported end of the beam is increased, the natural frequency of the system is lowered and the forces acting on the mass due to shock are also lowered. Exactly the same principle can be made to work in a meter mounted to the rear of its case, which in turn is front-mounted on the instrument panel. In addition, the panel itself can be made flexible for a further reduction of shock forces acting on the meter. Advantage can be taken of the ability of a thin flat plate to flex without serious deformation; if an instrument is mounted on the plate and its mounting studs are staggered with the plate's mounting studs, flexibility is obtained.

The foregoing principles lend themselves particularly to the design of optical elements; by the proper choice of brackets, most shock can be absorbed before it reaches the optical sight.

4. Avoid excessive flexibility. Excessive deflections, beyond the point at which permanent deformations may result, will cause failures; or they may cause elements of the system to impact against one another. If the natural frequencies become too low, then increased stiffness is necessary. Appropriate trade-off of damping and flexibility must be carefully considered in each design

problem.

5. Design for maximum energy storage. When an elastic member is deflected, it is storing energy. To ensure that the system can store as much energy as possible — and conversely to prevent any portion of the system from exceeding its elastic limit — all structural members should be designed to experience the same stress throughout (insofar as this is possible).

6. Avoid parallel connections of flexible and rigid members. A relatively flexible edge-wound resistor, for example, was surrounded with a brittle ceramic insulator made in *two* square-ended sections that fitted against each other end to end. When the core flexed, the insulator chipped where the sections joined, and failed longitudinally. When the insulator was redesigned in smaller sections with rounded ends, so that they fitted together in something resembling a ball-and-socket joint, it became flexible enough to avoid excessive bending stresses. In this case, the solution was to retain the parallel paths, but make the rigid one more flexible. In other cases, a series connection provides the best solution. Consider for example, the meter noted in paragraph 3. The dial can be attached to a bearing carried by the pole piece, which in turn can be attached to the rear of the instrument case; the case itself is attached by its front end to a mounting panel. This series connection of rigid and flexible elements is often an effective way to increase resistance to shock and vibration.

7. Apply residual compressive stress to the outer portions of members. When a member is subjected to a bending moment, failure tends to start with a crack at the surface under tension. The residual compressive stress will oppose the tensile stress and reduce it at its maximum point,

8. Avoid welding. Welding has not stood up well under the severe shock and vibration likely to be experienced by fire control equipment. Accordingly, it should not be used unless and until better techniques have been developed and proven. This rule, of course, applies especially to supporting members.

9. Do not depend on friction or gravity. These forces may be more than adequate under normal conditions but will be overcome by the high inertial forces of shock and vibration, Note that a nut-and-bolt attachment through an elongated or oversized bolt hole is essentially a friction attachment; when the bolt elongates elastically during shock or vibration, the two connected items will slide out of alignment. Attachments of this type should be replaced or supplemented with such positive holding devices as keys, pins, or body-bound bolts. Press and shrink fits are also unsuitable for shock-resistant applications. Similarly, clamps, bolts, or other positive holding devices should be used to hold chassis, batteries, tool boxes, etc., in place on moving vehicles, even though the item is heavy and has a low center of gravity,

10. Anticipate the worst. It is impossible to design completely against the practically unlimited shock that equipment may be subjected to in battle. But damage and danger to personnel can be substantially reduced by careful attention to auxiliary means of retention and by "overdesigning" mounts and fasteners to ensure that equipment will not fly about and cause secondary damage if it is overstressed.

5-3.4.3 Prevention of Shock in Bearings

Bearings -- particularly ball bearings, which are the type used most commonly in fire control equipment, because of their good performance relative to size -- tend to fail more often due to shock or blast effects than to fatigue. It is important, then, to minimize shock loads in the design. The ability of a bearing to tolerate shock relates to its basic static load rating, i.e., the static radial load that will permanently deform the ball or race by an amount equal to 10^{-4} X the ball diameter at the most heavily stressed contact point, While the degree of shock in a bearing depends both on the magnitude and duration of the shock force, a bearing can tolerate greater loads than the basic static load rating, provided the duration of the shock force does not cause brinelling to occur under the applied momentary load. For example, the dead-load capacity of a bearing before actual fracture or failure has been found to be eight times its static capacity. In practice, however, the following limits should be observed:

1. With no shock expected, the equivalent radial load should not exceed twice the

static capacity.

2. Where moderate shock loads may occur, the equivalent radial load (without shock) should not exceed the static capacity.

3. Where large shock forces may occur, the equivalent radial load (without shock) should not exceed one-half the static capacity.

Care must be taken that the bearing is not loaded up during installation. In most cases, by specifying push fits (rather than force fits), the designer can ensure that he, rather than the installer, will control the loading. Also, looser fits permit bearing movement to reduce brinelling caused by unidirectional vibration.

The basic methods prescribed below, derived from Reference 20, can be applied to increase shock resistance in bearings. Manufacturers' catalogs are often an excellent source of additional, detailed information.

1. Special extra-thick outer races. These races minimize outer-race distortion and lessen the danger of fracture, and may be used with cam followers, pulleys, index pawls, guide rollers, and other applications where continuous or frequent radial impacts will be sustained. However, they do not materially increase bearing capacity; they occupy more space; and they may be difficult to obtain in times of national emergency.

2. Springs and resilient materials. When properly mounted, preloaded springs and resilient materials will absorb the energy of shock and reduce the impact between bearing and housing. When unidirectional shock forces are anticipated, the bearing and its shock-protecting device should be mounted in the direction of axial shock-thrust so that all the bearing balls will share the load,

Springs cushion the effects of shock by resisting the tendency of the bearing to move as a single mass with its adjacent components. They are useful, for example, in potentiometers, indicators, and slightly preloaded devices. With jeweled pivot bearings, preloaded springs provide shock resistance and proper cone pressure, and they also extend the duration of the shock load, thereby increasing bearing life. Under some circumstances, springs can be used as backup for bearings on the same shaft, mounted in a cartridge or housing, but separated from one another in order to prevent damage to bearings from axial shock thrust in either axial direction. It should be noted that this method requires additional space for operation of the springs. Because spring stiffness is a function of the degree of shock anticipated, the designer should also check the natural frequency of the system under consideration against the operating frequency to avoid resonance.

Resilient materials can be used for shock absorption and also for reduction of noise and vibration. Rubber rings are effective against axial shock of bearings in gear trains, motors, and slightly preloaded devices; they absorb the shock transmitted through the shaft before brinelling takes place between balls and race. However, caution must be used in specifying such arrangements for fire control equipment; they may leak with changes in pressure or vibrations induced by explosions and rough terrain.

Alternatively, a rubber mount may be jacketed around the outer race so that the bearing is protected from shock in all directions, i. e. , in directions normal as well as parallel to the shaft.

The designer should make his selection of the thickness and elasticity of springs, rubber rings, and jackets on the basis of the operating requirements. Note that some resilient materials may deteriorate with the use of lubricants.

3. Preloading bearings. Bearings may be preloaded by various methods: for example, by grinding the bottom race of one bearing to reduce its width and applying the preload through two equal-length spacers— one for each bearing race — by means of a threaded collar that can be tightened until all clearances are taken up; or by matching pairs or "duplexing" of bearings, with the preload applied axially to either the inner or outer race. For specific recommendations, bearing manufacturer's catalogs should be consulted.

4. Specialized methods. These include floating units—for example, gyro units shock-mounted and suspended in a silicone fluid—and vibration isolators that make effective use of viscoelastic materials, such as rubber, nylon, rayon, and fibrous materials.

5-3.5 LUBRICATION

The best practice in fire control design is to keep lubrication to the minimum required for reducing friction, cooling, and protecting against corrosion; and to use permanent sealed lubrication systems wherever possible. Fire control systems tend to have rather light components, moving often at low speeds. High torque, applied for rather short periods, is typical of much fire control equipment. Except for heavy items, such as radar antennae or high-speed components, fire control equipment lends itself to minimizing lubrication which is desirable for the following reasons:

1. The difficulty of maintaining oils and greases at the proper viscosity in the extreme temperature ranges encountered by most fire control equipment.

2. The chance of human error or neglect. Men at the organizational level (where routine lubrication must be carried out), are relatively unskilled in the maintenance of fire control equipment. They could ruin conventional lubricated equipment by failing to put in the correct amount of the proper lubricant for the climatic conditions at the right times. (Over-lubrication can be extremely harmful to ball bearings, for example.)

3. Logistic problems. Lubricants are another item to transport and it may be difficult to keep a mobile unit supplied with the correct ones. Lubricants tend to deteriorate in long-term storage--a point of particular importance in military applications (see par 5-3.6).

4. Harmful effects. Lubricants must be kept away from many fire control items, particularly optical equipment, whose optical characteristics can be seriously distorted by even a thin film of oil.

5. Loss of plugs under field conditions, or loss of oil when equipment is being transported at altitude in an unpressurized cabin.

6. Change in environment. It is vital to recall that much of this equipment will be carried at altitude and landed or air-dropped for immediate use. It will thus undergo rapid changes in environment, with no opportunity to add or change lubricants before use.

For these reasons, the Army is omitting all lubrication fittings and oil holes from all fire control equipment except heavy or otherwise exceptional items. This action has been taken to prevent lubrication of such equipment — e.g., telescopes, mounts, and many other items — anywhere except in the shops where it is assembled; there, it is lubricated with grease conforming to Specification MIL-G-3278.

Plastics may often be used to good advantage to reduce the need for lubrication, provided their limitations are fully appreciated. Some applications are:

1. Combined plastic and metal gears
2. Sintered fluorocarbon plastics — e.g., nylon and Teflon — or nylon/graphite bearing surfaces
3. Teflon coating on metal.

However, some plastics tend to soak up moisture and expand with heat; they, therefore, should be used with caution where close tolerances must be maintained. Teflon is relatively stable; nylons vary in hygroscopic characteristics. Sintered bearings freeze in long-term storage and cannot be used for fits in the order of 0.0005 inch. Use molded rather than machined plastics because of expansion due to heat.

Where oils and greases are required, the following principles apply:

1. To meet wide variations in temperature, the designer should specify an oil with a pour point lower than the minimum operating temperature and with the highest viscosity that can be tolerated at the low temperature. When the temperature reaches operating values, the oil will approach the proper viscosity.

2. Roller bearings generally require lubricants of higher viscosity than ball bearings. Ball bearings often require a small amount of silicone or other specified oil. Note especially that stainless-steel ball bearings (now readily available) minimize the need for lubrication.

3. The use of grease instead of oil will often cut down maintenance, reduce the size of bearing enclosures, and maintain a more effective seal against contaminants. However, grease lubrication is not very effective for cooling and tends to shorten bearing life at high speeds.

4. It is extremely important to specify greases that will stand up under climatic extremes (see MIL-G-3278).

5. To prevent loss of lubricants at al-

titude or due to loss of a plug or other fitting, an integral, self-closing device — such as an Alemite or other fitting with a spring-loaded ball check — should be incorporated for adding lubricants. Such fittings also offer some protection against overlubrication.

6. Barriers should be employed to keep oils and greases away from optical devices. Volatile lubricants should not be used or kept anywhere in the vicinity of optical devices.

7. Finally, it is extremely important to choose lubricants that are chemically compatible with everything they will come in contact with--seals, gears, thrust rings, bearings and all other detail parts.

For further information on this subject, see Chapter 16 of Reference 21.

5-3.6 STORAGE

With many items of commercial equipment, the designer can specify storage under carefully controlled conditions and can set quite limited maximums on permissible time in storage. Fire control equipment, on the other hand, may be stockpiled for long periods and storage facilities vary from good warehouse protection to open storage in extreme climates. Three courses should be considered to ensure that equipment will be fit for use after long storage:'

1. Choose materials and components that will not age or corrode after long exposure to dust, humidity, extreme heat or cold; and pot or hermetically seal components that do not meet these requirements.

2. Design the equipment so that it can be easily disassembled, cleaned, and lubricated in forward areas by trained personnel to ready it for use.

3. Specify packing procedures that will preserve the equipment. Government specifications on various types of analogous equipment are a good source of information on procedures such as charging with inert gas, wax-dipping containers, canning, and using drying agents.

5-4 DESIGNING FOR MAINTAINABILITY

5-4. 1 BASIC PRINCIPLES

In designing for maintainability, it is important to understand what kind of maintenance and repair can be performed at each maintenance level. The design engineer should examine the following documents, as specified for the equipment being designed or for similar equipment:

1. The appropriate Department of the Army Supply Catalog which lists spare parts assigned to each maintenance level.

2. The maintenance allocation chart.

3. The specifications or instructions for preparation of technical manuals for the equipment, plus the actual technical manuals for similar equipment. These publications describe in detail the maintenance procedures performed at each level.

The situation differs for different types of equipment and operation. At a large, fixed installation, depot-level repair facilities for fire control materiel may be established on the premises. With fast, mobile combat units, maintenance may be limited over long periods to what the operator can perform.

In general, however, the Army carries out maintenance as follows:[22]

1. Organizational maintenance, performed by the using organization. This is limited to such tasks as preventive maintenance inspections (largely of a simple, visual nature), cleaning, servicing, lubricating, adjusting, and replacing certain easily disassembled parts. This level includes both operator maintenance (first echelon) and maintenance by organization maintenance personnel in small unit shops (second echelon).

2. Field maintenance, performed by units organized for direct support of one or more using groups. This level includes both mobile or easily moved shops in close support of combat units (third echelon) and more elaborately equipped fixed shops (fourth echelon). Field maintenance is generally limited to trouble-shooting, testing, and replacement of unserviceable piece parts, subassemblies, and assemblies.

3. Depot maintenance (fifth echelon). This level, operating in fixed installations, is normally capable of major overhauls. It can be considered equivalent to returning equipment to the factory.

To the design engineer, the Army maintenance setup — as well as the circumstances that dictate it — means that the equipment

must be designed and packaged so that:

1. All preventive maintenance necessary to keep the equipment in good operating condition can be performed by unskilled men operating under adverse conditions.

2. In the event of equipment malfunction, the trouble can be traced with a minimum of special instrumentation to a subassembly or component that can be easily replaced as a unit under combat conditions; or, the trouble can be temporarily bypassed or corrected by simple adjustments with simple tools, clearly graduated knobs and the like.

3. Vital subassemblies can be repaired rapidly by replacement of defective parts at the field-maintenance level, for quick return to the combat unit.

4. Complete disassembly and overhaul of the equipment need be performed only at widely separated intervals, to avoid excessive long-distance shipments to remote depots and to avoid overtaxing depot facilities.

A number of steps can be taken to achieve these goals, and to increase ease of maintenance and decrease maintenance time at all echelons. They must be considered at every stage of design; Army fire control materiel that cannot be maintained properly under the conditions of use cannot be considered satisfactory, and a drastic redesign may be required if the equipment is found in the end to be wanting in this respect.

Further information can be found in Reference 21. A good succinct discussion is made by Wood.[23] The paragraphs which follow briefly summarize various aspects of the subject.

5-4.2 ACCESSIBILITY

Accessibility can be properly designed into materiel only if the following points are analyzed correctly:

1. What routine maintenance will be required? Filling and drain plugs, fuses, lamps, oil filters, grease fittings, optical lenses requiring frequent cleaning, meters, gages, levelometers and the like used in checking the condition of the equipment, and any other item used in preventive maintenance or requiring frequent replacement should be easily accessible. A hard-to-reach item is likely to be neglected.

2. What trouble shooting will be required at the organizational maintenance level — i.e., under combat conditions — to restore the equipment rapidly to operating condition?

3. Which components, assemblies, and subassemblies will be removed and replaced as units at the organizational maintenance level? Which will be adjusted at this level?

4. Similarly, what trouble shooting will be carried out at the field-maintenance echelons, and what parts and subassemblies are subject to removal or adjustment at that level?

A good analysis of accessibility requirements in the light of these considerations will often result in a more compact design since the design engineer will no longer be providing access to items that will only be handled during a complete depot disassembly.

To meet the access requirements revealed in the preliminary analysis, the design engineer should consider the points which follow. While obvious in themselves, they often require a great deal of thought and care to ensure proper accessibility.

1. Safety. Good access means that the maintenance man can perform whatever operations are necessary without risking exposure to electric shock, moving parts, and other hazards. This is particularly important for maintenance that may be carried out under adverse conditions in the field. The designer must ask himself what the operating status of equipment will be during the particular maintenance task. If the equipment may be operating or energized, the source of hazard should be isolated from accesses; if not, then an interlock or warning plate may suffice.

With small units (which comprise most fire control equipment), safety can be ensured by separating adjustment and check points from high-voltage components by insulation barriers.

Note that under some circumstances--particularly with a wet operator handling wet equipment in rain or snow--voltages in the order of 110 volts can be fatal..

2. Access for removal, replacement, and inspection. Components, and modular subassemblies that are to be removed as units should be mounted and fastened so that

they can be removed easily, with no interfering components in either the fire control equipment itself or adjacent equipment. Much fire control equipment consists of compact chassis of various configurations. Consideration should be given to mounting electronic controls and components on the inside of the front panel or on a hinged interior panel that can be swung open to expose all items that may require adjustment or replacement; or they may be installed in parallel modules, each one removable from the front or rear without removing any other components. In large control chassis, pull-out drawers (with safety stops) may provide the best access. Arrangements of this type also facilitate visual inspection of major circuits. Care must be taken in all such arrangements to retain the "dead front" features of the equipment, to avoid shock.

By such devices as the use of captive screws and spring-loaded fasteners, the design engmeer may be able to apply some of the same principles to the design of mechanical and optical devices.

3. Degrees of accessibility. Controls used in normal operation should be accessible at all times, and (with their associated indicators) should be installed where they can be operated with comfort and ease (see par 5-5.1.3). Other items — such as battle-short switches — are for emergency use, to be operated only by the officer in charge or someone designated by him. Similarly, some items are to be adjusted or repaired only by specially trained technicians. In industry, items of the last two types are often protected by locks, with authorized persons holding the keys. In fire control equipment, where adjustments or emergency operations may have to be performed rapidly under fire, locks should not be used on any item that will require operation or maintenance at the organization level. Instead, such items should be protected with warning signs and should be installed so that a simple intervening operation is required for access, to prevent casual operation--removing a cap to operate an emergency switch, for example, or removing a panel to make adjustments.

4. Human factors. Ease of access reduces the probability of human error. If the maintenance man must remove an interfering component in order to perform routine maintenance or get at an unserviceable part, the additional operations increase the opportunities he has to make a mistake. All too often, designers include ready access to an item but do not allow enough room for the operator to perform the necessary tasks efficiently, or force him to use his hands at odd angles or to perform delicate operations blind (because he cannot reach and see the object at the same time), or provide gages or dial markings that are not easily visible or subject to misinterpretation. A very large body of human engmeering data has been accumulated to assist the designer (see par 5-5.1.3 and Ref 21), and with the help of this and the use of common sense the designer can greatly increase the ease of maintenance.

5. Environment. If maintenance must be performed in very cold weather (with gloves), driven rain, snow, or dust, or in the dark, ease of access becomes especially important. At the same time, consideration must be given to protecting vulnerable parts from exposure while an access opening is removed.

6. Access opening covers. The general rule is this: specify the easiest type to remove and replace that will fulfill the requirements of strength, protection of the equipment, and safety of personnel. In increasing order of difficulty of removing, they are: (a) no cover at all; (b) hinged covers, preferably with bottom-mounted hinges or with built-in supports to prop the cover open; (c) cover plates with quick-opening captive fasteners (these may be used where there is no swinging room for a hinged cover); (d) screwed-on covers, with the minimum number and largest size of screws that strength considerations permit. Doors and covers may be spring-loaded to ensure greater tightness against dust and moisture.

For dial faces and other visual inspection points, a plastic window or break-resistant glass window may be specified, depending on the anticipated optical deterioration of the plastic under field conditions and the stresses that will be imposed on the window. A metal cover plate with quick-opening captive fasteners is suggested for extreme environments where plastic or glass may not stand up.

7. Location and geometry of access openings. While rectangular or round open-

ings at the points nearest the items to be reached may be desirable in most larger equipment, the designer must also consider (a) whether an odd-shaped opening may not be best suited for the particular task or for removing an odd-shaped component, and (b) whether the opening is from the angle that leads to the easiest face of the component to work with. Where visibility may be a problem, a window may be provided in addition to the access opening.

5-4.3 STANDARDIZATION OF PARTS

As pointed out in par 5-3.1.2, the use of nonstandard or newly designed components may decrease equipment reliability. Accordingly, it may also increase the required maintenance.

Nonstandard components contribute to equipment or system failure in the first place because their low demand requires long storage time, a factor that brings about deterioration. In the second place, with more parts to become familiar with, maintenance men will have more difficulty installing, handling, and maintaining the equipment. Finally, small-quantity production of nonstandard items is characterized by lack of uniformity, and their use not only makes replacement spares more difficult to obtain but also burdens maintenance support with additional parts. The high failure rate of nonstandard components has led to a Department of Defense Standardization Program to guide equipment designers.

In the development of complex fire control systems or equipments, the designer should apply standardization at all stages of design, and specify standard components and subsystems wherever practical. The result will be ease and rapidity in replacing and interchanging parts and subsystems under all conditions.[24]

5-4.4 INTERCHANGEABILITY

In addition to standardization, the designer should work for maximum interchangeability of components and subassemblies. This means that for different parts of the same equipment, the same components should be used wherever they will meet the requirements (even though the functions of assemblies in which they are installed may be quite different). It also means that components that have been used in other fire control equipment should be specified for the one being designed wherever possible.

Maximum use of interchangeable parts leads to (1) efficient, uniform maintenance procedures, (2) fewer repair parts to support the equipment at every maintenance echelon, and (3) fewer failures, because components that have been time-tested in fire control applications are used to the maximum extent.

The interchangeability concept should be carried out for all subassemblies and components that will be replaced as units at any maintenance level (and especially at the organizational echelons). For example, if an amplifier is always going to be replaced as a unit, it doesn't matter if the tubes within different units differ, provided the electrical and physical characteristics of the units are the same in every respect that is important in the application; but if the amplifier may be repaired by replacing tubes, they too should be interchangeable.

Great progress has been made in recent years in standardizing parts through the Federal Stock System. Each Federal Stock Number (FSN) represents a component, subassembly, or assembly with inputs, outputs, and physical characteristics that are uniform in all important respects. Thus, in most applications, items with the same FSN may be considered interchangeable, even though they may be manufactured by different companies and may differ in unessential details. Each FSN is backed with a complete description of its characteristics, called an Item Description. The items, their stock numbers, and description patterns are listed in Reference 25. Manufacturers' part numbers may imply a similar interchangeability, though the situation depends on how rigorously the individual company maintains its parts documentation system.

5-4.5 STANDARDIZATION OF TOOLS

It is also important for the design engineer to design equipment that can be maintained and overhauled with as few tools as the end requirements permit, and with standard tools wherever possible. This is par-

ticularly important in the maintenance to be performed at the organization and field levels; but at the depot level, also, the use of elaborate test sets, special jigs, and the like can often be avoided if thought is given to this matter during design.

The tools that will be required should, in fact, be considered as an integral part of all phases of design. Often a design concept may have to be discarded early because the end result will be a requirement for maintenance or test equipment that is too heavy or elaborate for use in the field.

Unnecessary tools can often be eliminated by small changes in the design, For example, it may be discovered that by specifying a wider variety of fasteners than necessary, the designer is adding to the number of hand tools required. The minimum number of fasteners consistent with strength should be used, and they should be as uniform as possible.

A detailed discussion of the types of tools for fire control materiel maintenance is given in Reference 21.

5-4.6 AIDS TO DIAGNOSIS

In some electronics equipments, it has been estimated that 80 percent of down time is spent in diagnosing the trouble.[23] Accordingly, a prime responsibility of the design engineer is to make trouble shooting as quick and easy as he can. In order to do this, he must first predict where troubles are most likely to occur, the best ways to locate them, and the probable relative frequency of their occurrence. This should be done as early in the detail design stage as possible because (1) he may at this time discover the need for some long-lead-time test equipment and (2) trouble shooting procedures will affect not only the location of check points but also may affect the arrangement of subassemblies, as explained below.

Predicting troubles may be quite difficult, particularly for the design engineer; he often tends to predict troubles that, because he is aware of them, he has already designed out of the system. It may be well to enlist the aid of a maintenance engineer or technician with much field experience and to consult field reports.

The next step is to determine the instruments and equipment necessary to trace the trouble to (1) a modular or easily removable subassembly at the organization level and (2) piece parts or lower subassemblies at the field level. For step 1, built-in aids should be supplied wherever possible; for step 2, various items of special instruments and test equipment may be used. However, in either case, it is important to tailor the procedures to make use of equipment available at the echelon in question (see par 5-4. 1).

The type of diagnostic aid varies with the equipment. Examples include built-in boresighting aides with easily moved knobs in elevation and azimuth; valves for isolating portions of fluid systems, and gages for reading pressure, liquid level, etc., in the isolated portions; jack points on terminal points in relays, switches, P-C cards, etc., as appropriate; and in elaborate systems (such as computers), automatic checking systems. Details should not be overlooked, such as illuminated dials with adjustable luminance to suit varying ambient lighting conditions and meet blackout requirements. As noted in par 5-4. 2, every step should be taken to make checkpoints accessible.

It was pointed out earlier (par 5-3. 1. 1) that redundancy may be a practical means of increasing reliability. Similarly, the inclusion of parallel or standby circuits will greatly facilitate trouble shooting; independent checkout circuits should also be incorporated wherever practical.

At some point, it may become apparent that the equipment as originally conceived was not arranged for optimum maintenance. For example, personnel at the organizational level may only be able to trace a trouble down to a subsystem that cuts across two or more replaceable physical assemblies. In the detail design, the logical subdivisions from a trouble-shooting and repair point of view should be made to coincide with the physical units that can be replaced easily.

5-4.7 SIMPLIFYING REPAIR

Repair is simplified by providing accessibility (*see* par 5-4. 2) and by providing the most easily removable fasteners consistent with other design requirements. Items that are subject to breakage or frequent replacement should be particularly easy to replace.

For example, levelometer vials should never be designed as integral units with castings, but should be removable as a unit, with a standard screw driver. Particular attention should be paid to wiring harnesses, piping systems, and the like, to insure that a minimum number of disconnections need be made in removing and replacing a component or subassembly. The ultimate in achieving these goals is modular (or unitized) construction.

5-4.7.1 Modular Construction

Modular construction is the division of the equipment into plug-in or otherwise easily removable modules. It reduces and simplifies maintenance by reducing the need for special skills, tools, and test equipment. In fact, when properly applied, it achieves the chief aim of maintainability: it increases the availability of equipment by reducing down-time to a minimum. It also practically ensures that the level of maintenance demanded from each echelon will be commensurate with the equipment and skills available there. Modular construction further lends itself to mass production and contributes to interchangeability.

Modular design is particularly applicable to complex electronic equipment but it is also applicable to complex optical and mechanical equipment. It generally is not suited to simple, light-weight equipment (which is better replaced as a unit). Furthermore, in any equipment, certain repairs are best made by replacing detail parts, e.g., fuses, rather than modular subassemblies.

The designer should divide the equipment into the smallest number of modules that will still preclude the need for special skills, tools, and test equipment — in other words, design to a point just short of requiring skills, tools, and test equipment beyond the capabilities of the echelon of maintenance. As noted in par 5-4.6, the division into modules is closely tied in with the degree to which a trouble can be diagnosed.

For optimum equipment packaging, the designer should, if possible, devise modules approximately cubic in form. All the component parts that are to fit within the module should be approximately uniform in size and shape. Each module should be composed of component parts that are most conducive to the performance of a given €unction. While it may be desirable that they should also be capable of multiple usage, this goal is less important than the others. If possible, the components of a module should have the same "wear out" life.

It is also recommended that the designer arrange the modular units to permit operational testing when the units are removed from the equipment. For ease of handling, these units should be sufficiently light and small that each can be carried by one man. The designer should provide handles, properly designed, for modular units weighing 10 pounds or more.

5-4.7.2 The Throw-Away Concept

With the increased use of mass-produced modular subassemblies and components, the throw-away concept--replacing a defective or malfunctioning inodule and discardingit instead of repairing it in or out of the equipment--has become increasingly practical. Though it seems wasteful on the surface, this approach has inherent advantages that in the end may result in a substantial net savings,[26] as well as simplified and more effective maintenance. These advantages are:

1. More compact design. A subassemblythat is to be repaired economically must incorporate accessibility to the individual detail parts, with a minimum of removal of interferingparts. With the throw-away concept, this requirement is deleted; the subassemblycan be packagedin as little as half the volume and can be constructed from the insideout in the order mostconducive to efficient manufacture.

2. Simplified logistics. This is attained not only because compact items are easier to ship but because the complex problems are avoided of returning a subassembly to the correct maintenance echelon and ensuring that the detail parts are stocked there to make repairs.

3. Improved reliability. The throw-away concept permits increased use of sealingand potting to protect delicate and sensitive components from moisture, dust, solar radiation, and shock and vibration. This ad-

vantage is particularly pertinent to fire control equipment. It has been estimated that throw-away design reduces failures by 40 percent.

Clearly this concept is not frugal or otherwise beneficial when detail parts are expensive, difficult to manufacture, or in short supply, or under many other circumstances. The economics can become quite complex but they must be analyzed as carefully as possible before the concept is accepted or rejected. To improve the economics of this concept, the components within a module should have approximately the same "wear out" life wherever possible. For details on cost factors, see Reference 21.

5-4.7.3 Ease of Maintenance in Extreme Environments

The fire control designer must keep in mind that his equipment must be able to operate effectively in the environmental extremes listed in par 5-2.2 through 5-2.4. This means that the equipment must be designed so that relatively untrained men can perform field maintenance to keep it operating under such conditions as arctic cold, rain, snow, blowing sand, tropical heat, and the vibrations encountered on moving vehicles or in the vicinity of guns firing. The difficulties may be increased by gloves and other protective clothes that will be worn in some environments.

The rules and suggestions for ease of maintenance and operation throughout this chapter were in large part devised to help overcome the problems of extreme environment. In addition, it may be necessary to make special provisions for maintaining equipment in particular environments, such as:

1. Portable shelters or semitrailer installations for certain fire control systems or elements of systems, with provisions for heating and cooling.
2. Heaters and blankets to keep equipment in standby, "ready-to-operate" condition in extreme cold, and to protect elements that might be damaged by freezing.
3. Integral protective caps for optical elements, meter faces, and other instruments and components that are vulnerable to blown sand, snow, sleet, etc.
4. Fans directing the flow of air from hot equipment away from the operator.

5-5 DESIGNING FOR EASE OF OPERATION AND FOR SAFETY

5-5.1 HUMAN FACTORS IN MATERIEL DESIGN

5-5.1.1 Basic Principles

Military equipment has become so complex, precise, and fast acting in recent years that it threatens to exceed the abilities of humans to operate it. Human engineers, who are psychologists specializing in man-machine relationships, have become increasingly involved in designing equipment so that it can be operated effectively. Human engineering relies on basic and applied psychological research--basic research in the capabilities and limitations of human faculties, applied research in the behavior of these faculties in specific situations.

Psychological research differs from research in other fields in its dependence on the subjective reactions of humans, For example, one can determine the wavelength and energy of light from a source by objective investigation; but the related properties of color and brightness are subjective, and exist only in the eye (and brain) of the beholder. Similar relationships exist between the wavelength of sound (objective) and its pitch (subjective), the energy of sound and its loudness, thermal energy and the sense of hotness or coldness, etc.

It must not be thought, however, that psychological research is in the end subjective and opinionated. For example, a scale of brightness has been established by recording the observations of a large enough cross section of humans to be statistically valid. Since each point on the scale can be defined in terms of purely physical values, the same brightness can be recreated at any time and the scale becomes a permanent objective research tool. The same techniques have been applied to other visual functions, to hearing, muscular capabilities, etc. Once the basic scales have been established, more refined research can be carried out. For example, the ability of individuals or special groups of people to discriminate between

small changes in brightness can be measured under various conditions of preadaptation.

The branches of human engineering that are of chief interest to the designer of mechanical, electrical, and optical equipment are:

1. VISION

Light discriminations

Brightness sensitivity (the ability to detect a dim light)

Brightness discrimination

Color discrimination

Spatial discriminations

Visual acuity (the ability to see small objects or distinguish details or changes in contour)

Distance judgment (particularly depth perception--the ability to distinguish differences in distance)

Movement discrimination

Temporal discriminations

For example, the ability to distinguish the individual flashes of flickering lights

Dark adaptation

The ability of the eye to perform at night after various periods of adaptation. This field of investigation cuts across the above three, measuring various discriminations under the special conditions of dim light.

2. AUDITION

Pitch discrimination

Loudness discrimination

3. MOTOR PERFORMANCE

The ability to perform tasks involving bodily movements, including:

Speed

Accuracy

Force (for example, the force the operator can exert in pushing a lever)

4. PROPRIOCEPTION

The ability to sense through certain bodily receptors (muscles, tendons, semicircular canals, etc.), the position of the body and its movements in space and time.

5. SKIN SENSITIVITY

The sense of touch, of particular importance to the engineer designing fire control equipment that must be operated effectively during blackouts.

In addition to the data available on the foregoing faculties and their interrelationships, the engineer has at his disposal considerable information on how they are affected by learning, intelligence, and special conditions such as fatigue.

There are three possible ways of overcoming the human problems in operating equipment:

1. Choose operators who are peculiarly suited for the equipment. The Army is, of course, selective in its recruiting--men with gross visual, auditory, or other defects are not accepted--but it does not normally give highly refined tests, nor can it be ultraselective in assigning the ideal man to the task. This problem will be multiplied in time of war. *So* while the designer can take comfort in the fact that he does not have to consider the handicapped, he must still design for operators whose faculties may be average or perhaps somewhat below average.

2. Train operators for their specific tasks. This can be an effective way to overcome human engineering problems but, as already explained (see par 5-1.1), the Army must depend largely on relatively untrained personnel.

3. Design the equipment for ease of operation (the best solution). The design engineer cannot expect to become a human engineer — i. e. , a graduate psychologist with considerable post-graduate training — but he has resources that can help him solve many of the problems of designing for the human operator. These include: (a) common sense, (b) publications presenting psychological data for use by design engineers, and (c) the skills of human engineers.

5-5.1.2 Use of Common Sense

Common sense is often neglected in design. A familiar display of the lack of it is the automobile designer's use of identical knobs for lights, windshield wipers, ventilator, and so forth, with the positions of the knobs alternated from one model of car to another. By using radically different shapes of knobs (particularly for critical operations) and keeping them in the same relative position that they occupied on other, similar equipment, the designer can contribute quite simply to ease of operation. A few other examples of this kind of thinking are:

1. Design for visibility. (a) Place gages

where they canbe readby the operator in the normal position while he is operating associated controls; (b)provide lighting that is adjustable for easy reading throughout the ambientlight range (including desert sunshine), with luminous dials as necessary for reading in the dark; (c) make dials, gages, etc., as uncluttered as possible — if operations require that a gage be read to the nearest milliamp, don't fill it up with gradations to the 10ths or hundredths of a milliamp; and (d) use logical gradations — a dial marked off in fourths can be extremely hard to interpret if readings must be estimated in, *e.g.*, tenths.

2. Locate controls logically. Controls used most often and those used €or the most critical operations should be within easy reach, and should be grouped so that the operator is required to move in only one direction in performing a sequence of operations.

3. Design for the normal range of human faculties.[21] A common error is to design for the average man, whereas good human engineering practice (and common sense) dictates that one should design for all operators whose body measurements fall within the limits acceptable to the Army. Such operations as keeping optical sights or radar antennae aligned on a target, for example, may be performed by tall men, short men, men with long arms, or men with short arms, as well as men falling close to the average. Adjustments should he provided so that the individual can position the equipment for maximum ease and comfort.

5-5. 1.3 Applying Human Engineering Data

Common sense can help the design engineer avoid the more simple and obvious pitfalls. He can go much farther toward providingease of operation by consulting one of several texts prepared by human-engineering specialists for practical use by equipment designers. The Handbook of Human Engineering Data[27] contains introductory explanations of the basic principles, plus a great deal of tabular data on most aspects of human engineering, presented so that the "layman" can interpret and use them. Vision in Military Aviation,[28] in its early chapters, expounds on the principles of vision and instructs design engineers on how to apply the basic visual curves to practical problems, The later chapters contain many examples of visual performance as related to specific equipments and situations; much of this information is useful to the fire control designer as well as the aircraft designer. The Human Engineering Guide[29] is less specific than the other two but is useful for introducingthe design engmeer to the field of human engineering.

In addition, many articles and research reports have been written that apply directly to human factors in military equipment. The fire control equipment designer may be able toobtain data of a very practical nature that apply directly to his problem. In addition to the more general Armed Forces document centers, there is a Department of Defense human-engineering information and analysis service project at Tufts University's Institute of Psychological Research, Medford, Massachusetts, which maintains up-to-date indexes and abstracts, produces annual bibliographies, and has 25,000 documents on file.

When the designer is in doubt--particularly when he is designing radically new equipment that involves vision, motor performance, or a combination of these--he would do well to consult directly with a human-engineering specialist.*

It cannot be emphasized too strongly that equipment, no matter how carefully designed, is no more effective than its operator. Equipment that cannot be operated and maintained easily and effectively is a failure, even though it is electrically, mechanically, and optically perfect. A weapon is only as good as its operators and maintenance men.

5-5. 1.4 Designing Optical Systems for Maximum Visibility

The use of good human- and design-engineering practices for fire control optical equipment is particularly important because it will often be necessary to use the equipment under conditions of very poor illumination.

The light-energy loss during transmission through a single uncoated air-and-glass

* If this type of assistance is desired, the Director of the Human Engineering Laboratories, Aberdeen Proving Ground, Maryland should be contacted.

surface is estimated to range from 4% to 6% of the total amount of light energy passing through. While the loss through one surface may not be significant, the accumulative effect of several such surfaces along the optical path may be very important. Coatings producing low-reflection characteristics should be applied to most optical surfaces to improve light transmission. (See Reference 30 and Reference 21.) Exceptions are:

1. Coatings should not be applied to such optical elements as reticles, field lenses, and collective lenses that lie in or very near the focal plane of the optical assembly.

2. Exterior surfaces should never be coated; the exposed coating will deteriorate or may be damaged during cleaning.

Optical coatings can be applied by evaporated-film techniques or by chemical preparation. Certain evaporated films, such as magnesium fluoride, are insoluble in water but are soluble in some other common chemicals. Chemical coatings — e. g., silica or calcium fluoride — in general manifest a greater resistance to various solvents than evaporated films.

Reflection-reducing coatings must be chemically resistant as well as durable. Rain water, dust, salt water, soluble chemicals, and oil from fingerprints tend to increase optical reflections and deteriorate the coating. High-temperature baking of the coating in a vacuum increases coating permanence. For example, magnesium-fluoride films so treated become hard-baked coatings that are highly durable and possess good film efficiency as well as resistance to humidity and salt spray.

Combined hard-baked magnesium-fluoride and titanium films result in low-reflectance films and partial reflectors that are highly efficient, durable, and substantially nonabsorbing. Tests have indicated that the effect of the temperature of baking on the refractive index of such films is very marked; the higher the temperature, the higher the refractive index and the greater the durability of the film.

Optical design is covered in MIL-HDBK-141.

5-5.2 SAFETY

Fire control equipment presents only the usual kinds of hazards to personnel but these hazards are intensified by the extreme environments in which the equipment will be operated and maintained. The design engineer must apply good safety practices with the utmost rigor, not only to protect personnel but to permit them to carry out their duties rapidly and confidently.

The design engineer should be familiar with such sources as the codes, rules, and regulations of the National Bureau of Standards, the American Standards Association, and the National Electrical Manufacturers' Association (NEMA).

Some of the more pertinent practices in fire control design are described in the paragraphs below.

5-5.2.1 Electrical Safety[21]

1. Provide each piece of electrical or electronic equipment with a main power switch, designed so that it will not arc,

2. Where possible, provide interlock switches to cut off power automatically from the exposed portion of the equipment when an access door or cover is removed. Where conditions permit, bypass switches may be used on equipment that must be serviced with the power on.

3. Since it may be imperative to keep equipment operating in an emergency while it is being serviced, provide a battle-short switch to render the interlocks inoperative. This should be located at the station of the most responsible operator, and designed and labeled so that it will not be operated indiscriminately.

4. Maintain enclosures, exposed parts, and chassis at ground potential; do not (except with very-low-voltage circuits) use them as conductors to complete a circuit. (Single conductor circuits create inter-circuit interference as well as a hazard to personnel and a single insulation break may short the system.) Be particularly careful that the grounding system is reliable under conditions of shock and vibration. While techniques vary with the type of equipment, one method is to ground chassis via welded terminals, grounding lugs, and screws (with nuts and lockwashers); and to ground the enclosure itself with a throughbolt to which the common ground from the chassis is connected. This connection should be clearly labeled

Enclosure Ground and should have an external safety strap. Be sure to provide electronic test equipment (VTVM's, oscilloscopes, etc.) with grounding pigtails, and to connect panel-mounted jacks for test and other supplementary connections to the grounded leg of the circuit. With high potentials (lasers, for example), a large-area contact with ground is required.

5. Providefuses on the load side of all leads to the main power switch and elsewhere as necessary to protect equipment and personnel.

6. Provide high-voltage warning signs at access doors, covers, and wherever else a hazard exists. It is particularly important that they be legible in poor light and will not fade or deteriorate in severe climates.

7. Provide a positive-acting means of discharging high-grade filter capacitors, such as an automatically actuated shorting bar, in medium- and high-voltage power supplies.

8. Remember that even rather low voltages, such as 115-volt service, can be lethal to a wet soldier operating equipment in mud or water.

5-5.2.2 Mechanical Safety[21]

1. Be particularly careful to avoid sharp edges and protruding hardware on which a man could cut himself. Conduit, pipe, and components should not interfere with normal motions during operation and maintenance. Specify smooth surfaces and rounded corners.

2. Locate shields to protect against rotating or oscillating parts. Warning signs and safety switches or interlocks should be placed at the shields if an unusual hazard evists.

3. Provide handles for portable equipment and modules that will be removed and replaced as units.

4. Provide prominent warning lights or other safety devices on cooling and ventilation systems for equipment that is subject to rapid overheating.

5. Provide safety-limit stops for cabinets, etc., that are designed to be pulled part way out for servicing, so that they cannot be pulled out all the way and dropped.

5-5.2.3 Fire Protection

Use noncombustible materials wherever possible, particularly as enclosures for motors, inductors, capacitors, and other possible sources of fire. Be especially careful to avoid specifying materials that produce toxic fumes in tanks, shelters, and other enclosed spaces. With electric and electronic equipment, it is vital to specify a fire extinguisher using CO_2 or other nonconductive fluid.

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